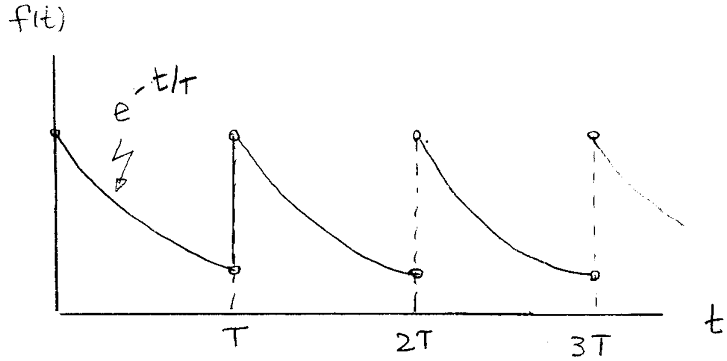


(a) $f(t) = e^{-t/T}, \quad 0 < t < T, \quad f(t+T) = f(t)$

<u>Complex Fourier Series</u>

$$f(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} ; \qquad \omega_0 \equiv \frac{2\pi}{T}$$

With

$$C_n = \frac{1}{T}\int_0^T f(t)\, e^{-jn\omega_0 t}\, dt$$

When $n=0$,

$$C_0 = \frac{1}{T}\int_0^T e^{-t/T} dt = -e^{-t/T}\Big|_0^T = 1 - \frac{1}{e} \approx 0.632$$

When $n \neq 0$,

$$C_n = \frac{1}{T}\int_0^T e^{-t/T} e^{-jn\omega_0 t} dt$$

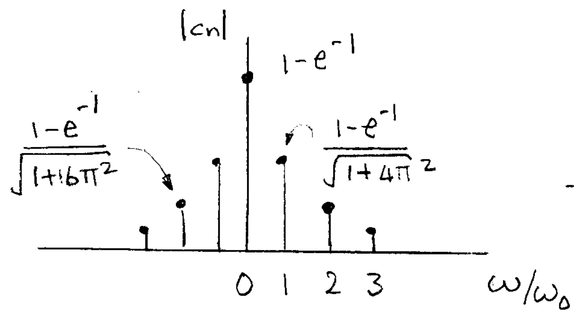
$$= \frac{1}{T}\int_0^T e^{-(\frac{1}{T}+jn\omega_0)t} dt = \frac{-1}{T(\frac{1}{T}+jn\omega_0)} e^{-(\frac{1}{T}+jn\omega_0)t}\Big|_0^T$$

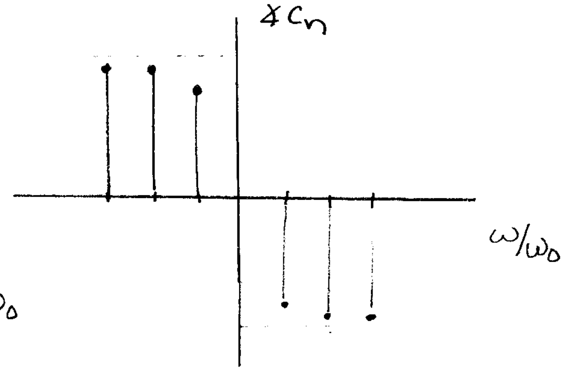
$$= \frac{1}{1+2\pi nj}\left[1 - e^{-(\frac{1}{T}+jn\omega_0)T}\right]$$

$$= \frac{1}{1+2\pi nj}\left[1 - e^{-(1+j2\pi n)}\right] ; \quad e^{-j2\pi n} = 1$$

$$= \frac{1-e^{-1}}{1+2\pi nj}$$

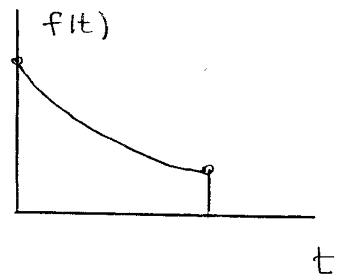
(b) $|c_n| = \dfrac{1-e^{-1}}{\sqrt{1+4\pi^2 n^2}}$, $\measuredangle c_n = -\tan^{-1}(2\pi n)$





(c) If $f(t)$ is padded with zero outside $0<t<T$,

$$f(t) = \begin{cases} e^{-t/T}, & 0<t<T \\ 0 & \text{otherwise} \end{cases}$$



Fourier transform

$$\bar{F}(\omega) = \int_0^{\infty} f(t)e^{-j\omega t}\,dt$$

$$= \int_0^{T} e^{-t/T}\cdot e^{-j\omega t}\,dt = \int_0^{T} e^{-(\frac{1}{T}+j\omega)t}\,dt$$

$$= \frac{-1}{\frac{1}{T}+j\omega}\, e^{-(\frac{1}{T}+j\omega)t}\Big|_0^T$$

$$= \frac{T}{1+j\omega T}\left[1-e^{(1+j\omega T)}\right]$$