ME 374, System Dynamics Analysis and Design
Homework 5

Distributed: 4/28/2008, Due: 5/9/2008
(There are 6 problems in this set.)

1. This is an exam problem of the Spring Quarter of 2006. Figure 1 shows a linear graph of a motor driving a heavy rotor. The electric circuit of the motor consists of a voltage source $V_s(t)$ and a resistor with resistance R. The rotor has mass moment of inertia J. The motor is modeled as an ideal transformer with $T = k_a i$ and $V = k_a \Omega$, where i and V are the current and voltage of the motor and T and Ω are the torque and angular velocity of the rotor. Answer the following questions.

   (a) Determine the driving point impedance $Z(s)$.

   (b) In an impedance test, the voltage is varied sinusoidally, i.e., $V_s(t) = v_0 \cos \omega t$, to measure impedance $Z(j\omega)$ along the pure imaginary axis. Roughly sketch the magnitude of $Z(j\omega)$ with respect to frequency ω.

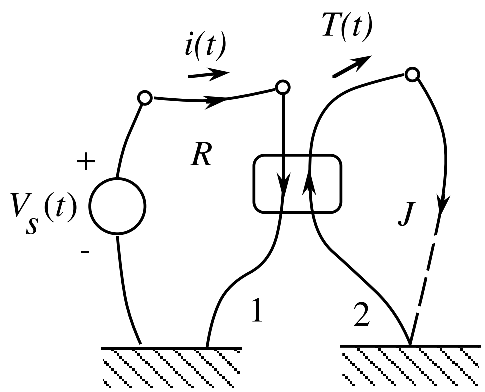

Figure 1: Linear graph of a motor driving a heavy rotor

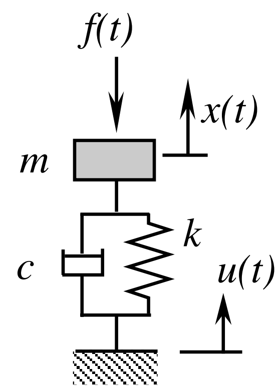

Figure 2: A simple model for isolation tables

2. Engineer X starts a new business building and selling vibration isolation tables. In principle, isolation tables can be modeled as a spring-mass-damper system as shown in Fig. 2, where m, c, and k are the mass, damping coefficient, and stiffness, respectively. Also, $u(t)$ is the motion of the floor, $x(t)$ is the displacement of the isolation table, and $f(t)$ is an external force acting on the table. For the isolation table to function well, we would like to minimize $x(t)$ as much as possible. Answer the following questions.

   (a) When $f(t)$ is not present, the equation of motion of the isolation table is

$$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + kx(t) = c\frac{du(t)}{dt} + ku(t) \tag{1}$$

   Derive the transfer function $H(s)$ from $u(t)$ to $x(t)$.

(b) Engineer X wants to do some experiments to know the transfer function $H(s)$ better. To do so, Engineer X needs to use a large shaker to generate $u(t)$. This is very expensive for a start-up company. Therefore, engineer X measures the driving point impedance $Z(s)$ with an input force $f(t)$ using a hammer, which does not cost a lot of money. Derives the driving point impedance when $u(t) = 0$. What useful information do you get from the impedance $Z(s)$? What advantage does Engineer X get by finding $Z(s)$ instead of $H(s)$?

3. This is an exam problem of the Winter Quarter of 2000. Consider the vehicle suspension model in Fig. 3 with velocity input $V_s(t)$.

   (a) Determine the driving point impedance $Z(s)$ of the system.

   (b) Let f_B be the damping force in the damper. Determine the transfer function $H(s)$ from the input velocity V_s to f_B. How can you use the impedance $Z(s)$ to determine the poles of $H(s)$?

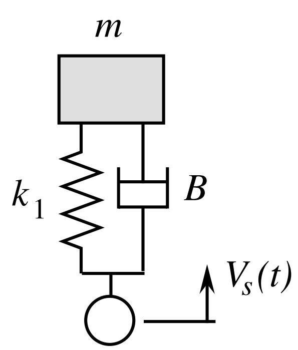

Figure 3: A spring-mass-damper model for a vehicle wheel

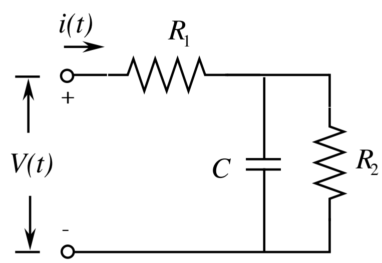

Figure 4: A simple RC-circuit

4. This is an old exam problem that I gave in Spring Quarter of 1998. Consider the circuit in Fig. 4. Answer the following questions.

   (a) Determine the driving point impedance $Z(s)$ of the circuit.

   (b) Let v_c be the voltage across the capacitor C. Derive the transfer function $H(s)$ from V to v_c in terms of the impedance $Z(s)$ and R_1. (Hint: Use Kirchhoff's voltage law or loop equation to relate V, v_c, i and R_1.)

5. This is an old exam problem that I gave in Spring Quarter 1999. Consider the simple spring mass system as shown in Fig. 5. The mass is subjected to fluid with damping coefficient B. Answer the following questions.

   (a) Determine the input impedance of the system.

   (b) Determine the transfer function from F to the displacement x_m of the mass. What is the relationship between the transfer function and the impedance?

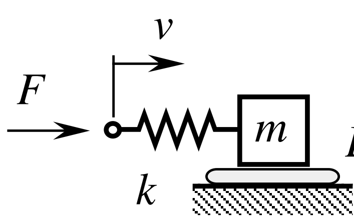

Figure 5: A simple spring-mass-damper model

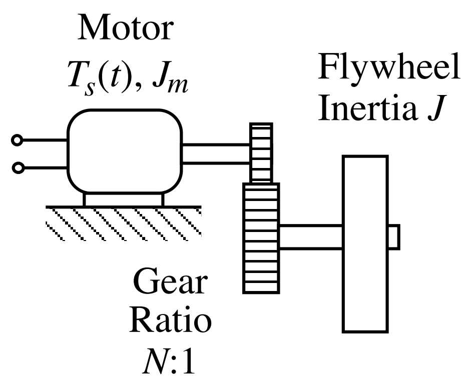

Figure 6: A motor drive system

6. Consider the rotational system shown in Fig. 6 in which an electric motor is used to control a large inertia through a gear train that provides a speed reduction of $N:1$. When the motor is driven by a current source, its output torque is proportional to the current and may be considered as an input to the mechanical subsystem. Answer the following questions.

   (a) Form a linear graph model for the system, considering the motor as a torque source T_s, the motor inertia J_m, the gear train as an ideal transformer with a ratio of $N:1$, and the flywheel inertia J. All damping in the system may be neglected.

   (b) Determine the effective impedance viewed by the motor; that is, reflect the flywheel inertia through the gear train and combine it with that of the motor to form an equivalent impedance driven by the motor.

   (c) As seen by the motor, how do the flywheel inertia and the motor inertia compare? How does the gear train modify the flywheel inertia viewed by the motor?