ME 374, System Dynamics Analysis and Design
Homework 7

Distributed: 5/12/2008, Due: 5/23/2008

(There are 6 problems in this set.)

1. This is an old exam problem that I gave in Spring Quarter of 1998. A simple seismometer is shown in Fig. 1. As the base moves vertically, the relative movement between the mass m and the drum is recorded by a moving pen. Assume that the earthquakes may be modeled as a sinusoidal vertical motion $u(t) = A\sin(\omega t)$.

   (a) If the input is displacement $u(t)$ and the output is $y(t)$, show that the ODE is

   $$m\ddot{y} + c\dot{y} + ky = -m\ddot{u} \tag{1}$$

   If the input is acceleration $a(t) \equiv \ddot{u}(t)$, show that the ODE is

   $$m\ddot{y} + c\dot{y} + ky = -ma(t) \tag{2}$$

   (b) Let $G_u(\omega)$ be frequency response function from $u(t)$ to $y(t)$, and let $G_a(\omega)$ be frequency response function from $a(t)$ to $y(t)$. Determine $G_u(\omega)$ and $G_a(\omega)$. You can leave them as fractions of two complex numbers.

   (c) Consider the magnitude plot in Fig. 2. Does it correspond to $|G_u(\omega)|$ or $|G_a(\omega)|$?

   (d) Estimate the natural frequency ω_n of the system from Fig. 2.

   (e) Recall that the damping ratio ζ of the system is given by

   $$\zeta = \frac{c}{2m\omega_n} \tag{3}$$

   Derive the peak amplitude $|G(\omega_n)|$ in terms of ζ. Use the peak amplitude of Fig. 2 to estimate ζ.

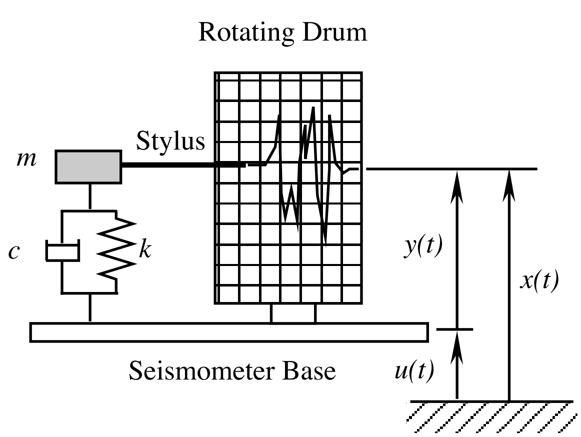

Figure 1: A simple seismometer

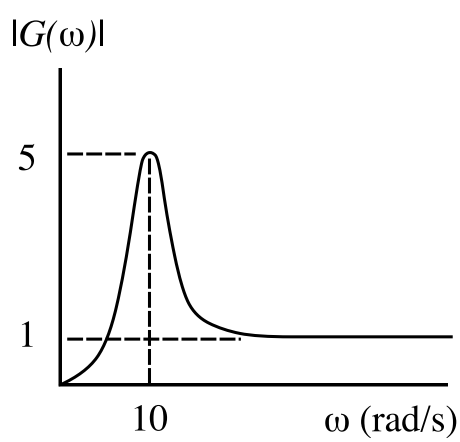

Figure 2: A magnitude plot

2. This is an old exam problem that I gave in the Winter Quarter of 2000. Consider the dental drill problem in our previous homework. The vibration of a dental drill is governed by the following differential equation

$$\ddot{x} + 0.1\dot{x} + 10^6 x = \dot{u}(t) \tag{4}$$

where $u(t)$ is the input force drilling on a tooth and $x(t)$ is the output vibration of the drill. Answer the following questions

   (a) Derive the frequency response functions $G(\omega)$, where ω is the frequency parameter. You can keep $G(\omega)$ in the complex form.

   (b) Determine the resonance frequency ω_n and resonance amplitude.

   (c) When $\omega << \omega_n$, estimate the magnitude of $G(\omega)$. How fast does it increase (in dB per decade)? When $\omega >> \omega_n$, estimate the magnitude of $G(\omega)$. How fast does it roll off (in dB per decade)?

   (d) Plot the magnitude and phase of $G(\omega)$ as a function of frequency ω.

3. In the class, we discussed a simple model of a car driving on a rocky road. The equation of motion is

$$m\ddot{x} + c\dot{x} + kx = c\dot{R}(t) + kR(t) \tag{5}$$

where $x(t)$ is the vibration response of the car and $R(t)$ is the excitation from the road. Moreover, m is the mass of the car, and k and c are the stiffness and damping of the suspension system, respectively.

   (a) In practical applications, human comfort level is proportional to the acceleration of the vehicle. Based on (5), derive the frequency response function from $R(t)$ to the acceleration $a(t) \equiv \ddot{x}(t)$. Call this frequency response function $G_a(\omega)$.

   (b) Plot the magnitude of $G_a(\omega)$ as a function of frequency ω.

   (c) Engineer X proposes to increase damping c to reduce vehicle vibration. Explain how increased damping c could affect the comfort level of the passengers based on the plot of $|G_a(\omega)|$.

4. This is an old exam problem of the Spring Quarter of 2006. This is also a real problem that we encountered in our research. Piezoelectric actuators have wide applications in many areas. Dynamic analysis of piezoelectric actuators consists of an electrical circuit and a mechanical structure; see Fig. 3. The electrical circuit is low-pass filter consisting of capacitance C from the piezoelectric material and the resistance R of the circuit. The cutoff frequency ω_c is $1/RC$. The mechanical structure has a resonance frequency ω_n. Answer the following questions.

   (a) Consider the case $\omega_c >> \omega_n$, plot the magnitude of the frequency response function of the complete piezoelectric actuator. Is the bandwidth limited by ω_c or ω_n?

   (b) Consider the case $\omega_c << \omega_n$, plot the magnitude of the frequency response function of the complete piezoelectric actuator. Is the bandwidth limited by ω_c or ω_n?

(c) Consider the case where the piezoelectric actuator is to be used a resonator, i.e., the actuator is driven sinusoidally at a single frequency to maximize the response amplitude. Which frequency will you use to drive the actuator? How would you choose ω_c and ω_n for resonators?

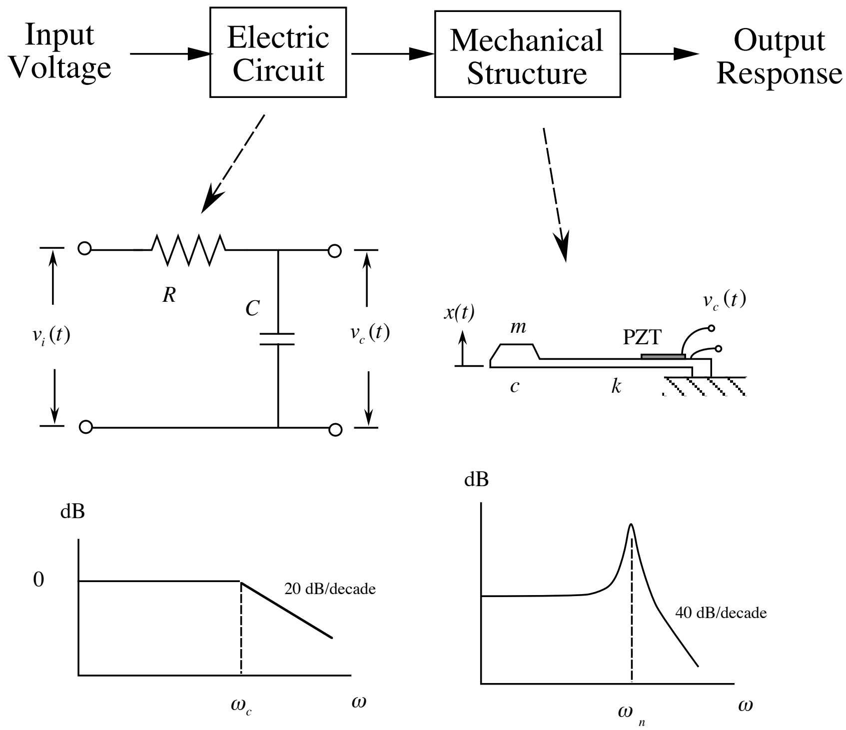

Figure 3: Block diagram showing PZT actuators

5. This is an old exam problem of the Spring Quarter of 2006. Figure 4 shows a tachometer measuring angular velocity $\Omega_s(t)$. The tachometer consists of a drag cup with angular damping coefficient c, a rotor with mass moment of inertia J, and a torsional spring with angular stiffness k. The angular position $\theta(t)$ of the rotor is governed by

$$J\ddot{\theta}(t) + c\dot{\theta}(t) + k\theta(t) = c\Omega_s(t) \tag{6}$$

Answer the following questions.

(a) Determine the frequency response function $G_k(\omega)$ from Ω_s to T_k, where $T_k = k\theta$ is the torque of the spring. Sketch the magnitude of $G_k(\omega)$ as a function of ω.

(b) Determine the frequency response function $G_\theta(\omega)$ from Ω_s to angular velocity $\dot{\theta}(t)$ of the rotor. Sketch the magnitude of $G_k(\omega)$ as a function of ω.

(c) To measure $\Omega_s(t)$, should one use T_k or $\dot{\theta}$ as output? Why? What is the bandwidth of the tachometer? What is the sensitivity (i.e., proportional constant) of the tachometer?

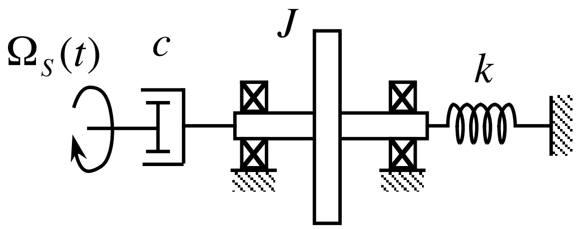

Figure 4: A tachometer measuring angular velocity

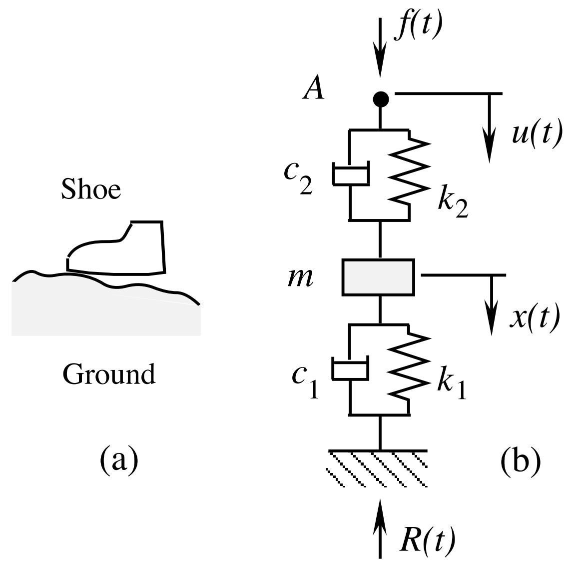

Figure 5: A mathematical model of a running shoe

6. Figure 5 shows a mathematical model of a running shoe. The shoe is modeled as a lumped mass m, two linear springs and two linear dampers. The springs have stiffness k_1 and k_2 and the dampers have damping coefficients c_1 and c_2. In addition, $f(t)$ is the force exerted by the foot on the shoe, $u(t)$ is the displacement of the foot, $x(t)$ is the displacement of the shoe, and $R(t)$ is the reaction of the ground to the shoe. Answer the following questions.

   (a) Derive the frequency response function $G(\omega)$ from $f(t)$ to $x(t)$ in terms of m, k_1, k_2, c_1, c_2, and ω. Plot the magnitude of the frequency response function $G(\omega)$ as a function of ω.

   (b) Dynamic stiffness $Z(\omega)$ of the shoe is defined as the frequency response function from $u(t)$ to $f(t)$. The dynamic stiffness $Z(\omega)$ is the stiffness of the system at different frequencies. When the dynamic stiffness is large, the shoe feels stiff. When the dynamic stiffness is small, the shoe feels soft. Derive the dynamic stiffness in terms of m, k_1, k_2, c_1, c_2, and ω. Plot the magnitude of the dynamic stiffness $Z(\omega)$ as a function of ω.

   (c) Transmissibility $Y(\omega)$ is the frequency response function from $f(t)$ to $R(t)$. Transmissibility is the ratio of forces transmitted from $f(t)$ to $R(t)$ at various frequencies. When the transmissibility is large, the shoes has large reaction force $R(t)$ meaning good rebound. Derive the transmissibility in terms of m, k_1, k_2, c_1, c_2, and ω. Plot the magnitude of transmissibility $Y(\omega)$ as a function of ω.

   (d) Assume that the running condition can be modeled as a periodic excitation of $f(t) = \sin \omega t$. Based on the results in the dynamic stiffness $Z(\omega)$ and transmissibility $Y(\omega)$ derived above, describe how you would run in order to end up with a soft shoe with a good rebound. Please explain your argument in detail.