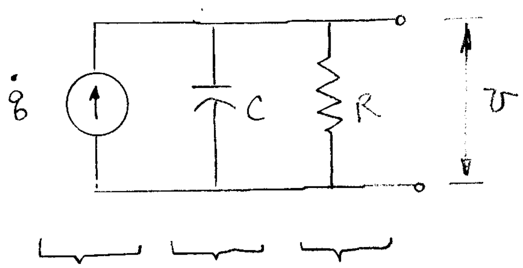
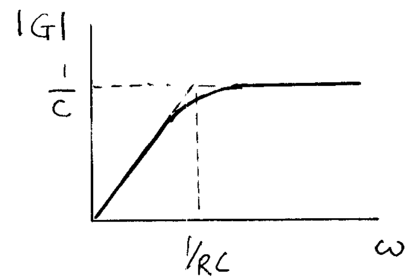


mechanical deformation | Piezo | Charge amplifier

$$C\frac{dv}{dt} + \frac{v}{R} = \dot{q}(t)$$

$$q(t) = Q_0 e^{j\omega t}, \quad v(t) = V_0 e^{j\omega t}$$

$$G(\omega) \equiv \frac{V_0}{Q_0} = \frac{jR\omega}{1 + j\omega RC}$$



(a) Input: $q(t) = \cos t + 2\sin 3t$

$\underline{G(\omega)}$: $C = 1\,F$, $R = 0.5\,\Omega$

$$\omega_c = \frac{1}{RC} = \frac{1}{(1)(0.5)} = 2 \text{ rad/s}$$

$$G(\omega) = \frac{j(0.5\omega)}{1 + 0.5\omega j} = \frac{\omega j}{2 + \omega j}$$

For $q_1(t) \equiv \cos t$, $\omega \equiv 1$,

$$G(\omega) = \frac{j}{2+j} = \frac{1}{\sqrt{5}} \measuredangle\left(\frac{\pi}{2} - \tan^{-1}\frac{1}{2}\right) = 0.447 \measuredangle 63.4^\circ$$

Response

$$v_1(t) = 0.447 \cos(t + 63.4^\circ)$$

For $q_2(t) \equiv 2\sin 3t$, $\omega \equiv 3$

$$G(\omega) = \frac{3j}{2+3j} = \frac{3}{\sqrt{13}} \measuredangle\left(\frac{\pi}{2} - \tan^{-1}\frac{3}{2}\right) = 0.832 \measuredangle 33.7^\circ$$

Response

$$v_2(t) = 1.664 \sin(3t + 33.7^\circ)$$

$$\Rightarrow v(t) = v_1(t) + v_2(t) = 0.447\cos(t + 63.4^\circ) + 1.664\sin(3t + 33.7^\circ)$$

(b) <u>Spectrum of $v(t)$</u>

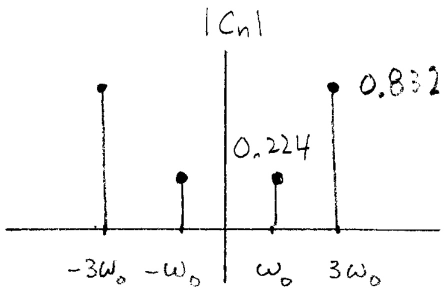
$$v(t) = (0.447)\left[\frac{e^{j(t+63.4^\circ)} + e^{-j(t+63.4^\circ)}}{2}\right]$$

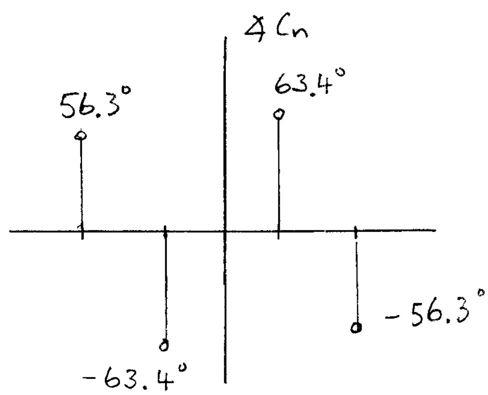
$$+ \frac{1}{2j}(1.664)\left[e^{j(3t+33.7^\circ)} - e^{-j(3t+33.7^\circ)}\right]$$

$$= \Bigg[\underbrace{\frac{0.447}{2}e^{j(63.4^\circ)}}_{C_1} e^{jt} + \underbrace{\frac{1.664}{2j}e^{j(33.7^\circ)}}_{C_3} e^{j3t}\Bigg] + c.c.$$

$C_1 = 0.1 + 0.2j$, $|C_1| = 0.2235$, $\measuredangle C_1 = 1.107$ (OR 63.4°)

$C_3 = 0.462 - 0.692j$, $|C_3| = 0.832$, $\measuredangle C_3 = -0.983$ (OR -56.3°)





(c) Corner frequency $= \frac{1}{RC} \equiv \omega_c$; $\omega_0 \equiv 1$

To have non-distorted signal, $\omega_0 > \omega_c$

Hence $\frac{1}{RC} < 1 \Rightarrow \frac{1}{R \cdot 1} < 1 \Rightarrow R > 1$ Ohm.