

Transmissibility

Assuming that the forcing function is harmonic in nature, we shall consider two cases of vibration transmission - one in which force is transmitted to the supporting structure, and one in which the motion of the supporting structure is transmitted to the machine.

(a) Force excitation

Consider the system shown in Figure 1, where $f(t)$ is the harmonic force acting on the system and $f_T(t)$ is the force transmitted to the supporting structure or base. The force transmitted through the spring and damper to the supporting structure is :

$$f_T(t) = kx + C_e \dot{x} \quad (1)$$

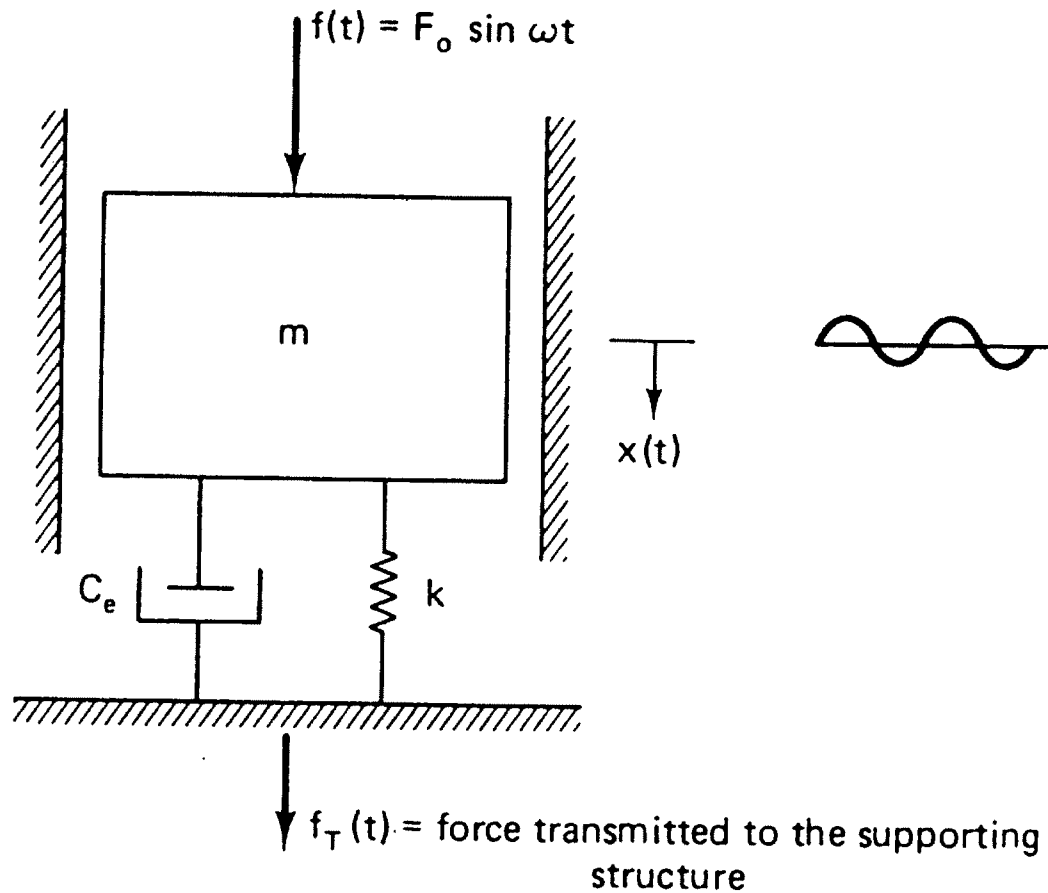


Figure 1 Force Excitation Model

The magnitude of this force as a function of frequency is :

$$\begin{aligned}
 F_T &= \sqrt{[kx(\omega)]^2 + [C_e \omega \dot{x}(\omega)]^2} \\
 &= kx \left(\omega \sqrt{1 + (C_e \omega / k)^2} \right) \quad (2)
 \end{aligned}$$

The oscillation magnitude as a function of frequency is :

$$x(\omega) = \frac{F_o}{\sqrt{(k - m\omega^2)^2 + (C_e \omega)^2}} \quad (3)$$

Substituting equation (3) into (2) :

$$T = \frac{F_T}{F_o} = \frac{\sqrt{1 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}} \quad (4)$$

T is defined as the **transmissibility** and represents the ratio of the amplitude of the force transmitted to the supporting structure to that of the exciting force.

(b) Motion excitation

The system that illustrates motion excitation is shown in Figure 2. The motion of the dynamic system is represented by the variable x and the harmonic displacement of the supporting base is represented by the variable y. The equation that describes the dynamics of the system is :

$$m\ddot{x} + C_b \left(\dot{x} - \dot{y} \right) + k(x - y) = 0 \quad (5)$$

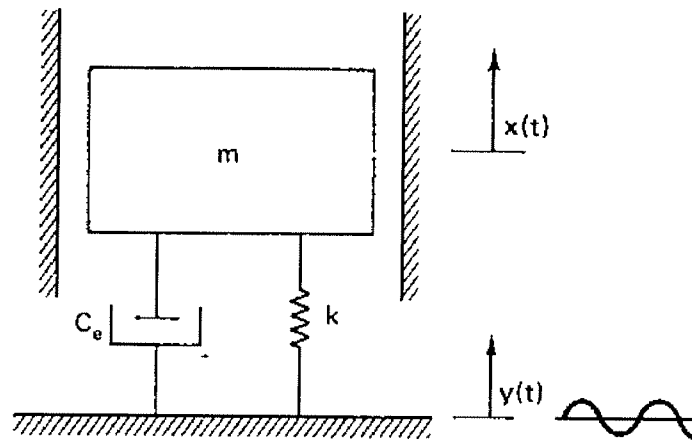


Figure 2 Motion Excitation Model

Then the ratio of the magnitudes of the displacements as a function of frequency, which is the transmissibility, is given by the expression

$$T = \frac{x}{y} = \frac{\sqrt{1 + [2\zeta(\omega/\omega_n)]^2}}{\sqrt{[1 - (\omega/\omega_n)^2]^2 + [2\zeta(\omega/\omega_n)]^2}} \quad (6)$$

Note that the transmissibility expressions for both force and motion excitation are identical. Therefore, it would appear that the engineering principles employed to protect the supporting structure under force excitation are the same as those used to protect the dynamic system from motion excitation.