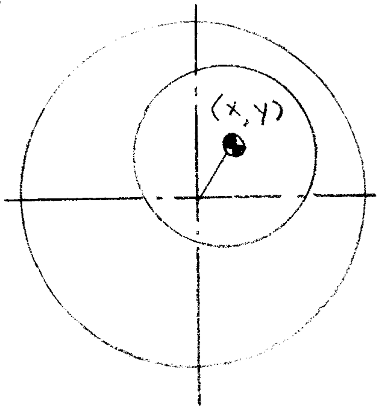
Hydrodynamic Spindle




$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} + \begin{bmatrix} k_1 & k_2 \\ -k_2 & k_1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f_x(t) \\ f_y(t) \end{bmatrix} \quad (1)$$

Define $z(t) \equiv x(t) + j\,y(t)$

$f(t) \equiv f_x(t) + j\,f_y(t)$

(a) $1^{st}$ Row of (1) $+ j \times 2^{nd}$ Row of (2) $\Rightarrow$

$$m\ddot{z} + c\dot{z} + (k_1 - jk_2)z = f(t) \quad (2)$$

Let $f(t) = f_0 e^{j\omega t}$, $z(t) = z_0 e^{j\omega t}$

$$\left[-m\omega^2 + j\omega c + (k_1 - jk_2)\right] z_0 = f_0$$

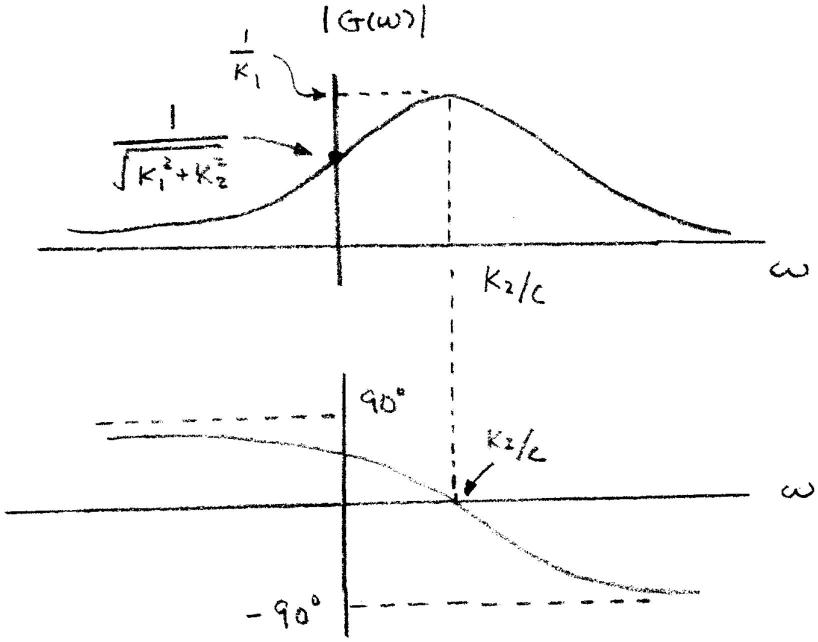
FRF:

$$G(\omega) = \frac{z_0}{f_0} = \frac{1}{(k_1 - m\omega^2) + j(\omega c - k_2)}$$

(b) for $\omega \ll \sqrt{\frac{k_1}{m}}$, $m\omega^2 \ll k_1$, we have

$$G(\omega) \approx \frac{1}{k_1 + j(\omega c - k_2)}, \quad |G| = \frac{1}{\sqrt{k_1^2 + (\omega c - k_2)^2}}$$

$$\measuredangle G(\omega) = -\tan^{-1}\frac{\omega c - k_2}{k_1}$$




Hence

$|G(\omega)| \neq |G(-\omega)|$ } because EOM (2) does not

$\measuredangle G(\omega) \neq -G(-\omega)$ } have real coefficients.

- Resonance occurs at $\omega = \frac{K_2}{C}$
- If $k_1 \to 0$, resonance amplitude goes to $\infty$
- At Resonance, $\measuredangle G = 0$, which can be used to identify resonance