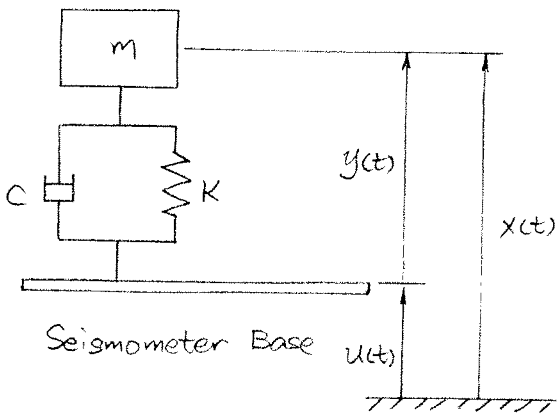
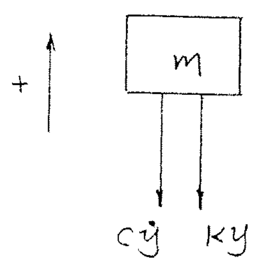
Problem 1



(a) F.B.D.



$$\Sigma F: \quad -c\dot{y} - ky = m\ddot{x}$$

$$\Rightarrow -c\dot{y} - ky = m(\ddot{y} + \ddot{u})$$

$$\Rightarrow \underline{m\ddot{y} + c\dot{y} + ky = -m\ddot{u}}$$ ANS.

If $a(t) \equiv \ddot{u}(t)$

$$\Rightarrow \underline{m\ddot{y} + c\dot{y} + ky = -ma}$$ ANS.

(b) Let $y(t) = Y(s)e^{st}$, $u(t) = U(s)e^{st}$

$$\Rightarrow ms^2 Y(s)\cancel{e^{st}} + cs Y(s)\cancel{e^{st}} + k Y(s)\cancel{e^{st}} = -ms^2 U(s)e^{st}$$

$$\Rightarrow H_u(s) = \frac{Y(s)}{U(s)} = \frac{-ms^2}{ms^2 + cs + k}$$

F.R.F.

$$G_u(\omega) = \left.\frac{-ms^2}{ms^2 + cs + k}\right|_{s=j\omega} = \underline{\frac{m\omega^2}{(k - m\omega^2) + c\omega j}}$$ ANS.

Let $a(t) = A(s)e^{st}$

$$m\ddot{y} + c\dot{y} + ky = -ma$$

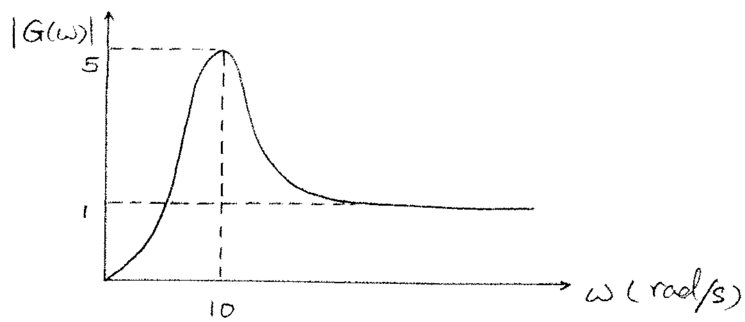
$$\Rightarrow ms^2 Y(s)e^{st} + csY(s)\cancel{e^{st}} + kY(s)\cancel{e^{st}} = -m A(s)\cancel{e^{st}}$$

$$\Rightarrow H_a(s) = \frac{Y(s)}{A(s)} = \frac{-m}{ms^2 + cs + k}$$

F.R.F.

$$G_a(\omega) = \left.\frac{-m}{ms^2 + cs + k}\right|_{s=j\omega} = \underline{\frac{-m}{(k - m\omega^2) + c\omega j}} \quad \text{ANS.}$$

(c)



$$\left.\begin{aligned} |G_u(\omega)| &= \frac{m\omega^2}{\sqrt{(k - m\omega^2)^2 + c^2\omega^2}} \\ \zeta &= \frac{c}{2m\omega_n} \Rightarrow c = 2\zeta m \omega_n \end{aligned}\right\}$$

$$\Rightarrow |G_u(\omega)| = \frac{m\omega^2}{\sqrt{(k - m\omega^2)^2 + 4\zeta^2 m^2 \omega_n^2 \omega^2}} = \frac{m\omega^2}{\sqrt{m^2(\frac{k}{m} - \omega^2)^2 + 4\zeta^2 m^2 \omega_n^2 \omega^2}}$$

$$= \frac{\omega^2}{\sqrt{(\omega_n^2 - \omega^2)^2 + 4\zeta^2 \omega_n^2 \omega^2}} = \frac{\omega^2}{\sqrt{\omega_n^4\left\{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2\right\}}}$$

$$= \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}}$$

$$|G_a(\omega)| = \frac{m}{\sqrt{(k-m\omega^2)^2 + c^2\omega^2}}, \qquad c = 2\zeta m \omega_n$$

$$\Rightarrow |G_a(\omega)| = \frac{m}{\sqrt{(k-m\omega^2)^2 + 4\zeta^2 m^2 \omega_n^2 \omega^2}} = \frac{\cancel{m}}{\sqrt{\cancel{m^2}\left(\frac{k}{m}-\omega^2\right)^2 + 4\zeta^2 \cancel{m^2}\omega_n^2\omega^2}}$$

$$= \frac{1}{\sqrt{(\omega_n^2-\omega^2)^2 + 4\zeta^2\omega_n^2\omega^2}} = \frac{1}{\sqrt{\omega_n^4\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2\right\}}}$$

$$= \frac{\frac{1}{\omega_n^2}}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}}$$

✓ Just look at case when $\omega \to \infty$.

$$|G_u(\omega)| \approx \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left(\frac{\omega}{\omega_n}\right)^4 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}} \approx \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\sqrt{\left(\frac{\omega}{\omega_n}\right)^4}} = \frac{\left(\frac{\omega}{\omega_n}\right)^2}{\left(\frac{\omega}{\omega_n}\right)^2} = 1$$

$$|G_a(\omega)| \approx \frac{\frac{1}{\omega_n^2}}{\sqrt{\left(\frac{\omega}{\omega_n}\right)^4 + 4\zeta\left(\frac{\omega}{\omega_n}\right)^2}} \approx \frac{\frac{1}{\omega_n^2}}{\sqrt{\left(\frac{\omega}{\omega_n}\right)^4}} = \frac{\frac{1}{\omega_n^2}}{\left(\frac{\omega}{\omega_n}\right)^2} = \frac{1}{\omega^2}$$

Since: $|G_u(\omega)| \approx 1$ when $\omega \to \infty$, it corresponds to flat zone of $|G(\omega)|$.

So: <u>Fig. 2 represents $|G_u(\omega)|$</u> ANS.

(d) Natural Frequency ω_n can be found from Fig. 2, the frequency corresponding to resonance peak is ω_n.

So: $\underline{\omega_n \approx 10 \text{ rad/s}}$ ANS.

(e) $$\zeta = \frac{c}{2m\omega_n} = \frac{c}{20m} \Rightarrow \underline{c = 20\zeta m}$$

Previously we know:

$$|G_u(\omega)| = \frac{m\omega^2}{\sqrt{(K-m\omega^2)^2 + c^2\omega^2}}$$

$$\Rightarrow |G_u(\omega_n)| = \frac{m\omega_n^2}{\sqrt{(\underbrace{K-m\omega_n^2}_{\to 0})^2 + c^2\omega_n^2}} = \frac{m\omega_n^{\cancel{2}}}{c\cancel{\omega_n}}$$

$$= \frac{\cancel{m}\omega_n}{20\zeta\cancel{m}} = \frac{10}{20\zeta} = 5 \quad \text{(resonanse amplitude)}$$

$$\Rightarrow \underline{\zeta = 0.1}$$ ANS.

Problem 2

Dental Drill:

$$\ddot{x} + 0.1\dot{x} + 10^6 x = \dot{u}(t)$$

(a) Let $x(t) = X(s)e^{st}$, $U(t) = U(s)e^{st}$

$$\Rightarrow s^2 X(s)\cancel{e^{st}} + 0.1 s X(s)\cancel{e^{st}} + 10^6 X(s)\cancel{e^{st}} = s U(s)\cancel{e^{st}}$$

$$\Rightarrow H(s) = \frac{X(s)}{U(s)} = \frac{s}{s^2 + 0.1 s + 10^6}$$

F.R.F.:

$$G(\omega) = \frac{s}{s^2 + 0.1 s + 10^6}\bigg|_{s=j\omega} = \frac{\omega j}{(10^6 - \omega^2) + 0.1\omega j}$$ ANS.

(b) $$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{10^6}{1}} = 1000 \text{ rad/s}$$ ANS.

$$|G(\omega)| = \frac{\omega}{\sqrt{(10^6 - \omega^2)^2 + 0.01\omega^2}}\bigg|_{\omega=\omega_n=1000}$$

$$= \frac{1000}{\sqrt{0.01 \times 1000^2}} = 10$$ ANS.

(c) $$|G(\omega)| = \frac{\omega}{\sqrt{(10^6-\omega^2)^2+0.01\omega^2}}$$

✓ when $\omega \ll \omega_n = 1000$

$$\Rightarrow |G(\omega)| \approx \frac{\omega}{\sqrt{(10^6)^2+0.01\omega^2}} \approx \frac{\omega}{\sqrt{(10^6)^2}} = \frac{\omega}{10^6}$$

$$dB = 20\log_{10}|G(\omega)| = 20\log_{10}\frac{\omega}{10^6} = 20(\log_{10}\omega + \log_{10}10^{-6})$$

$$= \underbrace{20}\log_{10}\omega - 120$$

↳ Increase about 20 dB/decade ANS

✓ when $\omega \gg \omega_n = 1000$

$$\Rightarrow |G(\omega)| \approx \frac{\omega}{\sqrt{(\omega^2)^2+0.01\omega^2}} \approx \frac{\omega}{\sqrt{\omega^4}} = \frac{1}{\omega}$$

$$dB = 20\log_{10}|G(\omega)| = 20\log_{10}\frac{1}{\omega} = 20(\log_{10}1 + \log_{10}\omega^{-1})$$

$$= 20(0-\log_{10}\omega) = \underbrace{-20}\log_{10}\omega$$

↓

Decrease about 20 dB/decade ANS.

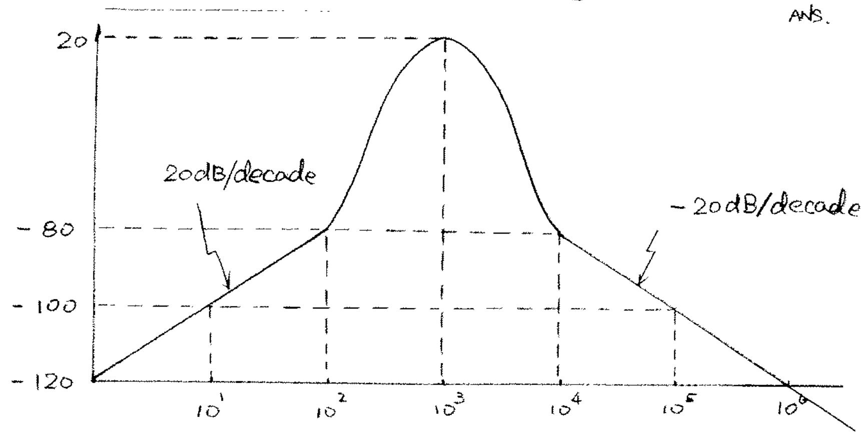
(d) plot in dB

when $\omega = \omega_n = 1000$

$dB = 20\log_{10}|G(\omega_n)|$

$= 20\log_{10}10$

$= 20$



Problem 3:

Car drives on a rocky road:

$$m\ddot{x} + c\dot{x} + kx = c\dot{R}(t) + kR(t)$$

(a) Let $x(t) = X(s)e^{st}$, $R(t) = R(s)e^{st}$

$$\Rightarrow ms^2 X(s)\cancel{e^{st}} + cs X(s)\cancel{e^{st}} + k X(s)e^{st}$$

$$= cs R(s)\cancel{e^{st}} + k R(s)\cancel{e^{st}}$$

$$\Rightarrow H(s) = \frac{X(s)}{R(s)} = \frac{cs + k}{ms^2 + cs + k}$$

$$a(t) = \ddot{x}(t) = s^2 X(s)e^{st} = A(s)e^{st}$$

So:

$$H_a(s) = \frac{A(s)}{R(s)} = \frac{s^2 X(s)}{R(s)} = \frac{s^2(cs+k)}{ms^2 + cs + k}$$

$$\Rightarrow G_a(\omega) = \left.\frac{s^2(cs+k)}{ms^2+cs+k}\right|_{s=j\omega} = \frac{-\omega^2(c\omega j + k)}{(k - m\omega^2) + c\omega j}$$

(b)

$$|G_a(\omega)| = \frac{\omega^2\sqrt{k^2 + c^2\omega^2}}{\sqrt{(k-m\omega^2)^2 + c^2\omega^2}}, \quad c = 2\zeta m\omega_n$$

$$\Rightarrow |G_a(\omega)| = \frac{\omega^2\sqrt{m^2\omega_n^4 + 4\zeta^2 m^2\omega_n^2\omega^2}}{\sqrt{m^2(\omega_n^2 - \omega^2)^2 + 4\zeta^2 m^2\omega_n^2\omega^2}}$$

$$= \frac{\omega^2\sqrt{\cancel{\omega_n^4}\left[1 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2\right]}}{\sqrt{\cancel{\omega_n^4}\left\{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2\right\}}}$$

✓ when $\omega \ll \omega_n$

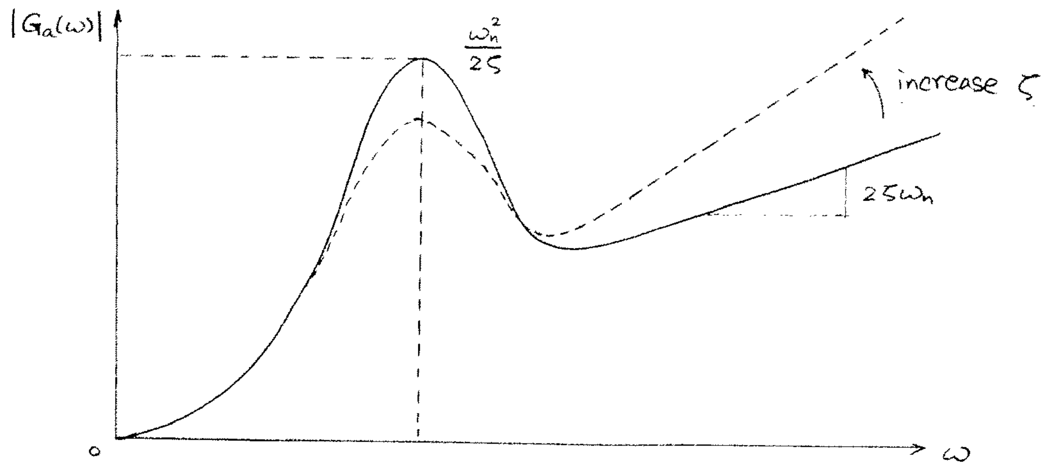
$$|G_a(\omega)| \approx \frac{\omega^2\sqrt{1}}{\sqrt{1^2+4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}} \approx \frac{\omega^2}{\sqrt{1}} = \underline{\omega^2}$$ ANS.

✓ when $\omega = \omega_n$

$$|G_a(\omega)| = \frac{\omega_n^2\sqrt{1+4\zeta^2}}{\sqrt{4\zeta^2}} \approx \underline{\frac{\omega_n^2}{2\zeta}}$$ ANS.

✓ when $\omega \gg \omega_n$

$$|G_a(\omega)| \approx \frac{\omega^2\sqrt{4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[\left(\frac{\omega}{\omega_n}\right)^2\right]^2+4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}} \approx \frac{\omega^2\cdot 2\zeta\left(\frac{\omega}{\omega_n}\right)}{\sqrt{\left(\frac{\omega}{\omega_n}\right)^4}} = \underline{2\zeta\omega_n\omega}$$ ANS.

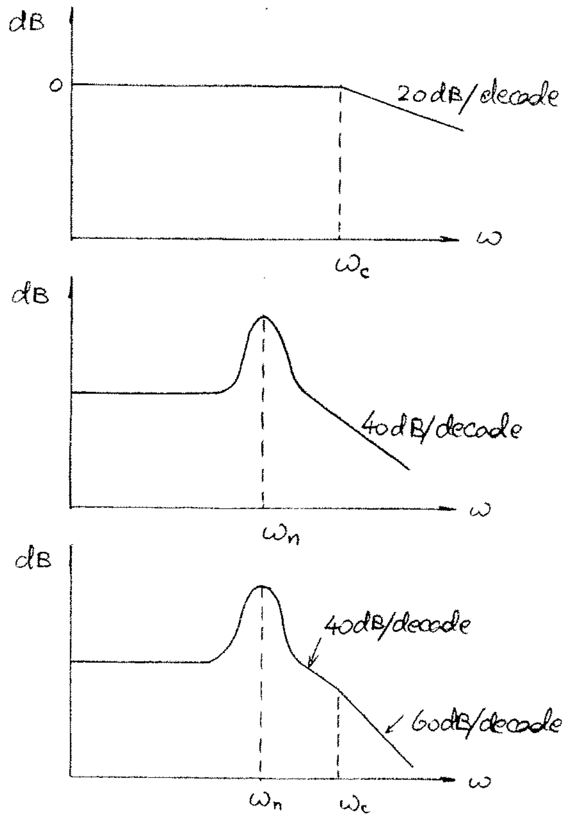


(c) According to equation $\zeta = \frac{c}{2m\omega_n}$, increase c will increase ζ, although this will decrease resonanse peak, it will increase $|G_a(\omega)|$ when $\omega \gg \omega_n$, so this is <u>NOT</u> a good idea.

Problem 4

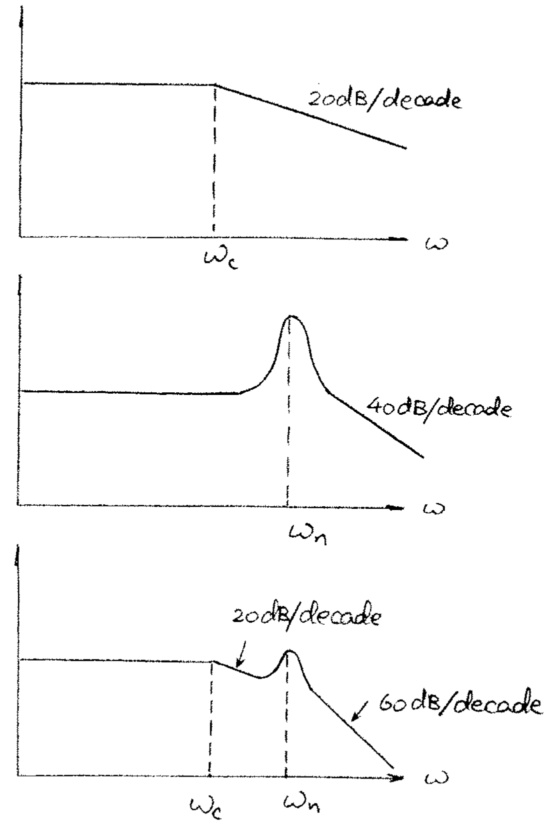
Piezoelectric actuator

(a) when $\omega_c \gg \omega_n$



✓ Bandwidth is limited by ω_n.

(b) when $\omega_c \ll \omega_n$



✓ Bandwidth is limited by ω_c.

(c) To maximize the response, drive system at it natural frequency ω_n.

To avoid resonance peak being distorted, choose $\omega_c \gg \omega_n$ setup.

Problem 5.

$$J\ddot{\theta}(t) + C\dot{\theta}(t) + K\theta(t) = C\Omega_s(t)$$

(a) Let $\theta(t) = \Theta(S)e^{st}$, $\Omega_s(t) = \Omega_s(S)e^{st}$

$$\Rightarrow JS^2\Theta(S)\cancel{e^{st}} + CS\Theta(S)\cancel{e^{st}} + K\Theta(S)\cancel{e^{st}} = C\Omega_s(S)\cancel{e^{st}}$$

$$\Rightarrow H(S) = \frac{\Theta(S)}{\Omega_s(S)} = \frac{C}{JS^2 + CS + K}$$

$$T_K(t) = \underbrace{T_K(S)}e^{st} = K\theta(t) = \underbrace{K\Theta(S)}e^{st}$$

So:

$$H_K(S) = \frac{T_K(S)}{\Omega_s(S)} = \frac{K\Theta(S)}{\Omega_s(S)} = \frac{KC}{JS^2 + CS + K}$$

$$\Rightarrow G_K(\omega) = \left.\frac{KC}{JS^2 + CS + K}\right|_{S=j\omega} = \underline{\frac{KC}{(K - J\omega^2) + C\omega j}}$$ ANS.

$$|G_K(\omega)| = \frac{KC}{\sqrt{(K - J\omega^2)^2 + C^2\omega^2}}$$

$$\Rightarrow |G_K(\omega)| = \frac{KC}{\sqrt{J^2(\omega_n^2 - \omega^2)^2 + 4\zeta^2 J^2\omega_n^2\omega^2}}$$

$$= \frac{KC}{J\omega_n^2} \cdot \frac{1}{\sqrt{[1 - (\frac{\omega}{\omega_n})^2]^2 + 4\zeta^2(\frac{\omega}{\omega_n})^2}}$$

$$= \frac{C}{\sqrt{[1 - (\frac{\omega}{\omega_n})^2]^2 + 4\zeta^2(\frac{\omega}{\omega_n})^2}}$$

✓ when $\omega \ll \omega_n$

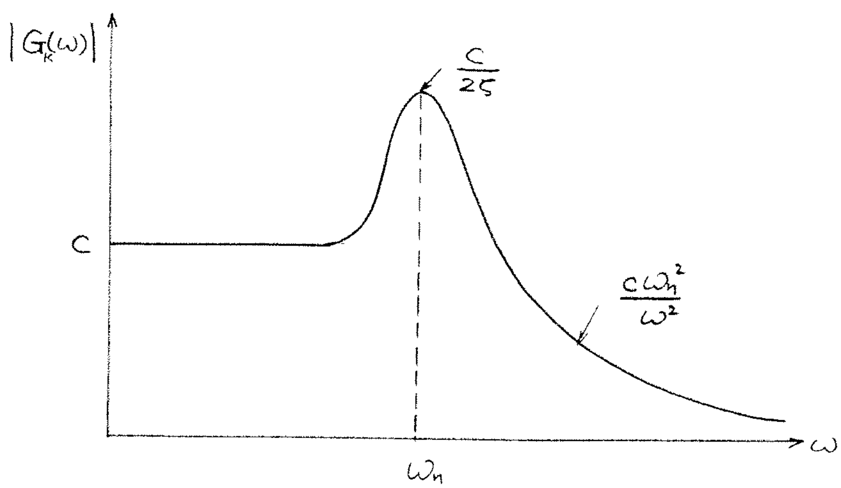
$$|G_K(\omega)| \approx \frac{C}{\sqrt{1+4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}} \approx \frac{C}{\sqrt{1}} = \underline{C} \quad \text{ANS.}$$

✓ when $\omega = \omega_n$

$$|G_K(\omega)| = \frac{C}{\sqrt{4\zeta^2}} = \underline{\frac{C}{2\zeta}} \quad \text{ANS.}$$

✓ when $\omega \gg \omega_n$

$$|G_K(\omega)| \approx \frac{C}{\sqrt{\left(\frac{\omega}{\omega_n}\right)^4+4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}} \approx \frac{C}{\sqrt{\left(\frac{\omega}{\omega_n}\right)^4}} = \underline{\frac{C\omega_n^2}{\omega^2}} \quad \text{ANS.}$$



(b) $$\dot{\Theta}(t) = \omega(S)\cancel{e^{st}} = S\,\Theta(S)\cancel{e^{st}}$$

So:

$$H_\Theta(\omega) = \frac{\omega(S)}{\Omega(S)} = \cancel{\frac{S\,\Theta(S)}{\Omega(S)}} = \frac{SC}{JS^2+CS+K}$$

$$\Rightarrow G_\Theta(\omega) = \left.\frac{SC}{JS^2+CS+K}\right|_{S=j\omega} = \underline{\frac{C\omega j}{(K-J\omega^2)+C\omega j}} \quad \text{ANS}$$

$$\left|G_\theta(\omega)\right| = \frac{C\omega}{\sqrt{(K-J\omega^2)^2 + C^2\omega^2}}$$

$$\Rightarrow \left|G_\theta(\omega)\right| = \frac{C\omega}{\sqrt{J^2(\omega_n^2-\omega^2)^2 + 4\zeta^2 J^2\omega_n^2\omega^2}}$$

$$= \frac{C\omega}{J\omega_n^2}\cdot\frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}}$$

✓ when $\omega \ll \omega_n$

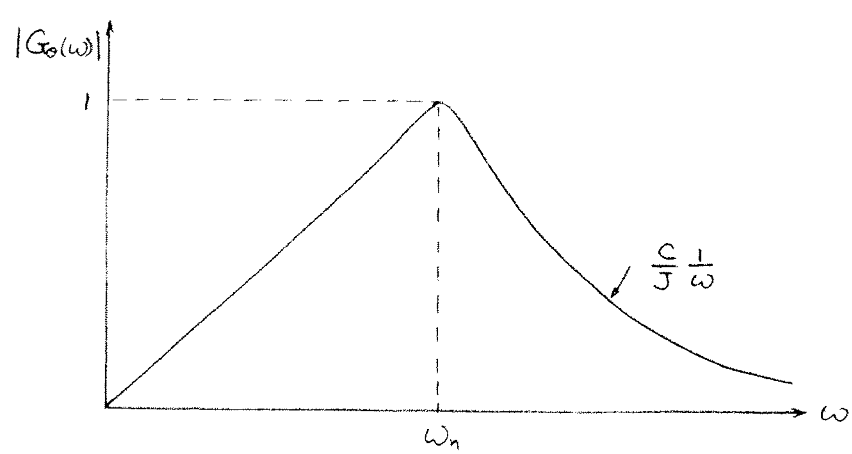
$$\left|G_\theta(\omega)\right| \approx \frac{C\omega}{J\omega_n^2}\cdot\frac{1}{\sqrt{1+4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}} \approx \frac{C\omega}{J\omega_n^2} = \frac{2\zeta \cancel{J}\omega_n\omega}{\cancel{J}\omega_n^{\cancel{2}}} = \frac{2\zeta}{\omega_n}\omega$$ Ans

✓ when $\omega = \omega_n$

$$\left|G_\theta(\omega)\right| = \frac{C\omega_n}{J\omega_n^2}\cdot\frac{1}{\sqrt{4\zeta^2}} = \frac{2\zeta \cancel{J}\omega_n^2}{\cancel{J}\omega_n^{\cancel{2}}\cdot 2\zeta} = 1$$ Ans

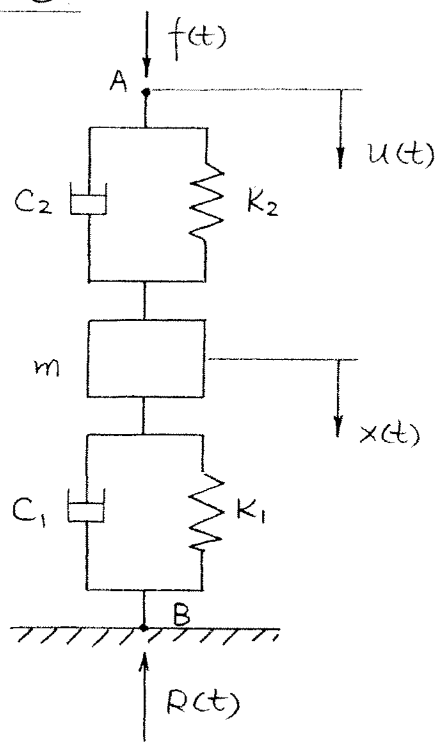
✓ when $\omega \gg \omega_n$

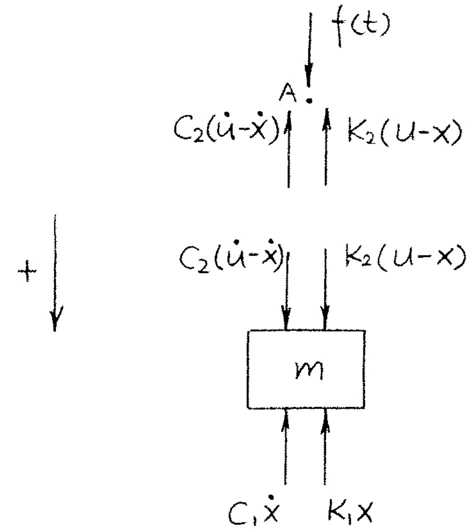
$$\left|G_\theta(\omega)\right| \approx \frac{C\omega}{J\omega_n^2}\cdot\frac{1}{\sqrt{\left(\frac{\omega}{\omega_n}\right)^4 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2}} \approx \frac{C\omega}{J\omega_n^2}\cdot\frac{1}{\sqrt{\left(\frac{\omega}{\omega_n}\right)^4}} = \frac{C\omega}{J\omega_n^2}\,\frac{\omega_n^2}{\omega^2} = \frac{C}{J}\frac{1}{\omega}$$



(C) To measure $\Omega_s(t)$, we should use T_K as output, because when $\omega \ll \omega_n$ $|G_K(\omega)|$ will be in a flat zone. The bandwidth of tachometer is ω_c, The sensitivity is constant for $|G_K(\omega)|$ flat zone, which is c.

Problem 6





(a) Point A: ΣF: $f(t) - C_2(\dot{u}-\dot{x}) - k_2(u-x) = 0$

Mass m: ΣF: $-C_1\dot{x} - K_1 x + C_2(\dot{u}-\dot{x}) + K_2(u-x) = m\ddot{x}$

$$\Rightarrow -C_1\dot{x} - K_1 x + f(t) = m\ddot{x}$$

$$\Rightarrow m\ddot{x} + C_1\dot{x} + k_1 x = f(t)$$

Let $x(t) = X(s)e^{st}$, $f(t) = F(s)e^{st}$

$$\Rightarrow mS^2 X(S)\cancel{e^{st}} + C_1 S X(S)\cancel{e^{st}} + K_1 X(S)\cancel{e^{st}} = F(S)\cancel{e^{st}}$$

$$\Rightarrow H_1(S) = \frac{X(S)}{F(S)} = \frac{1}{mS^2 + C_1 S + K_1}$$

$$\Rightarrow G(\omega) = \frac{1}{mS^2 + C_1 S + K_1}\Bigg|_{S=j\omega} = \frac{1}{(K_1 - m\omega^2) + C_1\omega j}$$

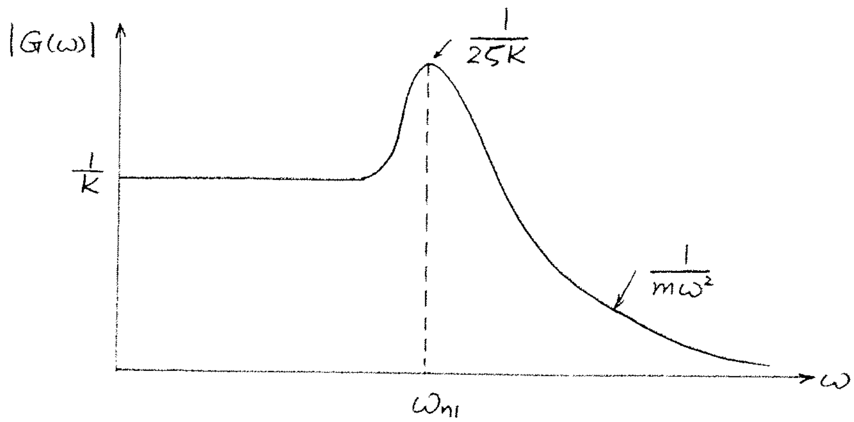
ANS.

$$|G(\omega)| = \frac{1}{\sqrt{(k_1 - m\omega^2)^2 + C_1^2\omega^2}} = \frac{1}{\sqrt{m^2\omega_{n1}^4\left\{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_n}\right)^2\right\}}}$$

✓ when $\omega \ll \omega_n$, $\quad |G(\omega)| \approx \dfrac{1}{m\omega_{n1}^2} = \dfrac{1}{K}$

✓ when $\omega = \omega_n$, $\quad |G(\omega)| = \dfrac{1}{2\zeta m\omega_{n1}^2} = \dfrac{1}{2\zeta K}$

✓ when $\omega \gg \omega_n$, $\quad |G(\omega)| \approx \dfrac{1}{m\omega_{n1}^2}\cdot\dfrac{1}{\sqrt{\left(\frac{\omega}{\omega_n}\right)^4}} = \dfrac{1}{m\omega^2}$



(b) $\quad f(t) - C_2(\dot{u} - \dot{x}) - K_2(u - x) = 0$

$\Rightarrow \quad C_2(\dot{u} - \dot{x}) + K_2(u - x) = f(t)$

Let $\quad u(t) = U(s)e^{st}, \quad x(t) = X(s)e^{st}, \quad f(t) = F(s)e^{st}$

$\Rightarrow C_2(sU(s)\cancel{e^{st}} - sX(s)\cancel{e^{st}}) + K_2(U(s)\cancel{e^{st}} - X(s)\cancel{e^{st}}) = F(s)\cancel{e^{st}}$

$\Rightarrow (C_2 s + K_2)U(s) - (C_2 s + K_2)X(s) = F(s)$

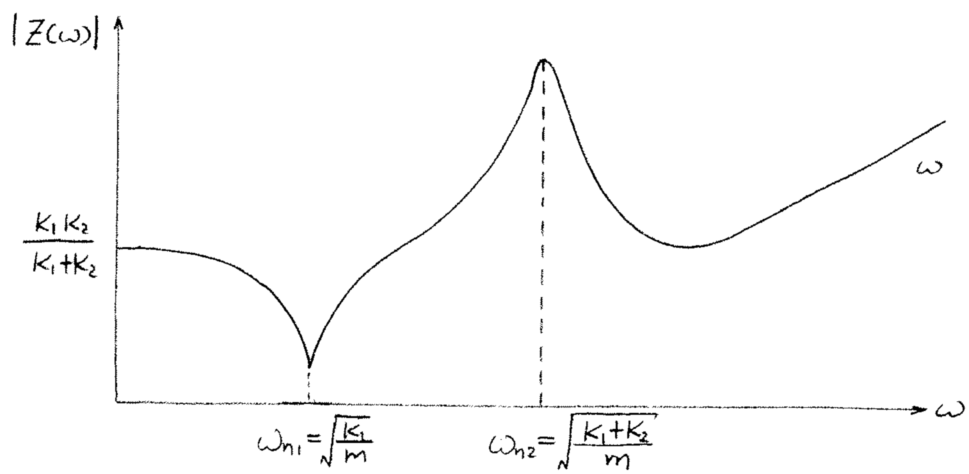
From Part (a)

$$X(s) = H_1(s)\cdot F(s) = \frac{1}{ms^2 + C_1 s + K_1} F(s)$$

$$\Rightarrow (C_2 s + K_2)\,U(s) - (C_2 s + K_2)\,\frac{1}{ms^2 + C_1 s + K_1}\,F(s) = F(s)$$

$$\Rightarrow H_2(s) = \frac{F(s)}{U(s)} = \frac{C_2 s + K_2}{1 + \dfrac{C_2 s + K_2}{ms^2 + C_1 s + K_1}}$$

$$\Rightarrow Z(\omega) = \frac{(K_2 + \omega C_2 j)\left[(K_1 - m\omega^2) + C_1 \omega j\right]}{\left[(K_1 + K_2) - m\omega^2\right] + (C_1 + C_2)\omega j}$$



(C) Point B: ΣF: $\quad C_1 \dot{x} + K_1 x - R(t) = 0$

Let $x(t) = X(s)e^{st}$, $\quad R(t) = R(s)e^{st}$

$$\Rightarrow C_1 s X(s)\cancel{e^{st}} + K_1 X(s) e^{st} = R(s) e^{st}$$

$$\Rightarrow H_3(s) = \frac{X(s)}{R(s)} = \frac{1}{C_1 s + K_1}$$

$$H_4(s) = \frac{R(s)}{F(s)} = \frac{\cancel{X(s)}}{F(s)} \cdot \frac{R(s)}{\cancel{X(s)}} = \frac{H_1(s)}{H_3(s)}$$

$$= \frac{C_1 s + K_1}{ms^2 + C_1 s + K_1}$$

$$\Rightarrow \; Y(\omega) = \frac{K_1 + C_1\omega j}{(K_1 - m\omega^2) + C_1\omega j}$$

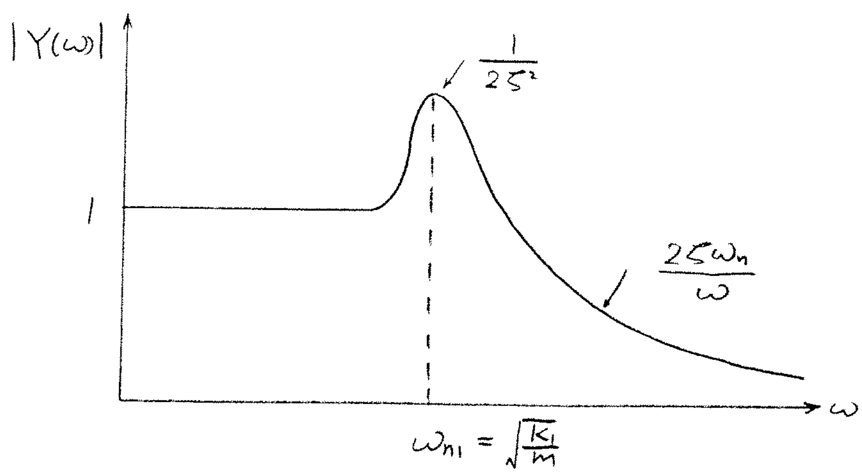
$$\text{So: } |Y(\omega)| = \sqrt{\frac{K_1^2 + C_1^2\omega^2}{(K_1 - m\omega^2)^2 + C_1^2\omega^2}} = \sqrt{\frac{\cancel{m^2}(\omega_{n1}^4 + 4\zeta^2\omega_n^2\omega^2)}{\cancel{m^2}(\omega_{n1}^2 - \omega^2)^2 + 4\zeta^2\cancel{m^2}\omega_n^2\omega^2}}$$

$$= \sqrt{\frac{\cancel{\omega_{n1}^4}\left[1 - 4\zeta^2\left(\frac{\omega}{\omega_{n1}}\right)^2\right]}{\cancel{\omega_{n1}^4}\left\{\left[1 - \left(\frac{\omega}{\omega_{n1}}\right)^2\right]^2 + 4\zeta^2\left(\frac{\omega}{\omega_{n1}}\right)^2\right\}}}$$

✓ when $\omega \ll \omega_n$, $|Y(\omega)| \approx 1$

✓ when $\omega = \omega_n$, $|Y(\omega)| = \sqrt{\frac{1 - 4\zeta^2}{4\zeta^2}} \approx \frac{1}{2\zeta^2}$

✓ when $\omega \gg \omega_n$, $|Y(\omega)| \approx \frac{2\zeta\omega_n}{\omega}$



(d) To get a soft shoe with good rebound:

soft shoe ; minimize the $|Z(\omega)|$ value.

good rebound, enlarge $|Y(\omega)|$ value.

Only choose when $\omega = \omega_{n1} = \sqrt{\frac{K_1}{m}}$ can fulfill the requirement.