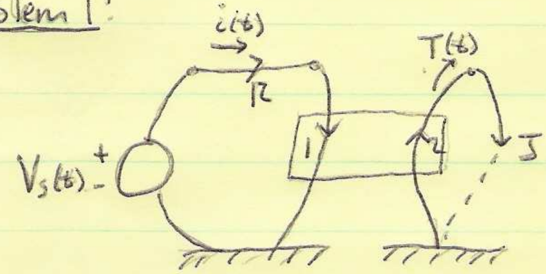
ME 374 Sp. 2008 HW #5 by Jason Frye

Problem 1:

a)

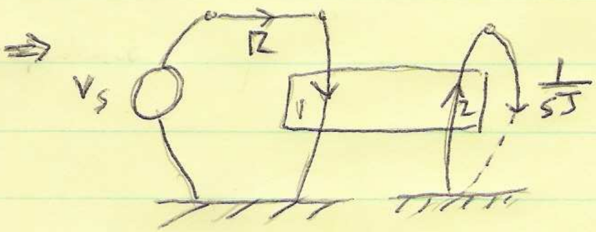


$V_1 = K_a \Omega_2$, $T_2 = K_a \cdot i_1$

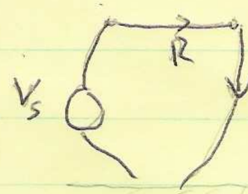
$$\Rightarrow \begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} K_a & 0 \\ 0 & \frac{1}{K_a} \end{bmatrix} \begin{bmatrix} \Omega_2 \\ T_2 \end{bmatrix}$$

$\Rightarrow$ Transformer: $TF = K_a$

$\Rightarrow$

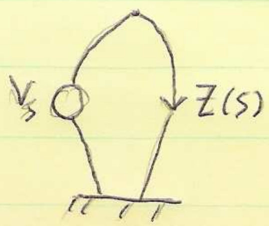


$\Rightarrow$



$Z_1(s) = (TF)^2 \cdot \frac{1}{sJ} = \frac{K_a^2}{sJ}$

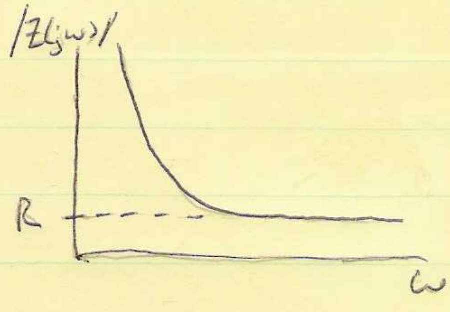
$\Rightarrow$



$$Z(s) = R + Z_1(s)$$
$$= R + \frac{K_a^2}{sJ}$$
$$\Rightarrow \boxed{Z(s) = \frac{K_a^2 + RJs}{Js}}$$

b) $Z(j\omega) = R + \frac{K_a^2}{jJ\omega}$

$= R - j\frac{K_a^2}{J\omega}$ $\Rightarrow |Z(j\omega)| = \sqrt{(R)^2 + \left(\frac{K_a^2}{J\omega}\right)^2}$

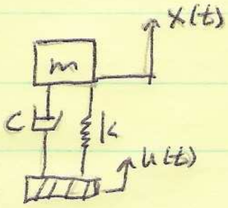


as $\omega \to 0$, $|Z(j\omega)| \approx \frac{K_a^2}{J\omega}$

as $\omega \to \infty$, $|Z(j\omega)| \approx R$

Problem 2:

a)



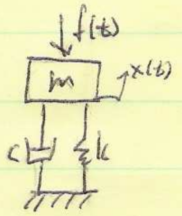
$$m\ddot{x} + c\dot{x} + kx = c\dot{u} + ku$$

$$u(t) = U(s)e^{st}, \quad x(t) = X(s)e^{st}$$

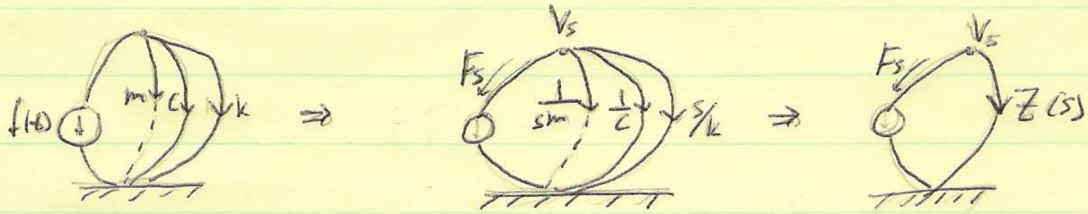
$$\Rightarrow (ms^2 + cs + k)X(s) = (cs + k)U(s)$$

$$\boxed{H(s) \equiv \frac{X(s)}{U(s)} = \frac{cs+k}{ms^2+cs+k}}$$

b)



Linear Graph Approach:



$$\frac{1}{Z(s)} = \frac{1}{1/sm} + \frac{1}{1/c} + \frac{1}{s/k}$$

$$= ms + c + k/s$$

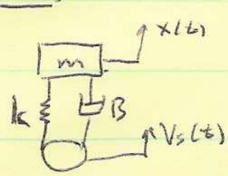
$$\frac{1}{Z(s)} = \frac{ms^2+cs+k}{s} \Rightarrow \boxed{Z(s) = \frac{s}{ms^2+cs+k}}$$

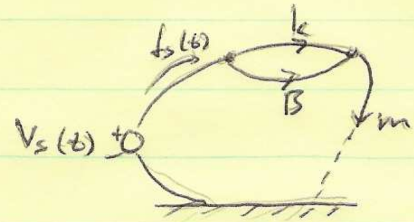
- The poles of $Z(s)$ are the same as the poles of $H(s)$. If Engineer X is interested in measuring the poles of $H(s)$, it is much cheaper to simply measure the poles of $Z(s)$ (which are the same as the poles of $H(s)$).

ME 374 Sp. 2008 HW #5 by Jason Frye

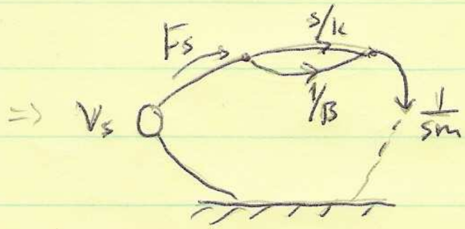
Problem 3:

a)

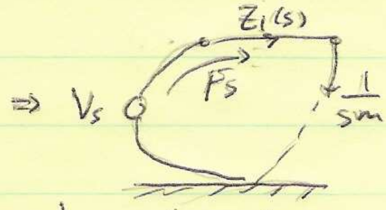




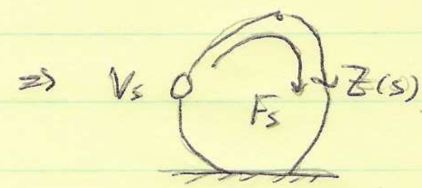
$\Rightarrow$



$\Rightarrow$



$\Rightarrow$



$$\frac{1}{Z_1(s)} = \frac{1}{s/k} + \frac{1}{1/B} = \frac{k}{s} + B = \frac{k+Bs}{s}$$

$$\Rightarrow Z_1(s) = \frac{s}{Bs+k}$$

$$Z(s) = Z_1(s) + \frac{1}{sm} = \frac{s}{Bs+k} + \frac{1}{sm}$$

$$\Rightarrow \boxed{Z(s) = \frac{ms^2+Bs+k}{ms(Bs+k)}}$$

b) Transfer function from $v_s(t)$ to f_B : $H(s) = \frac{F_B(s)}{V_s(s)}$

· Damper : $f_B = B \cdot v_B \Rightarrow H(s) = \frac{B \cdot V_B(s)}{V_s(s)}$

Recall : $Z_1(s) = \frac{V_1(s)}{F_1(s)}$; $V_1 = V_B$, $F_1 = F_s$

$$\Rightarrow Z_1(s) = \frac{V_B(s)}{F_s(s)} \Rightarrow V_B(s) = Z_1(s)\,F_s(s)$$

, Also: $Z(s) = \frac{V_s(s)}{F_s(s)}$

$$\Rightarrow V_s(s) = Z(s)\,F_s(s)$$

$$\Rightarrow H(s) = \frac{B \cdot V_B(s)}{V_s(s)} = \frac{B \cdot Z_1(s) \cdot \cancel{F_s(s)}}{Z(s) \cdot \cancel{F_s(s)}} = \frac{B \cdot Z_1(s)}{Z(s)}$$

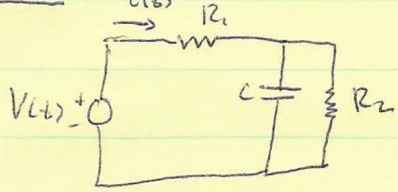
$$\Rightarrow H(s) = \frac{B\left(\frac{s}{Bs+k}\right)}{\left(\frac{ms^2+Bs+k}{ms(Bs+k)}\right)} = \boxed{\frac{Bms^2}{ms^2+Bs+k} = H(s)}$$

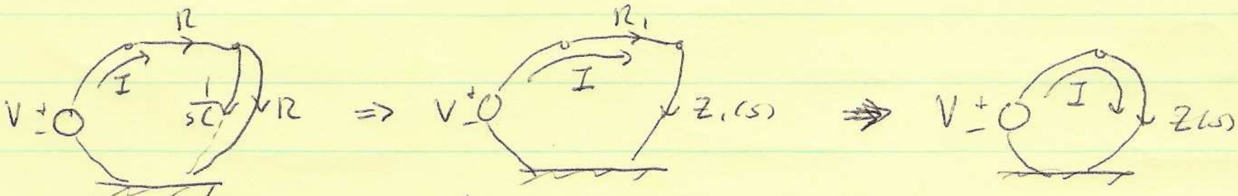
· The zeros of $Z(s)$ are the same as the poles of $H(s)$

ME 374 Sp. 2008 HW #5 by Jason Frye

Problem 4:

a)





$$\frac{1}{Z_1(s)} = \frac{1}{1/sC} + \frac{1}{R_2}$$

$$= sC + \frac{1}{R_2}$$

$$\frac{1}{Z_1(s)} = \frac{R_2Cs+1}{R_2}$$

$$\Rightarrow Z_1(s) = \frac{R_2}{R_2Cs+1}$$

$$Z(s) = R_1 + Z_1(s)$$

$$= R_1 + \frac{R_2}{R_2Cs+1}$$

$$\Rightarrow \boxed{Z(s) = \frac{R_1R_2Cs + R_1 + R_2}{R_2Cs+1}}$$

b) Transfer function from $V(t)$ to $v_c(t)$: $H(s) = \frac{V_c(s)}{V(s)}$

· ~~Kirchhatts~~ Kirchhoff's voltage law: $V - V_{R_1} - V_c = 0$

$$\Rightarrow V_c = V - V_{R_1}$$

$$\Rightarrow H(s) = \frac{V(s) - V_{R_1}(s)}{V(s)}$$

$$= 1 - \frac{V_{R_1}(s)}{V(s)}$$

$V_{R_1} = R_1 I$ $\Rightarrow \boxed{H(s) = 1 - \frac{R_1 I(s)}{V(s)} = 1 - \frac{R_1}{Z(s)}}$ $\left(Z(s) = \frac{V(s)}{I(s)} \Rightarrow \frac{1}{Z(s)} = \frac{I(s)}{V(s)} \right)$

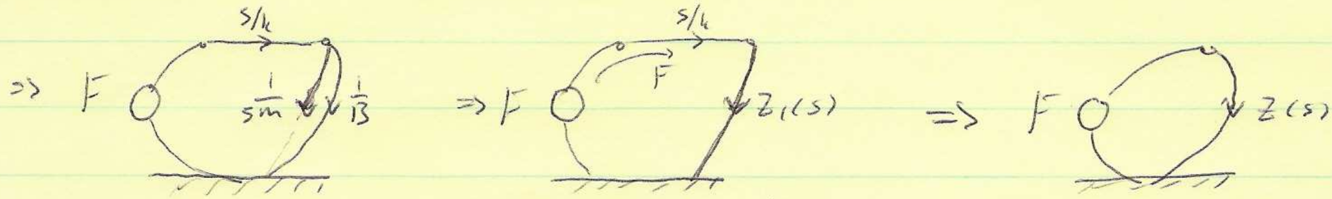
$\cancel{H(s) = 1 - R_1 \frac{1}{Z(s)}}$

$=$

ME 374 Sp. 2008 HW #5 by Jason Frye

Problem 5:

a) 



$$\frac{1}{Z_1(s)} = \frac{1}{1/sm} + \frac{1}{1/B} = ms + B \Rightarrow Z_1(s) = \frac{1}{ms+B}$$

$$Z(s) = \frac{s}{k} + Z_1(s) = \frac{s}{k} + \frac{1}{ms+B}$$

$$\Rightarrow \boxed{Z(s) = \frac{ms^2 + Bs + k}{k(ms+B)}}$$

b) Transfer function from $F(t)$ to $x_m(t)$: $H(s) = \frac{X_m(s)}{F(s)}$

- $x_m(t) = X_m(s)e^{st}$, $v_m = V_m(s)e^{st}$
- $V_m(t) = \frac{d}{dt}(x_m(t))$

$\Rightarrow V_m(s)e^{st} = \frac{d}{dt}\left(X_m(s)e^{st}\right) = sX_m(s)e^{st} \Rightarrow X_m(s) = \frac{1}{s}V_m(s)$

$\Rightarrow H(s) = \frac{1}{s} \cdot \frac{V_m(s)}{F(s)}$

$Z_1(s) = \frac{V_1(s)}{F_1(s)}$, $V_1(s) = V_m(s)$, $F_1(s) = F(s)$

$\Rightarrow Z_1(s) = \frac{V_m(s)}{F(s)}$

$$\Rightarrow H(s) = \frac{1}{s} \cdot \frac{V_m(s)}{F(s)} = \frac{1}{s} Z_1(s)$$

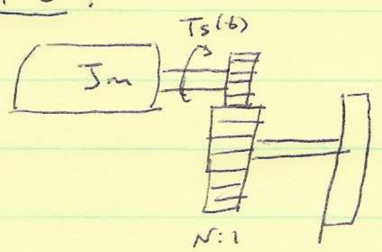
$$\Rightarrow \boxed{H(s) = \frac{1}{s(ms+B)}}$$

- $Z(s)$ and $H(s)$ share a pole at $s = \frac{-B}{m}$

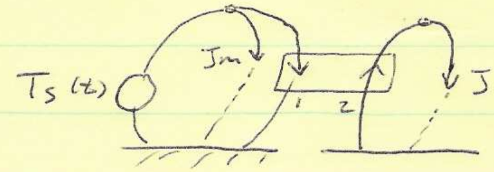
ME 374 Sp 2008 HW #5 by Jason Frye

Problem 6:

a)



$\Rightarrow$

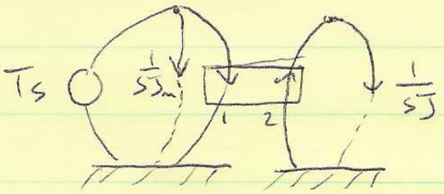


Gear Train: $\Omega_1 = N\Omega_2 \quad (\Omega_1 > \Omega_2)$

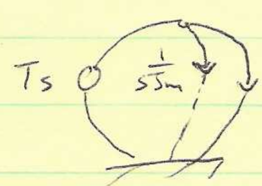
$T_1 = \frac{1}{N} T_2 \quad (T_1 < T_2)$

$\Rightarrow \begin{bmatrix} \Omega_1 \\ T_1 \end{bmatrix} = \begin{bmatrix} N & 0 \\ 0 & \frac{1}{N} \end{bmatrix} \begin{bmatrix} \Omega_2 \\ T_2 \end{bmatrix} \Rightarrow \underline{TF = N}.$

b)



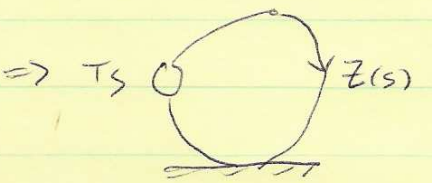
$\Rightarrow$



$Z_1(s) = (TF)^2 Z_2(s)$

$= N^2 \cdot \frac{1}{sJ}$

$\Rightarrow$



$$\frac{1}{Z(s)} = \frac{1}{1/sJ_m} + \frac{1}{N^2/sJ}$$

$$= sJ_m + \frac{1}{N^2} sJ$$

$$\frac{1}{Z(s)} = s\left(J_m + \frac{1}{N^2} J\right)$$

$$\Rightarrow \boxed{Z(s) = \frac{1}{s\left(J_m + \frac{1}{N^2} J\right)}}$$

c) The impedence for a rotational inertia element is $Z_J(s) = \frac{1}{sJ}$.

The effective impedence $Z(s)$ viewed by the motor looks like

$Z(s) = \frac{1}{sJ_{eff}}$, where $J_{eff} = J_m + \frac{1}{N^2} J$.

We see that, in J_{eff}, the flywheel inertia J is scaled by $\frac{1}{N^2}$.

Therefore, the gear train scales the flywheel inertia (by $\frac{1}{N^2}$), as viewed by the motor.