ME 374, System Dynamics Analysis and Design
Homework 3
Distributed: 4/14/2008 Due: 4/28/2008
(There are 5 problems in this set.)

1. This is an exam problem from the Spring Quarter of 2006. Consider an electrical high-pass filter whose dynamics is governed by

$$RC\dot{x} + x = RC\dot{u}(t) \tag{1}$$

where $u(t)$ is the input voltage and $x(t)$ is the output voltage. In addition, R and C are the resistance and capacitance of the filter, respectively. Answer the following questions.

   (a) Find the transfer function $H(s)$ from $u(t)$ to $x(t)$.

   (b) Consider the case when $R = C = 1$. If $u(t) = \sin t$, determine the magnitude and phase of the transfer function $H(s)$.

   (c) Consider the case when $R = C = 1$. If $u(t) = \sin t$, determine the magnitude and phase of the output $x(t)$.

   (d) Let the excitation be $u(t) = \cos \omega t$, where ω is the excitation frequency. In this case, the corner frequency of the high-pass filters is defined as the frequency ω so that the phase of $H(s)$ is $\pi/4$. Determine the corner frequency of the high-pass filter governed by (1).

2. Viscoelastic materials are often used in damping applications to absorb vibration. To characterize viscoelastic materials, it is common to use transfer functions of the materials with the strain being the input and stress as output. Figure 1 shows a model, called Burgers model, to simulate a viscoelastic material. Basically, the viscoelastic material is modeled as a combination of two springs and two dashpots. The stain of the spring-dashpot combination is ϵ and the stress of the material is σ.

   (a) Derive that the differential equation governing σ and ϵ is

$$\frac{k_1}{c_2}\sigma + \left(1 + \frac{k_1}{k_2} + \frac{c_1}{c_2}\right)\dot{\sigma} + \frac{c_1}{k_2}\ddot{\sigma} = k_1\dot{\epsilon} + c_1\ddot{\epsilon} \tag{2}$$

   where c_1, c_2, k_1, and k_2 are constants from Burgers model.

   (b) Derive the transfer function of the material using $\epsilon(t)$ as the input and $\sigma(t)$ as output.

   (c) Consider a viscoelastic material with $c_1 = c_2 = 1$ and $k_1 = k_2 = 10$. Now the material is tested using a tensile machine. If a constant strain $\bar{\epsilon}$ is applied to the material, what is the steady-state stress σ of the material? In other words, what is the forced response of the material after the transient response has died out?

   (d) For the same viscoelastic material, what would be the steady-state stress σ, if the input strain is $\epsilon(t) = \cos 10t$?

3. Figure 2 shows the poles and zeros of an unknown transfer function $H(s)$, where the crosses are poles and the circles are zeros. Answer the following questions.

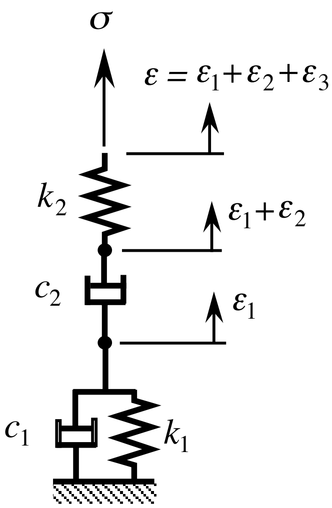

Figure 1: Bergers model of a viscoelastic material

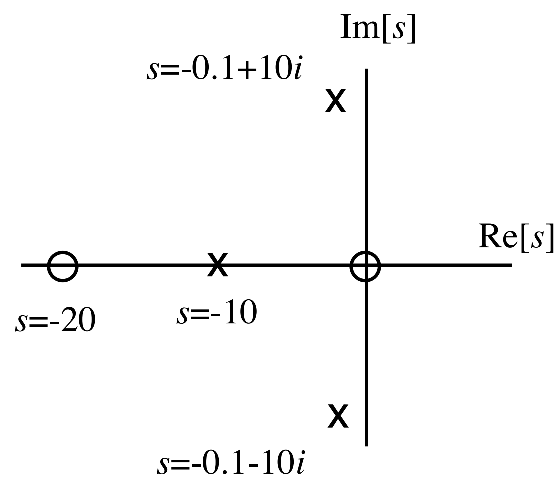

Figure 2: Pole-zero plot of an unknown transfer function

(a) Reconstruct the transfer function.

(b) What is the order of the ordinary differential equation governing the system response? What are the characteristic roots of the ordinary differential equations?

(c) If the input is $\cos\omega t$, where ω is a parameter that can be varied arbitrarily. What frequency ω will cause the system to have minimal response and why? What frequency ω will cause the system to have maximal response and why?

4. Consider the system shown in Fig. 3, where the cylinder of radius r and mass m is pulled through a massless spring with spring constant k and a massless dashpot with damping coefficient c. Assume that the cylinder rotates freely about its axis and that the input displacement $u(t)$ is known. The equation of motion governing x is

$$\frac{3}{2}m\ddot{x} + c\dot{x} + kx = c\dot{u} + ku \tag{3}$$

The input is $u(t)$ and the output is $x(t)$.

(a) Determine the transfer function $H(s)$ relating $u(t)$ to $x(t)$.

(b) Assume zero initial conditions. Determine the output $x(t)$ through the transfer function, when input $u(t) = e^{-2t}$.

(c) What is the magnitude and the phase of $H(s)$ when input $u(t) = e^{-2t}$?

(d) Determine the poles and zeros of the system. Discuss the cases when $c^2 > 6mk$ and $c^2 < 6mk$.

(e) Plot the poles and zeros on the complex s plane for the two cases.

5. This is an exam problem of the Winter Quarter of 2000. The vibration of a dental drill is governed by the following differential equation

$$\ddot{x} + 0.1\dot{x} + (10^6 + k)x = \dot{u}(t) \tag{4}$$

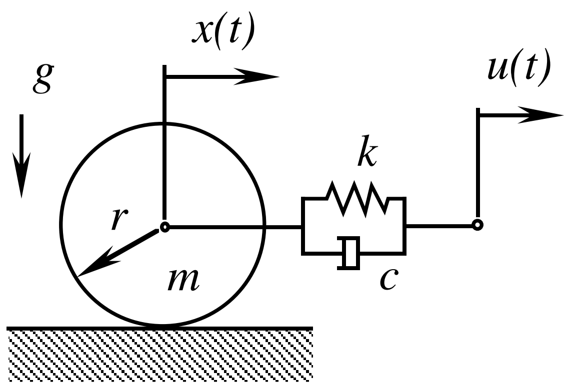

Figure 3: Rolling cylinder excited by $u(t)$

where $u(t)$ is the input force drilling on a tooth and $x(t)$ is the output vibration of the drill. In addition, k is a constant stiffness parameter, which depends on the bearings used in the dental drill.

(a) Determine the transfer function $H(s)$.

(b) Assume that $k = 2 \times 10^6$. If $u(t) = \cos 1000t$, what is the magnitude and phase of $H(s)$?

(c) A dentist's nightmare is to have the drill shaking violently while drilling a tooth. Assume that the dentist drill is operating at 1000 Hz (or 6280 rad/s). What stiffness parameter k do you want to avoid? Otherwise, you will never sell the dental drill.

(d) Due to design constraints (e.g., costs, dimensions, vendors, and fatigue life), you choose bearings with $k = 2 \times 10^6$. Originally, the dental drill is designed to operate at 350 Hz (or 2200 rad/s), but you want to reduce the vibration further. Give two alternatives to make your design better. Explain why.