ME 374, System Dynamics Analysis and Design
Homework 9

Distributed: 5/26/2008, Due: 6/6/2008

(There are 4 problems in this set.)

1. This is an old exam problem of the Spring Quarter 2006. Figure 1 shows a suspension system consisting of a mass m, a linear damper with damping coefficient c, a spring with stiffness k, and a massless roller A. The mass rides along a guide rod while the roller rolls over an unknown terrain. The displacement of the roller is $y(t)$ and the displacement of the spring/damper connection is $x(t)$. Answer the following questions.

   (a) The impulse response function from $y(t)$ to $x(t)$ is

   $$h(t) = e^{-t/\tau} \tag{1}$$

   where τ is the time constant of the system. Derive the frequency response function from $y(t)$ to $x(t)$. (Hint: Recall that impulse response function and frequency response function are Fourier transform pairs.)

   (b) Assume that the frequency response function from $y(t)$ to $x(t)$ takes the form of

   $$G(\omega) = \frac{1}{1 + 10\omega j} \tag{2}$$

   Derive and plot the magnitude of the frequency response function.

   (c) From the frequency response function given in (2), determine if the suspension is a first-order system or a second-order system. Explain why. What is the bandwidth of the suspension system?

   (d) Figure 2 is the magnitude of the output spectrum of $x(t)$. Based on the $G(\omega)$ defined in (2), determine the magnitude of the input spectrum of $y(t)$.

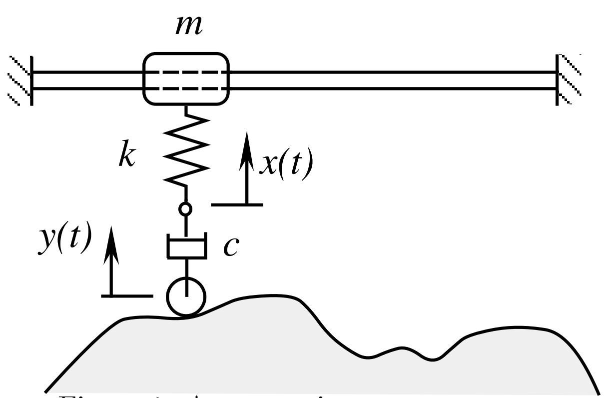

Figure 1: A suspension system on a terrain

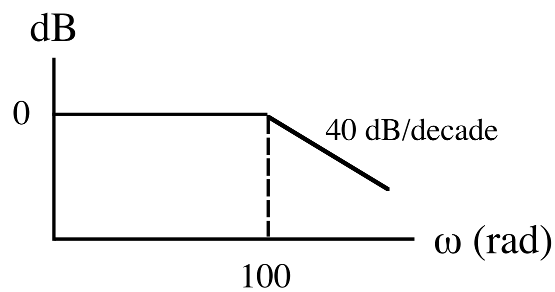

Figure 2: Magnitude of the output spectrum of $x(t)$

2. This is an old exam problem that I gave in the Spring Quarter of 1998. My student Henry Bittner designed an automated hammer to excite spinning disks in the labs. A typical force

history generated by the hammer is modeled mathematically by

$$f(t) = \begin{cases} 1, & 0 < t < T \\ 0, & \text{Otherwise} \end{cases} \tag{3}$$

where T is the duration of the hammer impact.

(a) Find the Fourier transform $F(\omega)$ of the input force $f(t)$.

(b) Determine the amplitude of $F(\omega)$ and it should look like Fig. 3. Show that the amplitude is zero when

$$\omega = \frac{2\pi}{T}, \frac{4\pi}{T}, \ldots \tag{4}$$

(c) Now Henry wants to use this hammer to excite a spinning disk whose frequency response function is shown in Fig. 4. If the hammer is designed so that $T = 2.5$ ms, would Henry excite the 400-Hz resonance of the disk? Why? Would Henry excite the 500-Hz resonance of the disk? Why?

(d) If $T = 1$ ms, calculate the amplitude of the output spectrum at 400 Hz and 500 Hz.

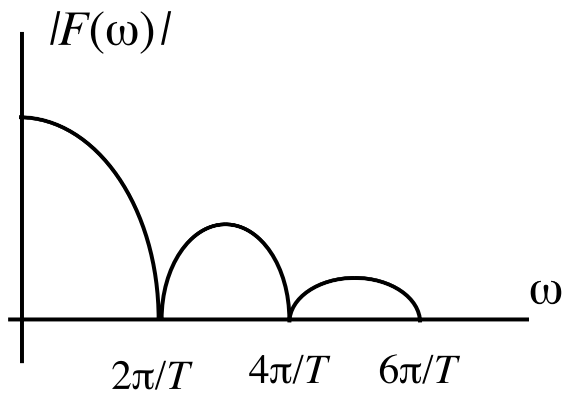

Figure 3: Amplitude of $F(\omega)$

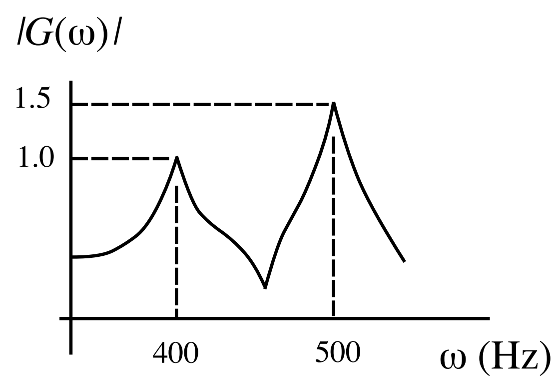

Figure 4: Frequency response function

3. The spring-mass-damper system with base motion in Fig. 5 is a simple model for many applications, such as isolation table and earthquake response. The mass is m, the viscous damping has coefficient c, and the spring has spring constant k. The ground experience a displacement $y(t)$. The displacement of the mass from equilibrium is $x(t)$. In order to improve the performance, a control force $f(t)$ is also applied to the mass.

(a) Derive the equation of motion.

(b) When the system is passive (i.e., the control force is absent), determine the frequency response function $G(\omega)$ from $y(t)$ to $x(t)$. Identify the undamped natural frequency, resonance amplitude, and how fast the frequency response function rolls off as the frequency is above the natural frequency.

(c) Consider the case of spring force cancellation. In this case, the control force is $f(t) = -ky(t)$. Determine the frequency response function $G_{sp}(\omega)$ from $y(t)$ to $x(t)$. Identify the undamped natural frequency, resonance amplitude, and how fast the frequency response function rolls off as the frequency is above the natural frequency.

(d) Consider the case of damping force cancellation. In this case, the control force is $f(t) = -c\dot{y}(t)$. Determine the frequency response function $G_{damp}(\omega)$ from $y(t)$ to $x(t)$. Identify the undamped natural frequency, resonance amplitude, and how fast the frequency response function rolls off as the frequency is above the natural frequency.

(e) In consulting firm A, the model is to simulate response of a building during a earthquake. Therefore, the motion $y(t)$ is the ground motion from the earthquake. The motion $x(t)$ is the absolute displacement of the building. In this case, the goal is to minimize the motion of the building relative to the ground at the resonance. If you were the engineer in charge of the case, would you use spring force cancellation or damping force cancellation? Explain why.

(f) In consulting firm B, the model is to simulate response of an isolation table subjected to the ground motion $y(t)$. Therefore, the motion $x(t)$ is the absolute displacement of the isolation table. In this case, the goal is to minimize the absolute motion of the isolation table. If you were the engineer in charge of the case, would you use spring force cancellation or damping force cancellation? Explain why.

(g) In the course of vibration control, the control unit suddenly broke and sent the following force pulses

$$f(t) = \begin{cases} \sin \omega_0 t, & 0 < t < \dfrac{2\pi}{\omega_0} \\ 0, & \text{Otherwise} \end{cases} \tag{5}$$

Determine the spectrum of $f(t)$ in the frequency domain. Plot the magnitude of the spectrum as a function of ω. Determine the condition when $f(t)$ presents insignificant effects to the response $x(t)$.

4. Figure 6 shows a simplified model to simulate a recording head flying over a rough disk surface in computer hard disk drives. The head has mass m and is supported by a suspension with stiffness k_1. Moreover, the moving disk surface will generate an air bearing lifting the head slightly above the disk surface (e.g., in the order of 20 nm). The air bearing is simplied as a linear spring with stiffness k_2 and damping coefficient c. Let $x(t)$ be the roughness of the disk surface and serve as the input excitation to the head/suspension system. Moreover, $y(t)$ is the relative displacement of the head to the disk. In real hard disk drive applications, we want to keep $y(t)$ almost constant, so that the head can follow the disk surface to perform read/write operations.

(a) Show that the equation of motion is

$$m\ddot{y} + c\dot{y} + (k_1 + k_2)\, y = -m\ddot{x} - k_1 x \tag{6}$$

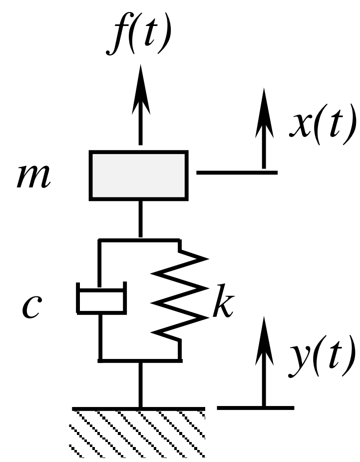

Figure 5: A spring-mass-damping model for vibration control

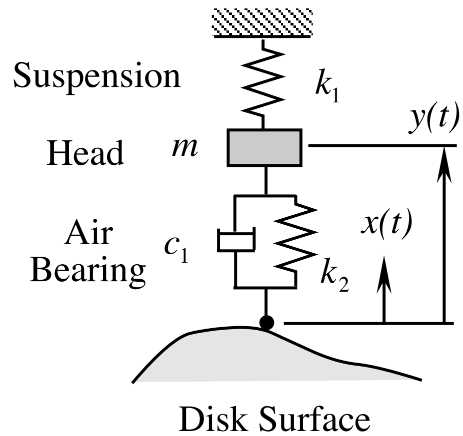

Figure 6: Suspension in computer hard disk drives

(b) Derive the frequency response function. Plot the magnitude and phase of the frequency response function. In plotting the frequency response function, let's define

$$\omega_1 = \sqrt{\frac{k_1}{m}}, \qquad \omega_2 = \sqrt{\frac{k_1 + k_2}{m}} \tag{7}$$

Describe the motion of the head in the following three frequency ranges: $0 < \omega < \omega_1$, $\omega_1 < \omega < \omega_2$, and $\omega_2 < \omega$.

(c) If we want to design the disk drive so that the head can follow the disk surface for a wide frequency range, how should we choose k_1, k_2, and m?

(d) Now consider the case when the head flies over an isolated bump on the disk. The excitation from the bump to the head appears in the form of

$$x(t) = \begin{cases} h \sin \pi t / T, & 0 < t < T \\ 0, & t > T \end{cases} \tag{8}$$

where h is the height of the bump and T is the time needed to fly over the bump. Find $X(\omega)$, which is the Fourier transform of $x(t)$ in (8). Plot the amplitude of $X(\omega)$ with respect to frequency ω. Identify the frequencies where $|X(\omega)|$ is zero.

(e) Given the design in part (c), what is the minimum T the disk can have without significantly exciting the head into large vibration?