ME 374, System Dynamics Analysis and Design
Homework 1

Distributed: 3/31/2008, Due: 4/11/2008

(There are 4 problems in this set.)

1. Consider the following three matrices

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -6 & -7 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix} \tag{1}$$

   For each of these three matrices,

   (a) determine their eigenvalues and eigenvectors,

   (b) verify your eigenvalues and eigenvectors through Matlab command [V, D] = eig ($\mathbf{A}$), and

   (c) form the modal matrix and diagonalize $\mathbf{A}$ via the similarity transformation. If you cannot diagonalize $\mathbf{A}$, please transform $\mathbf{A}$ to a Jordan form.

2. Consider an ellipse governed by

$$F \equiv 34x^2 - 24xy + 41y^2 = c \tag{2}$$

   where c is a constant and $F \equiv 34x^2 - 24xy + 41y^2$ is called a *quadratic form*. Please use the following steps to determine the major and minor axes of the ellipse.

   (a) First, we need to realize that the quadratic form F can be written as

$$F = \begin{pmatrix} x \\ y \end{pmatrix}^T \mathbf{A} \begin{pmatrix} x \\ y \end{pmatrix} \tag{3}$$

   where $\mathbf{A}$ is a symmetric square matrix given by

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{4}$$

   Since $\mathbf{A}$ is symmetric, $a_{12} = a_{21}$. Compare (2) and (3) to identify matrix $\mathbf{A}$.

   (b) Calculate the eigenvalues (λ_1, λ_2) and eigenvectors ($\mathbf{u}_1$, $\mathbf{u}_2$). Normalize the eigenvectors so that their magnitude is unity. Form the modal matrix $\mathbf{M} \equiv [\mathbf{u}_1, \mathbf{u}_2]$.

   (c) Consider a coordinate transformation defined as

$$\begin{pmatrix} x \\ y \end{pmatrix} \equiv \mathbf{M} \begin{pmatrix} x' \\ y' \end{pmatrix} \tag{5}$$

   Note that the coordinate transformation in (5) is a rotation of the coordinates from xy to $x'y'$. Draw the $x'y'$ coordinate systems on the xy plane. What is the angle between the two coordinate systems?

(d) Find $\mathbf{M}^{-1}$ and show that

$$\mathbf{M}^{-1} = \mathbf{M}^T \tag{6}$$

(e) Substitute (5) into (2) and (3) and make use of (6) to obtain

$$F = \lambda_1 x'^2 + \lambda_2 y'^2 = c \tag{7}$$

Find the magnitude of the major and minor axes of the ellipse. What are the directions of the major and minor axes?

3. Consider the following matrix

$$\mathbf{A} = \begin{bmatrix} 3 & 6 & 6 \\ 1 & 4 & 3 \\ -1 & -3 & -2 \end{bmatrix} \tag{8}$$

(a) Find the eigenvalues and eigenvectors of **A**. Note that the eigenvalues are repeated and you should obtain two eigenvectors corresponding to the repeated eigenvalues.

(b) Form the modal matrix to diagonalize **A**.

(c) Student X thinks that it is impossible to diagonalize **A**. Instead, one can only reduce **A** to a Jordan form. The reason is that **A** is not symmetric and has repeated eigenvalues. It is well known that a non-symmetric matrix with repeated eigenvalues will lead to a Jordan form. Do you agree with Student X? Why?

4. Consider the following matrix with

$$\mathbf{A} = \begin{bmatrix} 1 & k \\ -k & 2 \end{bmatrix} \tag{9}$$

where k is a positive real number.

(a) Determine the value of k so that the eigenvalues of **A** are repeated.

(b) Determine the corresponding eigenvectors.

(c) Form the modal matrix to diagonalize **A**. If you cannot diagonalize **A**, reduce it to a Jordan form.