ME 374, System Dynamics Analysis and Design
Homework 6
Distributed: 5/5/2008, Due: 5/16/2008
(There are 4 problems in this set.)

1. This is an old exam that I gave in the Winter Quarter of 1998. Piezoelectric materials are often used as sensor materials because they transform mechanical deformation to electrical charge. Figure 1 shows an electric circuit of a piezoelectric sensor. The piezoelectric material is modeled as a current source in parallel with a capacitor. The resistance models the impedance of an amplifier. The goal is to make the output voltage $v(t)$ proportional to the input charge $q(t)$, so that we can use the voltage $v(t)$ to measure the charge $q(t)$. The ODE governing the input and output is
$$RC\frac{dv}{dt} + v = R\frac{dq}{dt} \tag{1}$$
where $q(t)$ is the input and $v(t)$ is the output.
   (a) Determine the frequency response function $G(\omega)$. You can leave it as a fraction of two complex numbers.
   (b) Find the magnitude of the frequency response function $G(\omega)$.
   (c) Show the asymptotic behavior of $G(\omega)$ when $\omega \ll 1/RC$ and when $\omega \gg 1/RC$.
   (d) Plot the magnitude of $G(\omega)$. (Not necessarily the Bode plot.)
   (e) For what frequency range can you use this sensor?

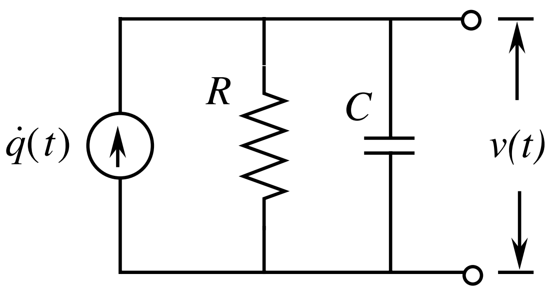

Figure 1: A model of piezosensors

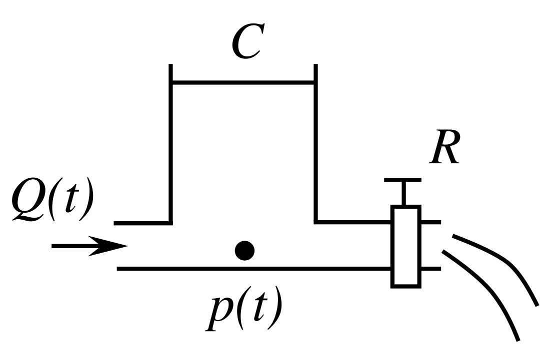

Figure 2: a hydraulic system

2. This is an old exam problem that I gave in Winter Quarter of 2001. Consider the hydraulic system shown in Fig. 2. The tank has a fluid capacitance C and the drain has a linear resistance R. In addition, the system is driven by a through-variable source with prescribed input flow rate $Q(t)$. When the input flow rate $Q(t)$ is a constant Q_0, the system reaches a steady-state. In this state, the flow rate $q_C^{(0)}$ into the tank is zero, the flow rate $q_R^{(0)}$ through the drain is Q_0, and the pressure p_0 at the bottom of the tank is RQ_0. Now consider the case when the input flow rate fluctuates with $Q(t) = Q_0 + u(t)$, where
$$u(t) = A\sin\omega t \tag{2}$$

In (2), A and ω are the amplitude and frequency of the fluctuation, respectively. As a result of $u(t)$, the pressure at the bottom of the tank also fluctuates with $RQ_0 + p(t)$, the flow rate into the tank fluctuates with $q_C(t)$, and the flow rate through the drain fluctuates with $Q_0 + q_R(t)$. Moreover, the fluctuation components are related through

$$q_C(t) = C\frac{dp(t)}{dt} \tag{3}$$

and

$$q_R(t) = \frac{1}{R}p(t) \tag{4}$$

Furthermore, the equation governing the pressure fluctuation is

$$RC\frac{dp(t)}{dt} + p(t) = Ru(t) \tag{5}$$

where $u(t)$ is the fluctuation of the input flow rate defined in (2).

(a) Derive the frequency response function $G_p(\omega)$ from $u(t)$ to $p(t)$ using (5). You can leave the frequency response function $G_p(\omega)$ in the complex form.

(b) Plot the magnitude of $|G_p(\omega)|$ as a function of ω. Is it a low-pass filter or high-pass filter? What is the frequency range in which the tank pressure $p(t)$ does not fluctuate significantly?

(c) The frequency response function $G_C(\omega)$ from $u(t)$ to $q_C(t)$ is

$$G_C(\omega) = \frac{jRC\omega}{1 + jRC\omega} \tag{6}$$

Plot the magnitude of $|G_C(\omega)|$ as a function of ω.

(d) Use the result of part (c) to answer this question. Safety regulations require that the fluction amplitude of $q_C(t)$ be less than $A/2$ for $\omega < 200$ Hz, where A is the excitation amplitude in (2). What is the range of RC? Explain why.

3. This is an old exam problem that I gave in the Spring Quarter of 2004. Micro-electro-mechanical systems (MEMS) are emerging technology that uses semiconductor processes to fabricate tiny sensors and actuators. One type of MEMS actuator is thermal drives consisting of a silicon base beam and a metal electrode with large resistance; see Fig. 3(a). When the electric voltage $V(t)$ is applied to the electrode, it generates heat $q(t)$ increasing the relative temperature $T(t)$ of the beam to the ambient fluid. Since silicon and metal have different coefficients of thermal expansion, the temperature change causes the motion $x(t)$. Figure 3(b) shows the block diagram describing the dynamics, where R and k are both constants. Moreover, the relationship between the temperature $T(t)$ and and input heat flow $q(t)$ satisfies

$$\rho v c\frac{dT(t)}{dt} + hAT(t) = q(t) \tag{7}$$

where ρ is the density of the beam, v is the volume of the beam, c is the specific heat of the beam, h is the heat transfer coefficient of the beam, and A is the surface area of the beam. Answer the following questions.

(a) Derive the frequency response function $G(\omega)$ from $q(t)$ to $T(t)$.

(b) Plot the magnitude $G(\omega)$ as a function of ω. Is it a low-pass filter or a high-pass filter? What is the bandwidth of $G(\omega)$? What is the time constant of the system?

(c) Two thermal drives A and B have identical materials. The size of A is twice as large as B in every dimension. Which thermal drive has a larger bandwidth and why?

(d) Thermal drive C has the following frequency response function

$$G(\omega) = \frac{0.01}{1 + j\tau\omega} \tag{8}$$

where τ is the time constant. In addition, the thermal drive has a corner frequency of 100 Hz. The background noise has a magnitude of 10^{-3}. When $|G(\omega)|$ is less than the background noise, one cannot measure the response of the thermal drive any more. Determine the frequency range where the response cannot be measured.

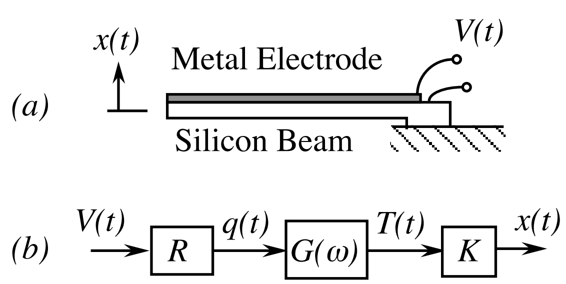

Figure 3: a thermal drive

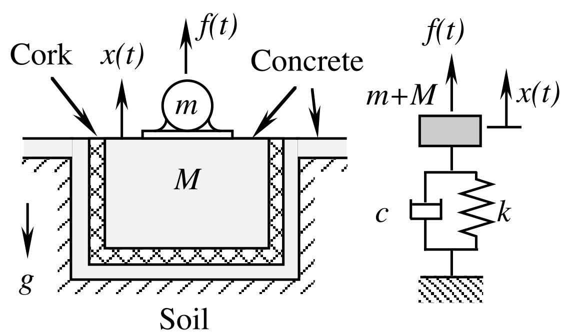

Figure 4: Vibration isolation of a machine on a foundation

4. Machines that are expected to transmit substantial static or dynamic forces through their pedestal are installed on foundations to isolate vibration. A usual arrangement is shown in Fig. 4. The machine of mass m, mounted on a massive foundation of mass M, rests directly on cork and soil that provides restoring and damping forces. As a result, the machine and the foundation system can be modeled as a single spring-mass-dashpot system shown in Fig. 4. The mass will be $m + M$, the stiffness k can be obtained from soil mechanics, and the damping c usually is very difficult to know exactly and greatly depends on the materials. Obviously, the gravitational acceleration g needs to be considered. Because of the heavy weight of the foundation and the machine, the system will have a static deflection δ when the system is mounted on the cork and soil. In addition, the motion of the heavy foundation mass is described by the displacement $x(t)$ with respect to the equilibrium position (i.e., static deflection δ). When the machine is on, the system is subjected to a sinusoidal force $f(t) = F_0 \cos \omega t$ resulting from the unbalance of the machine.

   (a) Formulate the equation of motion. Find the undamped natural frequency ω_n and the viscous damping factor ζ.

(b) Show that the undamped natural frequency of the system can also be found as

$$\omega_n = \sqrt{\frac{g}{\delta}} \tag{9}$$

where δ is the static deflection of the foundation mass.

(c) When the machine is turned on, we can consider a complex excitation

$$F(t) = F_0 e^{i\omega t} \tag{10}$$

Determine the response $x(t)$ of the machine and the foundation.

(d) The force transmitted to the soil is

$$F_T \equiv c\dot{x} + kx \tag{11}$$

Therefore, we can define the transmissibility as

$$G(i\omega) \equiv \frac{F_T}{F} \tag{12}$$

Derive $G(i\omega)$ and plot $|G(i\omega)|$ as a function of $\frac{\omega}{\omega_n}$. Determine when the transmissibility is less than 1.