ME 374, System Dynamics Analysis and Design
Homework 8
Distributed: 5/19/2008, Due: 5/30/2008
(There are 5 problems in this set.)

1. This is an old exam problem of the Spring Quarter of 2006. Consider a periodic function with period T and a mathematical expression

$$f(t) = \begin{cases} 1, & 0 < t < 3T/4 \\ 0, & 3T/4 < t < T \end{cases} \tag{1}$$

   (a) Expand $f(t)$ in (1) into a complex Fourier series. Derive the first four terms of the Fourier series, i.e., c_0 to c_3.

   (b) How would the coefficients c_{-1} to c_{-3} be related to c_1 to c_3?

   (c) If $T = 0.01$ second, plot the complex spectrum of $f(t)$ using results obtained in part (a).

2. In winter, ice can form on cables and power lines. When wind blows, the geometry of the ice substantially changes the air flow around the cable resulting in flow-induced vibration. This is known as galloping.

   (a) Consider a wind excitation given by

$$f(t) = \cos 1.5pt + 2\sin 2.5pt \tag{2}$$

   where p is a parameter controlled by the geometry of the ice. What is the fundamental frequency of $f(t)$ shown in (2)? Plot the complex spectrum of $f(t)$ as a function of frequency – both the magnitude and phase.

   (b) The frequency response function of the cable is approximated by

$$G(\omega) = \frac{j\omega}{m\left(\omega_n^2 - \omega^2\right)} \tag{3}$$

   where m is the inertia of the cable, ω_n is the natural frequency, and ω is the driving frequency. Based on the frequency response function $G(\omega)$ in (3), determine the output response in the time domain when $\omega_n = 2p$.

3. This is an old exam that I gave in the Spring quarter of 1998. Piezoelectric materials are often used as sensor materials because they transform mechanical deformation to electrical charge. Figure 1 shows an electric circuit of a piezoelectric sensor. The piezoelectric material is modeled as a current source in parallel with a capacitor. The resistance models the impedance of an amplifier. The goal is to make the output voltage $v(t)$ proportional to the input charge $q(t)$, so that we can use the voltage $v(t)$ to measure the charge $q(t)$. The frequency response function (Bode plot) of the circuit is shown in Fig. 2. It is basically a high-pass filter with a corner frequency of $1/RC$.

(a) If the output voltage of the sensor is periodic as shown in Fig. 3, i.e.,

$$v(t) = \begin{cases} 1, & 0 < t < T/2 \\ 0, & \text{Otherwise} \end{cases} \tag{4}$$

express $v(t)$ as a complex Fourier series.

(b) Plot the spectrum (magnitude and phase) of $v(t)$ from $-3\omega_0$ to $3\omega_0$, where ω_0 is the fundamental frequency.

(c) If $T = 1$ ms and $1/RC = 500$ Hz. How would the input charge $q(t)$ look like? Will it look like the periodic square wave in Fig. 3 without much distortion? Why?

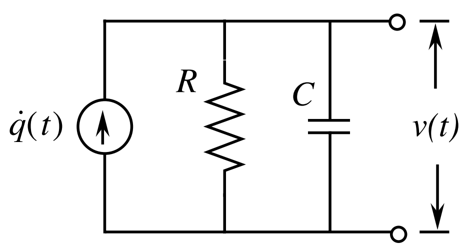

Figure 1: A model of piezosensors

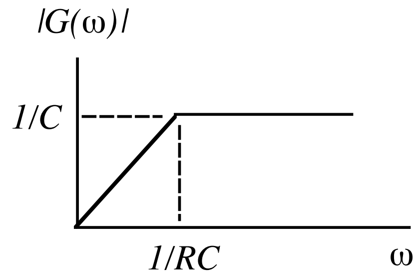

Figure 2: Frequency response function

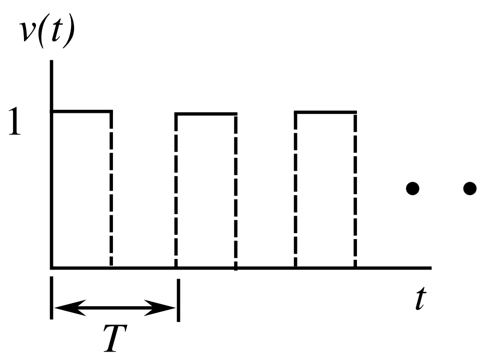

Figure 3: A periodic output $v(t)$

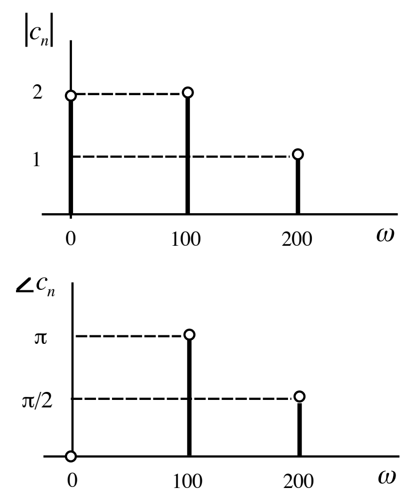

Figure 4: Complex spectrum of input excitation

4. This is an old exam problem of the Spring Quarter of 2006. Consider a hydraulic system whose frequency response function is given as

$$G(\omega) = \frac{j\omega}{1 + j\omega\tau} \tag{5}$$

where τ is the time constant of the system. The hydraulic system is subjected to an input whose complex spectrum (consisting of c_0, c_1 and c_2 components) is shown in Fig. 4. Let's consider the case when $\tau = 1$ second.

(a) Determine the output response in the time domain.

(b) How is the output response different from the input excitation given in the time domain?

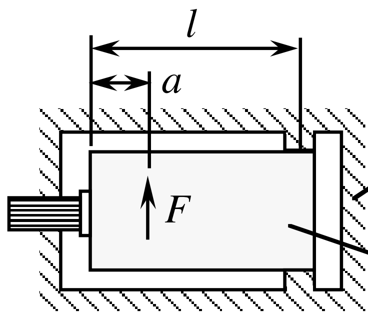

Figure 5: A disk drive in a computer server

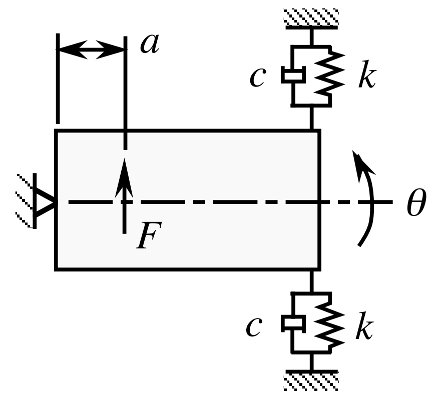

Figure 6: An 1-DOF model of disk drive

5. Consider a disk drive installed in a computer server as shown in Fig. 5. One end of the disk drive is connected to a flexible cable for data transfer, and the other end is spring loaded to the computer server. During a read/write process, a voice coil motor inside the disk drive slews read/write heads and their suspension system (not shown in Fig. 5) to read or write data. The actuation from the voice coil motor then exerts a force F exciting the disk drive. To analyze the motion, let's model the disk drive as a rigid-body and the flex cable connection as a hinged end; see Fig. 6. In addition, the other end of the drive is loaded with springs of stiffness k and viscous dampers of damping coefficient c. With this model, the disk drive undergoes rigid-body rocking and the motion can be described completely in terms of the rocking angle θ about the equilibrium condition. Moreover, application of Newton's law leads to the following equation of motion for θ.

$$I\ddot{\theta}(t) + 2cl^2\dot{\theta}(t) + 2kl^2\theta(t) = aF(t) \tag{6}$$

where I is the mass moment of inertia of the drive about the hinged point, a is the distance between the force and the hinge, and l is the length of the drive. Based on (6), answer the following questions.

(a) Derive the undamped natural frequency ω_n and viscous damping ratio ζ.

(b) Very often, the angular acceleration $\alpha(t) \equiv \ddot{\theta}(t)$ of the drive is of interest. Determine the frequency response function $G(\omega)$ from $F(t)$ to $\alpha(t)$. Plot the amplitude of $G(\omega)$ as a function of frequency. Explain the asymptotic behavior of $G(\omega)$.

(c) Consider an input excitation $F(t)$ from a periodic seek shown in Fig. 7. Mathematically, $F(t)$ is given by

$$F(t) = \begin{cases} 1, & 0 < t < \epsilon \\ 0, & \epsilon < t < T \end{cases}, \qquad F(t+T) = F(t) \tag{7}$$

where ϵ is the duration of the seek pulse and T is the period. Determine the Fourier series of $F(t)$.

(d) Assume that $\omega_n = 250$ Hz, $\zeta = 0.01$, $a/I = 1$, $T = 2.5$ ms, and $\epsilon = 1$ ms. Determine angular acceleration $\alpha(t)$ and plot $\alpha(t)$ as a function of time. Does the output $\alpha(t)$ have the same wave form as the input excitation $f(t)$? Why?

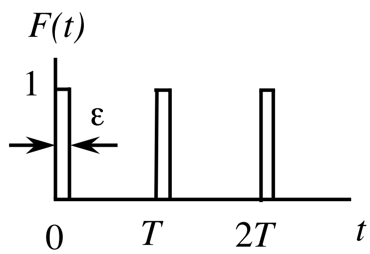

Figure 7: A periodic seeking excitation