18

ECONOMIC DECISION MAKING

18.1 INTRODUCTION

Throughout this book we have repeatedly emphasized that the engineer is a decision maker and that engineering design is a process of making a series of decisions over time. We also have emphasized from the beginning that engineering involves the application of science to real problems of society. In this authentic context, one cannot escape the fact that economics may play a role as big as, or bigger than, that of technical considerations in the decision making process of design. In fact, it sometimes is said, although a bit facetiously, that an engineer is a person who can do for $1.00 what any fool can do for $2.00.

The major engineering infrastructure that built this nation—the railroads, major dams, and waterways—required a methodology for predicting costs and balancing them against alternative courses of action. In an engineering project, costs and revenues will occur at various points of time in the future. The methodology for handling this class of problems is known as engineering economy or engineering economic analysis. Familiarity with the concepts and approach of engineering economy generally is considered to be part of the standard engineering toolkit. Indeed, an examination on the fundamentals of engineering economy is required for professional engineering registration in all disciplines in all states.

The chief concept in engineering economy is that *money has a time value*. Paying out $1.00 today is more costly than paying out $1.00 a year from now. A dollar invested today is worth a dollar plus interest a year from now. Engineering economy recognizes the fact that the *use of money* is a valuable asset. Money can be rented in the same way one can rent an apartment, but the charge for using it is called interest rather than rent. This time value of money makes it more profitable to push expenses into the future and bring revenues into the present as much as possible.

Before proceeding into the mathematics of engineering economy, it is important to understand where engineering economy sits with regard to related disciplines like economics and accounting. Economics generally deals with broader and more global issues than engineering economy, such as the forces that control the money supply and trade between nations. Engineering economy uses the interest rate established by the economic forces to solve more specific and detailed problems. However, it usually is a problem concerning alternative costs in the future. The accountant is more concerned with determining exactly, and often in great detail, what costs have been incurred in the past. One might say that the economist is an oracle, the engineering economist is a fortune teller, and the accountant is a historian.

18.2
MATHEMATICS OF TIME VALUE OF MONEY

If we borrow a present sum of money or principal P at a simple interest rate i, the annual cost of interest is $I = Pi$. If the loan is repaid in a lump sum F at the end of n years, the amount required is

$$F = P + nI = P + nPi = P(1 + ni) \tag{18.1}$$

where F = future worth
P = present worth
I = annual cost of interest
i = annual interest rate
n = number of years

If we borrow \$1000 for 6 years at 10 percent simple interest rate, we must repay at the end of 6 years:

$$F = P(1 + ni) = \$1000[1 + 6(0.10)] = \$1600$$

Therefore, we see that \$1000 available today is not equivalent to \$1000 available in 6 years. Actually, \$1000 in hand today is worth \$1600 available in only 6 years at 10 percent simple interest.

We can also see that the *present worth* of \$1600 available in 6 years and invested at 10 percent is \$1000.

$$P = \frac{F}{1 + ni} = \frac{\$1600}{1 + 0.6} = \$1000$$

In making this calculation we have discounted the future sum back to the present time. In engineering economy the term *discounted* refers to bringing dollar values *back in time* to the present.

18.2.1 Compound Interest

However, you are aware from your personal banking experiences that financial transactions usually use compound interest. In *compound interest,* the interest due at the end of a period is not paid out but is instead added to the principal. During the next period, interest is paid on the total sum.

$$\begin{aligned} &\text{First period:} && F_1 = P + P_i = P(1+i) \\ &\text{Second period:} && F_2 = P(1+i) + iP(1+i) = [P(1+i)](1+i) = P(1+i)^2 \\ &\text{Third period:} && F_3 = P(1+i)^2 + iP(1+i)^2 = P\left[(1+i)^2\right](1+i) = P(1+i)^3 \\ &n\text{th period:} && F_n = P(1+i)^n \end{aligned} \tag{18.2}$$

We can write Eq. (18.2) in a short notation that is convenient to use when the engineering economy relationships become more complex.

$$F_n = P(1+i)^n = P(F/P, i, n) \tag{18.3}$$

In Eq. (18.3) the function ($F/P, i, n$) has the meaning: Find the equivalent amount F given the amount P compounded at an interest rate i for n interest periods.

EXAMPLE 18.1 How long will it take money to double if it is compounded annually at a rate of 10 percent per year?

$$F = P(F/P, 10, n) \text{ but } F = 2P, \text{ (we want to find the doubling time)}$$
$$2P = P(F/P, 10, n)$$

Therefore, the answer clearly is found in a table of single-payment compound-amount factors at the year n for which $F_{PS} = 2.0$. Examining the table in Appendix 2 we see that, for $n = 7$, $F_{PS} = 1.949$ and, for $n = 8$, $F_{PS} = 2.144$. Linear extrapolation gives us $F_{PS} = 2.000$ at $n = 7.2$ years. We can generalize the result to establish the financial rule of thumb that the number of years to double an investment is 72 divided by the interest rate (expressed as an integer).

Usually in engineering economy, n is given in years and i is an annual interest rate. However, in banking circles the interest may be compounded at periods other than one year. Compounding at the end of shorter periods, such as daily, raises the effective interest rate. If we define r as the nominal annual interest rate and p as the number of interest periods per year, then the interest rate per interest period is $i = r/p$ and the number of interest periods in n years is pn. Using this notation, Eq. (18.2) becomes

$$F = P\left[\left(1 + \frac{r}{p}\right)^p\right]^n \tag{18.4}$$

Note that when $p = 1$, the above expression reduces to Eq. (18.2). Standard compound interest tables that are prepared for $p = 1$ can be used for other than annual periods. To do so, use the table for $i = r/p$ and for a number of years equal to $p \times n$. Alternatively, use the interest table corresponding to n years and an effective rate of yearly return equal to $(1 + r/p)^p - 1$.

TABLE 18.1
Influence of Compounding Period on Effective Rate of Return

Frequency of Compounding	No. Annual Interest Periods p	Interest Rate for Period, %	Effective Rate of Yearly Return, %
Annual	1	12.0	12.0
Semiannual	2	6.0	12.4
Quarterly	4	3.0	12.6
Monthly	12	1.0	12.7
Continuously	∞	0	12.75

If the number of interest periods per year p increases without limit, then $i = r/p$ approaches zero.

$$F = P \lim_{p \to \infty} \left(1 + \frac{r}{p}\right)^{pn} \tag{18.5}$$

From calculus, an important limit is $\lim_{x \to 0} (1 + x)^{1/x} = 2.7178 = e$. If we let $x = r/p$, then

$$pn = \frac{p}{r} rn = \frac{1}{x} rn$$

Since $p = r/x$, as $p \to \infty$, $x \to 0$, so Eq. (18.5) is rewritten as

$$F = P\left[\lim_{x \to \infty} (1 + x)^{1/x}\right]^{rn} = Pe^{rn} \tag{18.6}$$

Table 18.1 shows the influence of the number of interest periods per year on the effective rate of return.

18.2.2 Cash Flow Diagram

Engineering economy was developed to deal with financial transactions taking place at various times in the future. This can be best understood in terms of *cash flows.* Some of these will be cash inflows (receipts), like revenue from sale of products, reduction in operating cost, sale of used machinery, or tax savings. Others will be cash outflows (disbursements), such as the costs incurred in designing a product, the operating costs in making the product, and the periodic maintenance costs in keeping the factory running. The net cash flow is given by

$$\text{Net cash flow} = \text{cash inflows (receipts)} - \text{cash outflows (disbursements)} \tag{18.7}$$

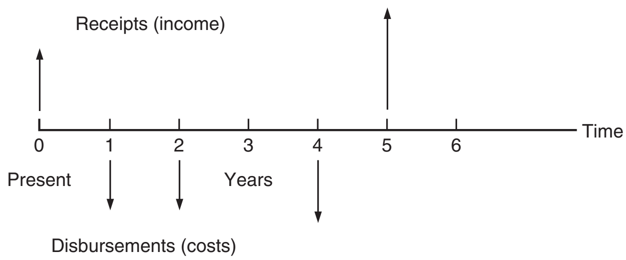


FIGURE 18.1
Cash flow diagram.

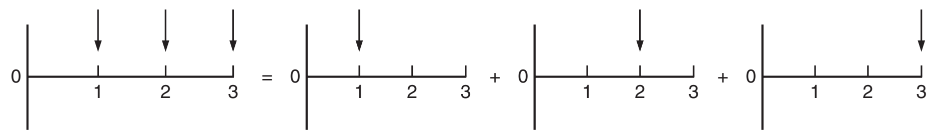


FIGURE 18.2
Equivalence of a uniform annual series.

Cash flows occur frequently and take place at varying times within the time period of the problem. In the cash flow diagram, Fig. 18.1, the horizontal axis represents time and the vertical axis is cash flow. Cash inflows are positive and are represented by arrows above the x-axis. Cash outflows are negative and are below the x-axis. It has been mentioned that engineering economy is chiefly concerned with assisting decision making about future financial decisions in an engineering project. Since future prediction of cash flows is likely to be imprecise, it is not worth carefully locating each cash flow on the diagram in time. Instead, the *end-of-period convention* is used in which the cash flows within a period are assumed to occur at the end or the interest period.

18.2.3 Uniform Annual Series

In many situations we are concerned with a uniform series of receipts or disbursements occurring equally at the end of each period. Examples are the payment of a debt on the installment plan, setting aside a sum that will be available at a future date for replacement of equipment, and a retirement annuity that consists of a series of equal payments instead of a lump sum payment. We will let A be the equal end-of-the-period payment that makes up the uniform annual series.

Figure 18.2 shows that if an annual sum A is invested at the end of each year for 3 years, the total sum F at the end of 3 years will be the sum of the compound amount of the individual investments A

$$F = A(1+i)^2 + A(1+i) + A$$

and for the general case of n years,

$$F = A(1+i)^{n-1} + A(1+i)^{n-2} + \cdots + A(1+i)^2 + A(1+i) + A \qquad (18.8)$$

Multiplying by $1 + i$, we get

$$F(1+i) = A(1+i)^n + A(1+i)^{n-1} + \cdots + A(1+i)^3 + A(1+i)^2 + A(1+i) \tag{18.9}$$

Subtracting Eq. (18.8) from Eq. (18.9):

$$\begin{aligned} (1+i)F &= A\left[(1+i)^n + \cancel{(1+i)^{n-1}} + \cdots + \cancel{(1+i)^3} + \cancel{(1+i)^2} + \cancel{(1+i)}\right] \\ F &= A\left[\cancel{(1+i)^{n-1}} + \cancel{(1+i)^{n-2}} + \cdots + \cancel{(1+i)^2} + \cancel{(1+i)} + 1\right] \\ F &= A\left[(1+i)^n - 1\right] \\ F &= A\frac{(1+i)^n - 1}{i} \end{aligned} \tag{18.10}$$

Equation (18.10) gives the future sum of n uniform payments of A when the interest rate is i. This equation may also be written:

$$F_n = A(F/A, i, n) \tag{18.11}$$

where F/A, i, n is the uniform-series compound amount factor that converts a series A to a future worth F.

By solving Eq. (18.10) for A, we have the uniform series of end-of-period payments, that, at compound interest i, provide a future sum F.

$$A = F\frac{i}{(1+i)^n - 1} \tag{18.12}$$

This type of calculation often is used to set aside money in a sinking fund to provide funds for replacing worn-out equipment, or for investing money to send a child to college.

$$A = F(A/F, i, n) \tag{18.13}$$

where (A/F, i, n) is the sinking fund factor. It sets up a future fund F by investing A each interest period n at a rate i.

By combining Eq. (18.2) with Eq. (18.10), we develop the relation for the present worth of a uniform series of payments A:

$$P = A\frac{(1+i)^n - 1}{i(1+i)^n} = A(P/A, i, n) \tag{18.14}$$

Solving Eq. (18.14) for A gives the important relation for capital recovery:

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P(A/P, i, n) \tag{18.15}$$

where (A/P, i, m) is the capital recovery factor. The A in Eq. (18.15) is the annual payment needed to return the initial capital investment P *plus* interest on that investment at a rate i over n years.

Capital recovery is an important concept in engineering economy. It is important to understand the difference between capital recovery and sinking fund. Consider the following example:

EXAMPLE 18.2 What annual investment must be made at 10 percent to provide funds for replacing a $10,000 machine in 20 years?

$$A = F(A/F, 10, 20) = \$10{,}000(0.01746) = \$174.60 \text{ per year put into the sinking fund}$$

What is the annual cost of capital recovery of $10,000 at 10 percent over 20 years?

$$A = P(A/P, 10, 20) = \$10{,}000(0.11746) = \$1174.60 \text{ per year for capital recovery}$$

We see that $(A/P, i, n) = (A/F, i, n) + i$

$$0.11746 = 0.01746 + 0.10000$$

Annual cost of capital recovery = annual cost of sinking fund + annual interest cost

$$\$1174.60 \quad = \quad \$174.60 \quad + 0.10(\$10{,}000)$$

With a sinking fund we put away each year a sum of money that, over n years, together with accumulated compound interest, equals the required future amount F. With capital recovery we put away enough money each year to provide for replacement in n years *plus we charge ourselves interest on the invested capital.* The use of capital recovery is a conservative but valid economic strategy. The amount of money invested in capital equipment ($10,000 in Example 18.2) represents an *opportunity cost,* since we are forgoing the revenue that the $10,000 could provide if invested in interest-bearing securities.

A summary of the compound interest relationships among F, P, and A is given in Table 18.2

Table 18.2 gives relationships for a uniform series of payments or receipts. Two other series often used in engineering economy are a gradient series in which the cash flow increases (or decreases) by a fixed increment at each time period, and a geometric series in which the cash flow changes by a fixed percentage at each time period.[1]

Using symbolic notation, as shown in Table 18.2, simplifies writing the equations and aids in making calculations. For example, many compound interest tables do not contain a table for determining A (sinking fund factor) when F is known. However, using the symbolic factors this can be obtained by simply multiplying factors.

$$A = F(A/F) = F(P/F)(A/P) \tag{18.16}$$

1. L. T. Blank and A. J. Tarquin, *Engineering Economy* 6th ed., McGraw-Hill, New York, 2004.

TABLE 18.2
Summary of Compound Interest Factors

Item	Conversion	Algebraic Relation	Factor	Factor Name
1	*P* to *F*	$F = P(1+i)^n$	$(F/P,i,n)$	Single payment, compound amount factor
2	*F* to *P*	$P = F(1+i)^n$	$(P/F,i,n)$	Single payment, present worth factor
3	*A* to *P*	$P = A\dfrac{(1+i)^n - 1}{i(1+i)^n}$	$(P/A,i,n)$	Uniform payment, present worth factor
4	*P* to *A*	$A = P\dfrac{i(1+i)^n}{(1+i)^n - 1}$	$(A/P,i,n)$	Capital recovery factor
5	*A* to *F*	$F = A\dfrac{(1+i)^n - 1}{i}$	$(F/A,i,n)$	Uniform series, compound amount factor
6	*F* to *A*	$A = F\dfrac{i}{(1+i)^n - 1}$	$(A/F,i,n)$	Sinking fund factor

18.2.4 Irregular Cash Flows

Payment at the Beginning of the Interest Period

In working with a uniform series of payments of receipts, A, it is conventional practice to assume that A occurs at the end of each period. However, sometimes a series of payments begins immediately so that the payments are made at the beginning of each time period, A_b.

As Fig. 18.3 shows, this is equivalent to increasing each annual payment by the interest earned in one period of the accumulation of interest. Thus, Eq. (18.10) would be written as

$$F = A_b(1+i)\left[\frac{(1+i)^n - 1}{i}\right] \tag{18.17}$$

Payments in Alternate Years

Figure 18.4 shows uniform payments in alternate years. One approach would be to consider this as three future payments and determine P as follows:

$$P = 100(P/F,\ 10,\ 2) + 100(P/F,\ 10,\ 4) + P(P/F,\ 10,\ 6) = 82.64 + 68.30 + 56.45 = \$207.39$$

An alternative approach is to consider the first annual payment to be a future payment over two years and determine the annual payment (sinking fund factor) to produce

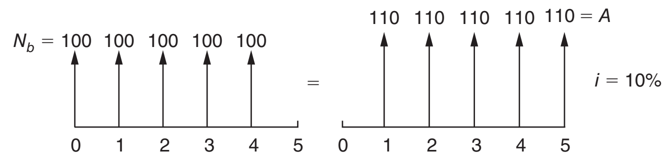


FIGURE 18.3
A uniform series paid at the beginning of the interest period, and the equivalent series paid at the end of the period.

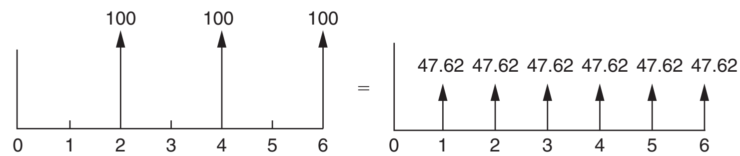


FIGURE 18.4
Conversion of payments every two years into annual payments.

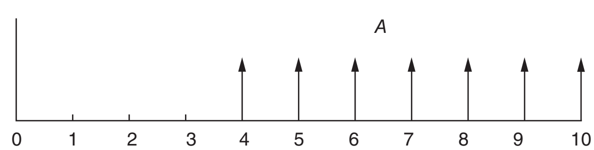


FIGURE 18.5
Finding present values of a uniform series that does not extend to time zero.

\$100. This would then be an annual payment paid over six years, since the payments are at the end of every two years, for six years total. $A = 100\ (A/F, 10, 2) = 100(0.4762) = \$47.62$)

$$P = 47.62(P/A, 10, 6) = 47.62(4.3553) = \$207.39$$

Uniform Payments Not Extending to Time Zero

Consider the uniform payments. A extending from years 4 to 10, Fig. 18.5. To find the present value, $P = A(P/A\ i, 7)$. This present value is located at the end of year 3, because the compound interest equations for the P/A factor assume that P will be determined one interest period prior to the first A in the series. Then to find the present value at time zero, P_3 must be discounted to the present. $P = F(P/F, i, 3)$ where $F = P_3$.

18.3
COST COMPARISON

Having discussed the usual compound interest relations, we now are in a position to use them to make economic decisions. A typical decision is which of two courses of action is less expensive when the time value of money is considered. Generally the rate of interest to be used in these calculations is set by the *minimum attractive rate of return,* MARR. This is the lowest rate of return a company will accept for investing its money. The MARR is established by the corporate finance officer based on current market opportunities for investing money or on the importance of the project to advancing the company.

18.3.1 Present Worth Analysis

When the two alternatives have a common time period, a comparison on the basis of present worth is advantageous.

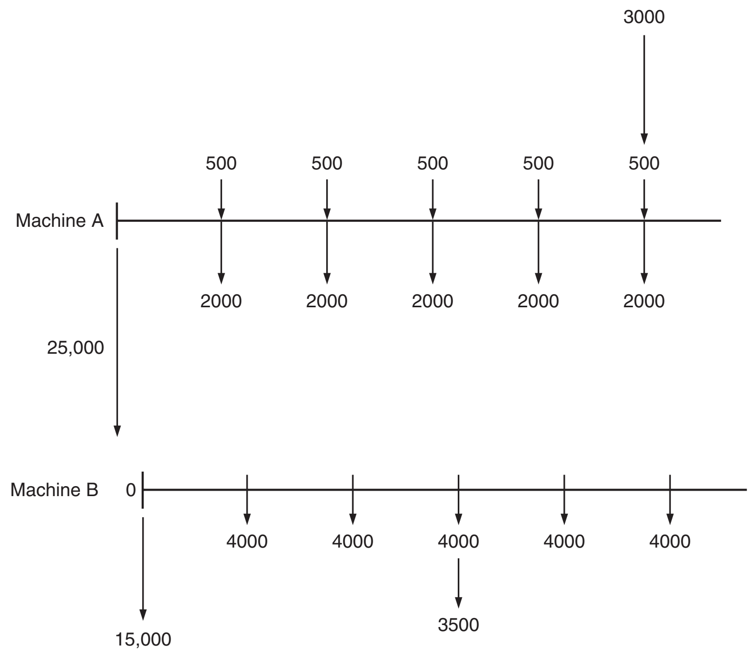
EXAMPLE 18.3 Two machines each have a useful life of 5 years. If money is worth 10 percent, which machine is more economical?

	A	B
Initial cost	\$25,000	\$15,000
Yearly maintenance cost	2,000	4,000
Rebuilding at end of third year	—	3,500
Salvage value	3,000	
Annual benefit from better quality production	500	

From the *cost* diagrams given on the next page we see that the cash flows definitely are different for the two alternatives. To place them on a common basis for comparison, we discount all costs back to the present time.

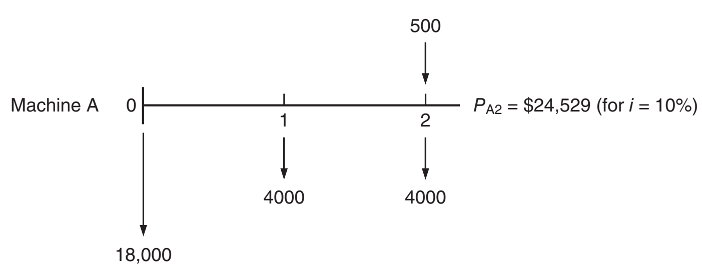
$$
\begin{aligned}
P_A &= 25{,}000 + (2000 - 500)(P/A, 10, 5) - 3000(P/F, 10, 5) \\
&= 25{,}000 + 1500(3.791) - 3000(0.621) = \$28{,}823 \\
P_B &= 15{,}000 + 4000(P/A, 10, 5)_5 + 3500(P/F, 10, 3) \\
&= 15{,}000 + 4000(3.791) + 3500(0.751) = \$32{,}793
\end{aligned}
$$

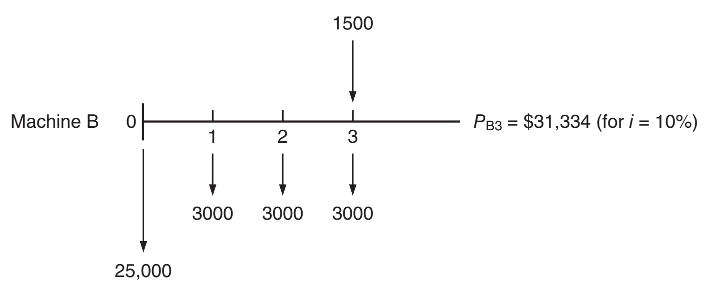
Machine A is more economical because it has the lower cost on a present worth basis. In this example we considered both (1) costs plus benefits (savings) due to reduced scrap rate and (2) resale value at the end of the period of useful life. Thus, we really determined



the *net present worth* for each alternative. We should also point out that present worth analysis is not limited to the comparison of only two alternatives. We could consider any number of alternatives and select the one with the smallest net present worth of costs.

In Example 18.3, both alternatives had the same life. Thus, the time period was the same and the present worth could be determined without ambiguity. Suppose we want to use present worth analysis for the following situation:





We cannot directly compare P_A and P_B because they are based on different time periods. One way to handle the problem would be to use a common 6-year period, in which we would replace machine A three times and replace machine B twice. This procedure works when a common multiple of the individual periods can be found easily, but a more direct approach is to convert the present worth based on a period n_1 to an equivalent P based on n_2 by[2]

$$P_{n_2} = P_{n_1} \frac{(A/P, i, n_1)}{(A/P, i, n_2)} \tag{18.18}$$

For our example, we convert P_B from a 3-year time period to a 2-year period.

$$P_{B_2} = P_{B_1} \frac{(A/P, i, n_1)}{(A/P, i, n_2)} = 31{,}334 \frac{(A/P, 10, 3)}{(A/P, 10, 2)} = 31{,}334 \left(\frac{0.40211}{0.57619} \right) = \$21{,}867$$

Since $P_A = \$24{,}529$, machine B is the more economical when compared on the basis of present worth *for equal time periods.*

18.3.2 Annual Cost Analysis

In the annual cost method, the cash flow over time is converted to an equivalent uniform annual cost or benefit. In this method no special procedures need be used if the time period is different for each alternative, because all comparisons are on an annual basis ($n = 1$).

Example	Machine A	Machine B
First cost	\$10,000	\$18,000
Estimated life	20 years	35 years
Estimated salvage	0	\$3000
Annual cost of operation	\$4000	\$3000

$$A_A = 10{,}000(A/P, 10, 20) + 4000 = 10{,}000(0.1175) + 4000 = \$5175$$
$$A_B = (18{,}000 - 3000)(A/P, 10, 35)_5 + 3000(0.10) + 3000 = \$4855$$

Machine B has the lower annual cost and is the more economical. Note that in calculating the annual cost of capital recovery for machine B we used the difference between the first cost and the salvage value; for it is only this amount of money that must be recovered. However, although the salvage value is returned to us, we are required to wait until the end of the useful life of the machine to recover it. Therefore,

2. For a derivation of Eq. (18.18), see F. C. Jelen and J. H. Black, *Cost and Optimization Engineering,* 2d ed., p. 28, McGraw-Hill, New York, 1983.

a charge for the annual cost of the interest on the investment tied up in the salvage value is made as part of the annual cost analysis.

Perhaps a more direct way to handle the case of machine B in the preceding example is to determine the equivalent annual cost based on the cash disbursements minus the annual benefit of the future resale value.

$$\begin{aligned} A_B &= 18{,}000(A/P, 10, 35)_{35} + 3000 - 3000(A/F, 10, 35)_{35} \\ &= 18{,}000(0.1037) + 3000 - 3000(0.0037) = \$4855 \end{aligned}$$

18.3.3 Capitalized Cost Analysis

Capitalized cost is a special case of present worth analysis. The capitalized cost of a project is the present value of providing for that project in perpetuity ($n = \infty$). The concept was originally developed for use with public works, such as dams and waterworks, that have long lives and provide services that must be maintained indefinitely. Capitalized cost subsequently has been used more broadly in economic decision making because it provides a method that is independent of the time period of the various alternatives.

We can develop the mathematics for capitalized cost quite simply from Eq. (18.18). If we let $n_2 = \infty$ and $n_1 = n$, then

$$P_\infty = P_n \frac{(A/P, i, n)}{(A/P, i, \infty)}$$

$$(A/P, i, n) = \frac{i(1+i)^n}{(1+i)^n - 1} \qquad (A/P, i, \infty) = \frac{i(1+i)^\infty}{(1+i)^\infty - 1} = i$$

Therefore, the capitalized cost K of a present sum P is given by

$$K = P_\infty = P\frac{(1+i)^n}{(1+i)^n - 1} = P(K/P, i, n) \tag{18.19}$$

Since most tables of compound interest factors do not include capitalized cost, we need to note that

$$(K/P, i, n) = (A/P, i, n)/i \tag{18.20}$$

In addition, the capitalized cost of an annual payment A is determined as follows:

$$P = A(P/A, i, n) \qquad K = P(K/P, i, n) = P\frac{(A/P, i, n)}{i} \text{ from Eq. } (18.20).$$

Substituting for P, dropping the notation for i and n: $K = A(P/A)\dfrac{(A/P)}{i} = \dfrac{A}{i}$ (18.21)

EXAMPLE 18.4 The capitalized cost is the present worth of providing for a capital cost in perpetuity; that is, we assume there will be an infinite number of renewals of the initial capital investment. Consider a bank of condenser tubes that cost \$10,000 and have an average life of 6 years. If $i = 10$ percent, then the capitalized cost is

$$K = (K/P, i, n) = P\frac{(A/P, i, n)}{i} = 10{,}000\frac{0.2296}{0.10} = \$22{,}960$$

We note that the excess over the first cost is 22,960 − 10,000 = \$12,960. If we invest that amount for the 6-year life of the tubes,

$$F = P(F/P, 10, 6) = 12{,}960(1.772) = \$22{,}960$$

Thus, when the tubes need to be replaced, we have generated \$22,960. We take \$10,000 to purchase a new set of tubes (we are neglecting inflation) and invest the difference (22,960 − 10,000 = 12,960) at 10 percent for 6 years to generate another \$22,960. We can repeat this process indefinitely. The capital cost is provided for in perpetuity.

EXAMPLE 18.5 Compare the continuous process and the batch process on the basis of capitalized cost analysis if $i = 10$ percent.

Solution

	Continuous Process	Batch Process
First cost	\$20,000	\$6000
Useful life	10 years	15 years
Salvage value	0	\$500
Annual power costs	\$1000	\$500
Annual labor costs	\$600	\$4300

Continuous process:

$$K = 20{,}000\frac{(A, P, 6, 10)}{0.10} + \frac{1000 + 600}{0.10}$$

$$K = 20{,}000\frac{0.1627}{0.10} + \frac{1600}{0.10} = \$48{,}540$$

Batch process:

$$K = 6000\frac{0.1315}{0.10} - 500\frac{1}{(1 + 0.10)^{15}}\frac{0.1315}{0.10} + \frac{4800}{0.10}$$

$$= 7890 - 500(0.2394)(1.315) + 48{,}000 = \$55{,}733$$

Note that the \$500 salvage value is a negative cost occurring in the fifteenth year. We bring this to the present value and then multiply by $(K/P, 10, 10) = (A/P, 10, 10)/.10$.

TABLE 18.3
Useful Excel Functions for Compound Interest Calculations

Function	Description
FV(*i*, *n*, *A*, PV, type)	Calculates future value, FV, given int. rate per period, no. of periods, constant payment amount, *A*, present value PV, type = 0 end of period payment; type = 1, beginning of period payment.
PV(*i*, *n*, *A*, FV, type)	Calculates present value PV, given *i*, *n*, periodic payments (−) or income (+) and future single payments or receipts.
NPV(*i*, Incl, Inc2 . . .)	Calculates net present value, NPV, of a series of irregular future incomes (+) or expenses (−) at periodic interest *i*.
PMT(*i*, *n*, PV, FV, type)	Calculates uniform payments *A* based on either a present value and/or a future value.
RATE(*n*, *A*, PV, FV, type, *g*)	Calculates interest rate per period. *g* requires a guess for *i*, about 10%
NOMINAL(effect *i*, npery)	Calculates the nominal annual interest rate given the effective rate and number of compounding periods per year, npery
EFFECT(non *i*, npery)	Calculates the effective interest rate given the nominal interest rate and npery.

Each of the three methods of cost comparison will give the same result when applied to the same problem. The best method to use depends chiefly on whom you need to convince with your analysis and which technique you feel they will be more comfortable with.

18.3.4 Using Excel Functions for Engineering Economy Calculation

The compound interest factors needed for engineering economy calculations can be determined on a calculator or looked up in the tables in all engineering economy textbooks.[3] Microsoft Excel provides an extensive menu of time value of money functions and other financial functions. When combined with the computational features of Excel and its "what if" capability, this makes an excellent general-purpose tool for engineering economic decision making. Table 18.3 gives a brief description of the most common functions for compound interest calculations. For details on using the functions, see the help pages in Excel or engineering economy texts.[4]

18.4
DEPRECIATION

Capital equipment suffers a loss in value over time. This may occur by corrosion or wear, deterioration, or obsolescence, which is a loss of economic efficiency because of technological advances. Therefore, a company should lay aside enough money each

3. Several tables of *F, P,* and *A* and their combination are given in the Appendix to this chapter.
4. L. T. Blank and A. J. Tarquin, op. cit, Appendix A.

year to accumulate a fund to replace the obsolete or worn-out equipment. This allowance for loss of value is called depreciation. Depreciation is an accounting expense on the income statement of the company. It is a non cash expense that is deducted from gross profits as a cost of doing business. In a capital-intensive business, depreciation can have a strong influence on the amount of taxes that must be paid.

$$\text{Taxable income} = \text{total income} - \text{allowable expenses} - \text{depreciation}$$

The basic questions to be answered about depreciation are: (1) what is the time period over which depreciation can be taken, and (2) how should the total depreciation charge be spread over the life of the asset? Obviously, the depreciation charge in any given year will be greater if the depreciation period is short (a rapid write-off).

The Economic Recovery Act of 1981 introduced the *accelerated cost recovery system* (ACRS) as the prime capital-recovery method in the United States. This was modified in the 1986 Tax Reform Act for Modified Accelerated Cost Recovery System (MACRS). The statute sets depreciation recovery periods based on the expected useful life. Some examples are:

- Special manufacturing devices; some motor vehicles 3 years
- Computers; trucks; semiconductor manufacturing equipment 5 years
- Office furniture; railroad track; agricultural buildings 7 years
- Durable-goods manufacturing equipment; petroleum refining 10 years
- Sewage treatment plants; telephone systems 15 years

Residential rental property is recovered in 27.5 years and nonresidential rental property in 31.5 years. Land is a nondepreciable asset, since it is never used up.

We shall consider four methods of spreading the depreciation over the recovery period n: (1) straight-line depreciation, (2) declining balance, (3) sum-of-the-years digits, and (4) the MACRS procedure. Only MACRS and the straight-line method currently are acceptable under the U.S. tax laws, but the other methods are useful in classical engineering economic analyses.

18.4.1 Straight-Line Depreciation

In straight-line depreciation an equal amount of money is set aside yearly. The annual depreciation charge D is

$$D = \frac{\text{initial cost} - \text{salvage value}}{n} = \frac{C_i - C_s}{n} \tag{18.22}$$

The *book value* is the initial cost minus the sum of the depreciation charges that have been made. For straight-line depreciation, the book value B at the end of the jth year is

$$B_j = C_i - \frac{j}{n}\left(C_i - C_s\right) \tag{18.23}$$

18.4.2 Declining-Balance Depreciation

The declining-balance method provides an accelerated write-off in the early years. The depreciation charge for the jth year D_j is a fixed fraction F_{DB} of the book value at the beginning of the jth year (or the end of year $j - 1$). For the book value to equal the salvage value after n years,

$$F_{DB} = 1 - \sqrt[n]{\frac{C_s}{C_i}} \tag{18.24}$$

and the book value at the beginning of the jth year is

$$B_{j-1} = C_i \left(1 - F_{DB}\right)^{j-1} \tag{18.25}$$

Therefore, the depreciation in the jth year is

$$D_j = B_{j-1} F_{DB} = C_i \left(1 - F_{DB}\right)^{j-1} F_{DB} \tag{18.26}$$

The most rapid write-off occurs for double declining-balance depreciation. In this case $F_{DDB} = 2/n$ and $B_{j-1} = C_i(1-2/n)^{j-1}$. Then

$$D_j = C_i \left(1 - \frac{2}{n}\right)^{j-1} \frac{2}{n}$$

Since the DDB depreciation may not reduce the book value to the salvage value at year n, it may be necessary to switch to straight-line depreciation in later years.

18.4.3 Sum-of-Years-Digits Depreciation

The sum-of-years-digits (SOYD) depreciation is an accelerated method. The annual depreciation charge is computed by adding up all of the integers from 1 to n and then taking a fraction of that each year, $F_{SOYD,j}$.

For example, if $n = 5$, then the sum of the years is $(1 + 2 + 3 + 4 + 5 = 15)$ and $F_{SOYD,2} = 4/15$, while $F_{SOYD,4} = 2/15$. The denominator is the sum of the digits; the numerator is the digit corresponding to the jth year when the digits are arranged in *reverse order.*

18.4.4 Modified Accelerated Cost Recovery System (MACRS)

In MACRS the annual depreciation is computed using the relation

$$D = qC_i \tag{18.27}$$

where q is the recovery rate obtained from Table 18.4 and C_i is the initial cost. In MACRS the value of the asset is completely depreciated even though there may be

TABLE 18.4
Recovery Rates q Used in MACRS Method

	Recovery Rate, q, %				
Year	$n = 3$	$n = 5$	$n = 7$	$n = 10$	$n = 15$
1	33.3	20.0	14.3	10.0	5.0
2	44.5	32.0	24.5	18.0	9.5
3	14.8	19.2	17.5	14.4	8.6
4	7.4	11.5	12.5	11.5	7.7
5		11.5	8.9	9.2	6.9
6		5.8	8.9	7.4	6.2
7			8.9	6.6	5.9
8			4.5	6.6	5.9
9				6.5	5.9
10				6.5	5.9
11				3.3	5.9
12–15					5.9
16					3.0

n = recovery period, years.

TABLE 18.5
Comparison of Depreciation Methods

	C_i = \$6000, C_s = \$1000, n = 5			
Year	**Straight Line**	**Declining Balance**	**Sum-of-Years-Digits**	**MACRS**
1	1000	1807	1667	1200
2	1000	1263	1333	1920
3	1000	882	1000	1152
4	1000	616	667	690
5	1000	431	333	690
6	—	—	—	348

a true salvage value. The recovery rates are based on starting out with a declining-balance method and switching to the straight-line method when it offers a faster write-off. MACRS uses a half-year convention that assumes that all property is placed in service at the midpoint of the initial year. Thus, only 50 percent of the first year depreciation applies for tax purposes, and a half year of depreciation must be taken in year $n + 1$.

Table 18.5 compares the annual depreciation charges for these four methods of calculation.

Microsoft Excel offers several functions for calculating depreciation: SLN (straight-line depreciation), DB (declining balance), DDB (double-declining balance), and SYD (sum-of-year-digits).

18.5
TAXES

Taxes are an important factor to be considered in engineering economic decisions. The chief types of taxes that are imposed on a business firm are:

1. *Property taxes:* Based on the value of the property owned by the corporation (land, buildings, equipment, inventory). These taxes do not vary with profits and usually are not too large.
2. *Sales taxes:* Imposed on sales of products. Sales taxes usually are paid by the retail purchaser, so they generally are not relevant to engineering economy studies of a business.
3. *Excise taxes:* Imposed on the manufacture of certain products like gasoline, tobacco, and alcohol. Also usually passed on to the consumer.
4. *Income taxes:* Imposed on corporate profits or personal income. Gains resulting from the sale of capital property also are subject to income tax.

Generally, federal income taxes have the most significant impact on engineering economic decisions. Although we cannot delve into the complexities of tax laws, it is important to incorporate the broad aspects of income taxes into our analysis.

The income tax rates are strongly influenced by politics and economic conditions. Currently the United States has a corporate *graduated tax schedule* as follows:

Taxable Income	Tax Rate
\$1–\$50,000	0.15
\$50,001–\$75,000	0.25
\$75,001–\$100,000	0.34
\$100,001–\$335,000	0.39
\$335,001–\$10 M	0.34
\$10M–\$15M	0.35
\$15M–\$18.3M	0.38
Over \$18.3 M	0.35

Most states and some cities and counties also have an income tax. For simplicity in economic studies a single effective tax rate is often used. This commonly varies from 35 to 50 percent. Since state taxes are deductible from federal taxes, the effective tax rate is given by

$$\text{Effective tax rate} = \text{state rate} + (1 - \text{state rate})(\text{federal rate}) \tag{18.28}$$

The chief effect of corporate income taxes is to reduce the rate of return on a project or venture.

$$\text{After-tax rate of return} = \text{before-tax rate of return} \times (1 - \text{income tax rate})$$
$$r = i(1 - t) \tag{18.29}$$

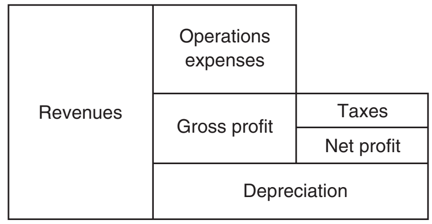


FIGURE 18.6
Distribution of corporate revenues.

Note that this relation is true only when there are no depreciable assets. For the usual case when we have depreciation, capital gains or losses, or investment tax credits, Eq. (18.29) is a rough approximation. The importance of depreciation in reducing taxes is shown in Fig. 18.6. The depreciation charge appreciably reduces the gross profit, and thereby the taxes. However, since depreciation is retained in the corporation, it is available for growing the enterprise.

EXAMPLE 18.6 High-Tech Pumps has a gross income in 1 year of \$15 million. Operating expenses (salaries and wages, materials, etc.) are \$10 million. Depreciation is \$2.6 million. Also, this year there is a *depreciation recapture* of \$800,000 because a specialized CNC machine tool that is no longer needed is sold for more than its book value. (*a*) Compute the company's federal income taxes. (*b*) What is the average federal tax rate? (*c*) If the state tax rate is 11 percent, what is the total income taxes paid?

(*a*) Taxable income (TI) = gross income − operating expenses − depreciation + depreciation recapture

$$\begin{aligned} \text{TI} &= 15 - 10 - 2.6 + 0.8 = \$3.2\text{M} \\ \text{Taxes} &= (\text{TI range})(\text{marginal rate}) \\ &= (50{,}000)0.15 + (25{,}000)0.25 + (25{,}000)0.34 \\ &\quad + (235{,}000)0.39 + (3.2\text{M} - 0.335\text{M})0.34 \\ &= 7500 + 6250 + 8500 + 91{,}650 + 974{,}100 = \$1{,}088{,}000 \end{aligned}$$

(*b*) Average federal tax rate $= \dfrac{1{,}088{,}00}{3{,}200{,}00} = 0.34$

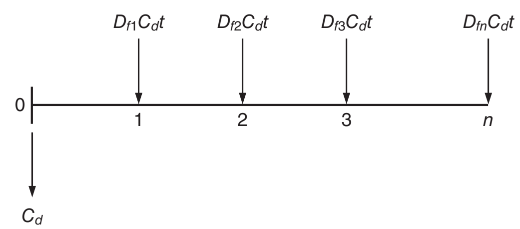
(*c*) From Eq. (18.28)

$$\text{Effective tax rate } = 0.11 + (1 - 0.11)(0.34) = 0.11 + 0.3026 = 0.4126$$

$$\text{Total income taxes } = 32{,}2000{,}000(0.4126) = \$1{,}320{,}320$$

Note that including state taxes makes a differences.

Consider a depreciable capital investment $C_d = C_i - C_s$. At the end of each year depreciation amounting to $D_f C_d$ is available to reduce the taxes by an amount $D_f C_d t$.



Note that the fractional depreciation charge each year D_f may vary from year to year depending on the method used to establish the depreciation schedule. See, for example, Table 18.5. The present value of this series of costs and benefits is

$$P = C_d - C_d t \left[\frac{D_{f_1}}{1+r} + \frac{D_{f_2}}{(1+r)^2} + \frac{D_{f_3}}{(1+r)^3} + \cdots + \frac{D_{f_n}}{(1+r)^n} \right] \tag{18.30}$$

The exact evaluation of the term in brackets will depend on the depreciation method selected.

EXAMPLE 18.7 A manufacturing company of modest size is considering an investment in energy-efficient electric motors to reduce its large annual energy cost. The initial cost would be \$12,000, and over a 10-year period it is estimated that the firm would save \$2200 annually in electricity costs. The salvage value of the motors is estimated at \$2000. Determine the after-tax rate of return.

Solution

First we will establish the before-tax rate of return. We need to determine the cash flow for each year. Cash flow, in this context, is the net profit or savings for each year. We shall use straight-line depreciation to determine the depreciation charge. Table 18.6 shows the cash flow results. The before-tax rate of return is the interest rate at which the before-tax cash flow savings just equals the purchase cost of the motors.

$$12{,}000 = 2200(P/A, i, 10) + 2000(P/F, i, 10)$$

We find the rate of return by trying different values of i in the compound interest tables. For $i = 14$ percent,

$$\begin{aligned} 12{,}000 &= 2200(5.2161) + 2000(0.2697) \\ &= 11{,}475 + 539 = 12{,}014 \end{aligned}$$

Therefore, the before-tax rate of return is very slightly more than 14 percent. To find the after-tax rate of return, we use the after-tax cash flow in Table 18.6. From Eq. (18.29) we estimate the after-tax rate of return to be 7 percent.

TABLE 18.6
Cash Flow Calculations for Example 18.7

Year	Before-Tax Cash Flow	Depreciation	Taxable Income	50% Income Tax	After-Tax Cash Flow
0	−12,000				−12,000
1 to 9	2,200	1000	1200	−600	1,600
10	2,200	1000	1200	−600	1,600
	2,000				2,000

$$12{,}000 = 1600(P/F, i, 10) + 2000(P/F, i, 10)$$

For $i = 6\%$:

$$12{,}000 = 1600(7.3601) + 2000(0.5584)$$
$$= 11{,}776 + 1117 = 12{,}893 \quad i \text{ too low}$$

For $i = 8\%$:

$$12{,}000 = 1600(6.7101) + 2000(0.4632)$$
$$= 10{,}736 + 926 = 11{,}662 \quad i \text{ too high}$$
$$i = 6\% + 2\% \frac{12{,}893 - 12{,}000}{12{,}893 - 11{,}662} = 6\% + 2\%(0.72)$$
$$i = 6 + 1.44 = 7.44\%$$

For tax purposes the expenditures that a business incurs are divided into two broad categories. Those for facilities and production equipment with lives in excess of one year are called capital expenditures; they are said to be "capitalized" in the accounting records of the business. Other expenses for running the business, such as labor and material costs, direct and indirect costs, and facilities and equipment with a life of one year or less, are ordinary business expenses. Usually they total more than the capital expenses. In the accounting records, they are said to be "expensed." The ordinary expenses are directly subtracted from the gross income to determine the taxable income, but only the annual depreciation charge can be subtracted from the capitalized expenses.

When a capital asset is sold, a capital gain or loss is established by subtracting the book value of the asset from its selling price. Frequently in our modern history, capital gains have received special treatment by being taxed at a rate lower than for ordinary income.

Investment in capital is a vital step in the innovation process that leads to increased national wealth. Therefore, the federal government frequently uses the tax system to stimulate capital investment. This most often takes the form of a tax credit, usually 7 percent but varying with time from 4 to 10 percent. This means that 7 percent of the purchase price of qualifying equipment can be deducted from the taxes that the firm owes the U.S. government. Moreover, the depreciation charge for the equipment is based on its full cost.

18.6 PROFITABILITY OF INVESTMENTS

One of the principal uses for engineering economy is to determine the profitability of proposed projects or investments. The decision to invest in a project generally is based on three different sets of criteria.

Profitability: Determined by techniques of engineering economy to be discussed in this section. Profitability is an analysis that estimates how rewarding in monetary terms an investment will be.

Financial analysis: How to obtain the necessary funds and what it will cost. Funds for investment come from three broad sources: (1) retained earnings of the corporation, (2) long-term commercial borrowing from banks, insurance companies, and pension funds, and (3) the equity market through the sale of stock.

Analysis of intangibles: Legal, political, or social consideration or issues of a corporate image often outweigh financial considerations in deciding on which project to pursue. For example, a corporation may decide to invest in the modernization of an old plant because of its responsibility to continue employment for its employees when investment in a new plant 1000 miles away would be economically more attractive.

However, in our free-enterprise system a major goal of a business firm is to maximize profit. It does so by committing its funds to ventures that appear to be profitable. If investors do not receive a sufficiently attractive profit, they will find other uses for their money, and the growth—even the survival—of the firm will be threatened.

Four methods of evaluating profitability are commonly used. Accounting rate of return and payback period are simple techniques that are readily understood, but they do not take time value of money into consideration. Net present value and discounted cash flow are the most common profitability measures in which time value of money is considered. Before discussing them, however, we need to look a bit more closely at the concept of cash flow.

Cash flow measures the flow of funds into or out of a project. Funds flowing in constitute positive cash flow; funds flowing out are negative cash flow. The cash flow for a typical plant construction project is shown in Fig. 18.7. From an accounting point of view, cash flow is defined as

$$\text{Cash flow} = \text{net annual cash income} + \text{depreciation}$$

You might consider cash income as "real dollars" and the depreciation an accounting adjustment to allow for capital expenditures. Table 18.7 shows how cash flow can be determined in a simple situation.

18.6.1 Rate of Return

The rate of return on the investment (ROI) is the simplest measure of profitability. It is calculated from a strict accounting point of view without consideration of the time

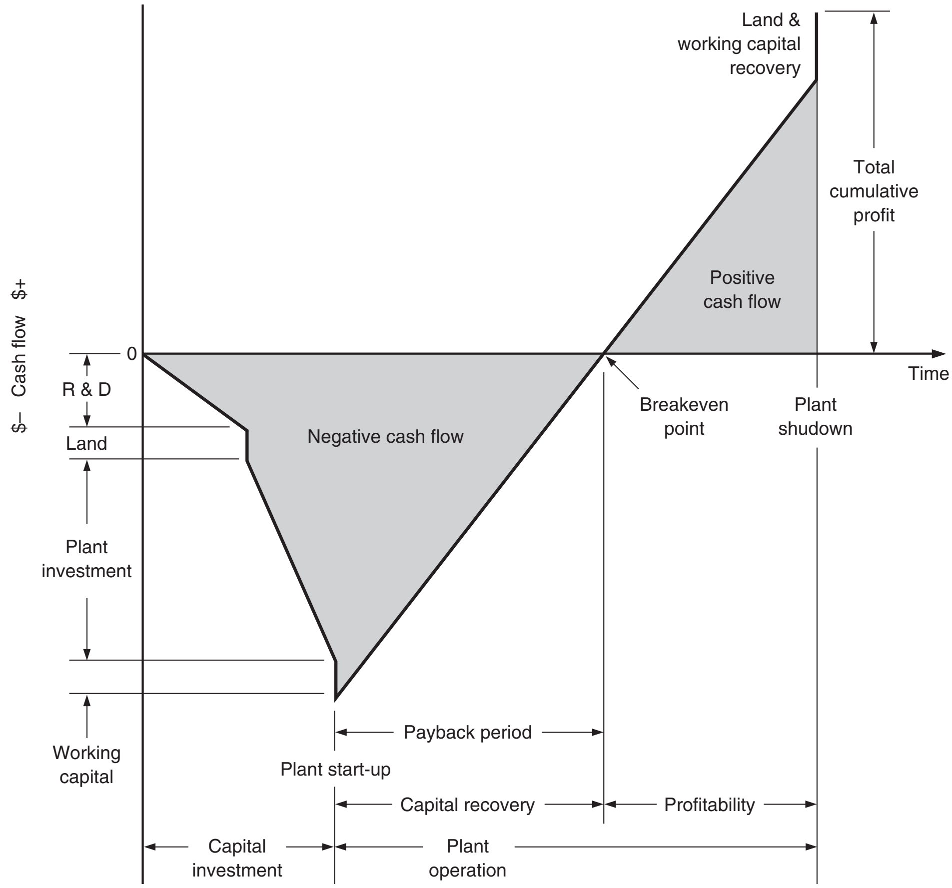


FIGURE 18.7
Typical costs in the cycle of a plant investment.

TABLE 18.7
Calculation of Cash Flow

(1) Revenue (over 1-year period)	$500,000
(2) Operating costs	360,000
(3) (1) − (2) = gross earnings	140,000
(4) Annual depreciation charge	60,000
(5) (3) − (4) = taxable income	80,000
(6) (5) × 0.35 = income tax	28,000
(7) (5) − (6) = net profit after taxes	52,000
Net cash flow (after taxes) (7) + (4) = 52,000 + 60,000	112,000

value of money. It is a simple ratio of some measure of profit or cash income to the capital investment. There are a number of ways to assess the rate of return on the capital investment. ROI may be based on (1) net annual profit before taxes, (2) net annual profit after taxes, (3) annual cash income before taxes, or (4) annual cash income after taxes. These ratios, usually expressed as percents, can be computed for each year or on the average profit or income over the life of the project. In addition, capital investment sometimes is expressed as the average investment. Thus, although the ROI is a simple concept, it is important in any given situation to understand clearly how it has been determined.

EXAMPLE 18.8 An initial capital investment is \$360,000 and has a 6-year life and a \$60,000 salvage value. Working capital is \$40,000. Total net profit after taxes over 6 years is estimated at \$167,000. Find the ROI.

Solution

$$\text{Annual net profit} = \frac{167{,}000}{6} = \$28{,}000$$

$$\text{ROI on initial capital investment} = \frac{28{,}000}{360{,}000 + 40{,}000} = 0.07$$

18.6.2 Payback Period

The payback period is the period of time necessary for the cash flow to fully recover the initial total capital investment (Fig. 18.4). Although the payback method uses cash flow, it does not include a consideration of the time value of money. Emphasis is on rapid recovery of the investment. Also, in using the method, no account is taken of cash flows or profits recovered after the payback period. Consider Table 18.8.

By the payback period criterion, project A is more desirable because it recovers the initial capital investment in 3 years. However, project B, which returns a cumulative cash flow of \$110,000, obviously is more profitable overall.

18.6.3 Net Present Worth

In Sec. 18.3, as one of the techniques of cost comparison, we introduced the criterion of net present worth (NPW).

$$\text{Net present worth} = \text{present worth of benefits} - \text{present worth of costs}$$

By this technique the expected cash flows (both + and −) through the life of the project are discounted to time zero at an interest rate representing the minimum acceptable return on capital, MARR. The project with the greatest positive value of NPW is preferred. NPW depends upon the project life, so strictly speaking the net present worths of two projects should not be compared if the projects have different service lives.

TABLE 18.8
Payback Period Example

	Cash Flow	
Year	**Project A**	**Project B**
0	$ −100,000	$ −100,000
1	50,000	0
2	30,000	10,000
3	20,000	20,000
4	10,000	30,000
5	0	40,000
6	0	50,000
7	0	60,000
	$10,000	$110,000
Payback period	3 years	5 years

Obviously, the value of NPW will be dependent upon the interest rate used for the calculation. Low interest rates will tend to make NPW more positive, for a given set of cash flows, and large values of interest will push NPW in a negative direction. There will be some value of i for which the sum of the discounted cash flows equals zero; NPW = 0. This value of i is called the internal rate of return, IRR.

18.6.4 Internal Rate of Return

In the beginning of this chapter we considered calculation methods that determined what sum of money at the present time, when invested at a given interest rate, *is equivalent* to a larger sum at a future time. Now with the internal rate of return, we find what interest rate makes the present sum and the future sum *equivalent.* This value of interest rate is called the *internal rate of return,* IRR. This is the rate of return for which the net present value equals zero. PW of benefits – PW of costs = 0.

If, for example, the internal rate of return is 20 percent, it implies that 20 percent per year will be earned on the investment in the project, in addition to which the project will generate sufficient funds to repay the original investment. Depreciation is considered implicitly in NPW and IRR calculations through the definition of cash flow.

Because the decision on profitability is expressed as a percentage rate of return in the IRR method, it is more readily understood and accepted by engineers and business people than the NPW method, which produces a sum of money as an answer. In the NPW method it is necessary to select an interest rate for use in the calculations, and that may be a difficult and controversial thing to do. But by using the IRR method, we compute a rate of return, called the internal rate of return, from the cash flows. One situation in which NPW has an advantage is that individual values of NPW for a series of subprojects may be added to give the NPW for the complete project. That cannot be done with the rate of return developed from IRR analysis.

EXAMPLE 18.9 A machine has a first cost of $10,000 and a salvage value of $2000 after a 5-year life. Annual benefits (savings) from its use are $5000, and the annual cost of operation is $1800. The tax rate is 50 percent. Find the IRR rate of return.

Solution

Using straight-line depreciation, the annual depreciation charge is

$$D = \frac{C_i - C_s}{n} = \frac{10{,}000 - 2{,}000}{5} = \$1600$$

The annual cash flow after taxes is the sum of the net receipts and depreciation.

$$\begin{aligned}(CF)_a &= (5000 - 1800)(1 - 0.50) + 1600(0.50) \\ &= 1600 + 800 = \$2400\end{aligned}$$

Year	Cash Now
0	−10,000
1	2,400
2	2,400
3	2,400
4	2,400
5	2,400 + 2,000 (C_s)

$$\text{NPW} = 0 = -10{,}000 + 2400(P/A, i, 5) + 2000(P/F, i, 5)$$

If i = 10 percent, NPW = +340; if i = 12 percent, NPW = −214. Thus, we have the IRR bracketed, and

$$\begin{aligned}i &= 10\% + (12\% - 10\%)\frac{340}{340 + 214} \\ &= 10 + 2\left(\frac{340}{564}\right) = 10 + 1.2 = 11.2\%\end{aligned}$$

The IRR function in Microsoft Excel can be used to quickly determine the internal rate of return. To use IRR, the net benefits or costs(-) are entered in a column of cells, one for each period. Enter a 0 for any period where there is no cash flow. Finally, enter a guess as the starting point for the calculation. For example: =IRR(A2:A8,5)

It is an important rule of engineering economy that *each increment* of investment capital must be justified on the basis of earning the minimum required rate of return.

EXAMPLE 18.10 A company has the option of investing in one of the two machines described in the following table. Which investment is justified?

	Machine A	Machine B
Initial cost C_i	\$10,000	\$15,000
Useful life	5 years	10 years
Salvage value C_s	\$2,000	0
Annual benefits	\$5,000	\$7,000
Annual costs	\$1,800	\$4,300

Solution

Assume a 50 percent tax rate and a minimum attractive rate of return of 6 percent. The conditions for machine A are identical with those in Example 18.9, for which $i = 11.2$ percent. Calculation of the IRR for machine B shows it is slightly in excess of the minimum rate of 6 percent. However, that is not the proper question. Rather, we should ask whether the *increment of investment* (\$15,000 − \$10,000) is justified. In addition, because machine B has twice the useful life of machine A, we should place them both on the same time basis (see Table 18.9).

TABLE 18.9
Cash Flow, Example 18.10

Year	Machine A	Machine B	Difference, B − A
0	−10,000	−15,000	−5,000
1	2,400	2,100	−300
2	2,400	2,100	−300
3	2,400	2,100	−300
4	2,400	2,100	−300
5	2,400 − 10,000 + 2,000	2,100	−300 + 8,000
6	2,400	2,100	−300
7	2,400	2,100	−300
8	2,400	2,100	−300
9	2,400	2,100	−300
10	2,400 + 2,000	2,100	−300 − 2,000

$$\text{NPW} = 0 = -5000 - 300(P/A, i, 10) + 8000(P/F, i, 5) - 2000(P/F, i, 10)$$

But, even at $i = \frac{1}{4}$ percent, NPW = −2009, and there is no way that the extra investment in machine B can be justified economically.

When only costs—not income (or savings)—are known, we can still use the IRR method for incremental investments, but not for a single project. We assume that the lowest capital investment is justified without being able to determine the internal rate of return, and we then determine whether the additional investment is justified.

EXAMPLE 18.11 On the basis of the data in the following table, determine which machine should be purchased.

	Machine A	Machine B
First cost	$3000	$4000
Useful life	6 years	9 years
Salvage value	$500	0
Annual operating cost	$2000	$1600

Solution

This solution will be based on cash flow before taxes. To place the machines on a common time frame, we use a common life of 18 years.

TABLE 18.10
Cash Flow, Example 18.11

Year	Machine A	Machine B	Difference, B − A
0	−3000	−4000	−1000
1 to 5	−2000	−1600	+400
6	−2000 − 2500	−1600	+400 + 2500
7 to 8	−2000	−1600	+400
9	−2000	−1600 − 4000	+400 − 4000
10, 11	−2000	−1600	+400
12	−2000 − 2500	−1600	+400 + 2500
13 to 17	−2000	−1600	+400
18	−2000 + 500	−1600	+400 − 500

$$\text{NPW} = 0 = -1000 + 400(P/A, i, 18) + 2500(P/F, i, 6) + 2500(P/F, i, 12)$$
$$- 400(P/F, i, 9) - 500(P/F, i, 18)$$

Trial and error shows that $i \approx 47$ percent, which clearly justifies purchase of machine B.

We have presented information on the four most common techniques for evaluating the profitability of an investment. The rate-of-return method has the advantage of being simple and easy to use. However, it ignores the time value of money and the consideration of cash flow. The payback period also is a simple method, and it is particularly attractive for industries undergoing rapid technological change. Like the rate-of-return method, it ignores the time value of money, and it places an undue emphasis on projects that achieve a quick payoff. The net present worth method takes both cash flow and time value of money into account. However, it suffers from the problem of ambiguity in setting the required rate of return, and it may present problems when projects with different service lives are compared. Internal rate of return has the advantage of producing an answer that is the real internal rate of return. The method readily permits comparison between alternatives, but it is assumed that all cash flows generated by the project can be reinvested to yield a comparable rate of return.

18.7 OTHER ASPECTS OF PROFITABILITY

Innumerable factors[5] affect the profitability of a project in addition to the mathematical expressions discussed in Sec. 18.6. The purpose of this section is to round out our consideration of the crucial subject of profitability.

We need to realize that profit and profitability are not quite the same concept. Profit is measured by accountants, and its value in any one year can be manipulated in many ways. Profitability is inherently a long-term parameter of economic decision making. As such, it should not be influenced much by short-term variations in profits. In recent years there has been a strong trend toward undue emphasis on quick profits and short payoff periods that work to the detriment of long-term investment in high-technology projects.

Estimation of profitability requires the prediction of future cash flows, which in turn requires reliable estimates of sales volume and sales price by the marketing staff and of material price and availability. The quadrupling of crude oil price in 2005 was a dramatic example of how changes in raw material costs can greatly influence profitability predictions. Similarly, trends in operating costs must be looked at carefully, especially with respect to whether it is more profitable to reduce operating costs through increased investment, as with automation.

The estimated investment in machinery and facilities that is required for the project is usually the most accurate component of the profitability evaluation. (This topic is considered in more detail in Chap. 16.) The depreciation method used influences how the expense is distributed over the years of a project, and that in turn determines what the cash flow will be. However, a more fundamental aspect of depreciation is the effect of writing off a capital investment over a long time period. As a result, costs are underestimated and selling prices are set too low. A long-term write-off combined with inflation results in insufficient cash flow to permit reinvestment. Inflation creates hidden expenses like inadequate allowance for depreciation. When depreciation methods do not allow for inflated replacement costs, those costs must be absorbed on an after-tax basis. Profit and profitability are overstated in an inflationary period.

A number of technical decisions are closely related to the investment policy and profitability. At the design stage it may be possible to ensure a level of product superiority that is more than that needed by the current market. Later, when competitors enter the market, the superiority would prove useful, but it is not achieved without an initial cost to profitability. Economics generally favor building as large a production unit as the market can absorb. However, this increased profitability is achieved at some risk to maintaining continuity of production should the unit be down for repairs. Thus, there often is a trade-off between the increased reliability of having a number of small units over which to spread the risk and a single large unit with somewhat higher profitability.

The profitability of a particular product line can be influenced by decisions of cost allocation. Such factors as overhead, utility costs, transfer prices between divisions of a large corporation, or scrap value often require arbitrary decisions for allocation

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5. F. C. Jelen, *Hydrocarbon Processing,* pp. 111–15, January 1976.

between various products. Thus, the situation often favors certain products and discriminates against others because of cost allocation policies. Sometimes corporations take a position of milking an established product line with a limited future (a "cash cow") in order to stimulate the growth of a new and promising product line. Another profit decision is whether to charge a particular item as a current expense or capitalize it to make a future expense. In a period of inflation there is strong pressure to increase present profitability by deferring costs into the future by capitalizing them. It is argued that a fixed dollar amount deferred into the future will have less consequence in terms of future dollars.

The role of the government in influencing profitability is very great. In the broader sense the government creates the general economic climate through its policies on money supply, taxation, and foreign affairs. It provides subsidies to stimulate selected parts of the economy. Its regulatory powers have had an increasing influence on profitability in such areas as pollution control, occupational health and safety, consumer protection and product safety, use of federal lands, antitrust, minimum wages, and working hours.

Since profitability analysis deals with future predictions, there is inevitable uncertainty. Incorporation of uncertainty or risk is possible with advanced techniques. Unfortunately, the assignment of the probability to deal with risk is in itself a subjective decision. Thus, although risk analysis (Sec. 18.10) is an important technique, one should always realize its true origins.

18.8 INFLATION

Since engineering economy deals with decisions based on future flows of money, it is important to consider inflation in the total analysis. From 1984 to 2006 the average inflation in consumer price has been 4.1 percent. However, there have been years (1974, 1979, and 1980) when the rate of inflation was in double digits.[6]

Inflation exists when prices of goods and services are increasing so that a given amount of money buys less and less as time goes by. Interest rates and inflation are directly related. The basic interest rate is about 2 to 3 percent higher than the inflation rate. Thus, in a period of high inflation, not only does the dollar purchase less each month but the cost of borrowing money also rises.

Price changes may or may not be considered in an economic analysis. For meaningful results, costs and benefits must be computed in comparable units. It would not be sensible to calculate costs in 1990 dollars and benefits in 2000 dollars.

Inflation is measured by the change in the Consumer Price Index (CPI), as determined by the U.S. Department of Labor, Bureau of Labor Statistics.[7] The CPI is reported monthly, based on a survey of the price of a "market basket" of goods and services purchased by consumers. In 1984 the CPI was re-centered at 100, and items for price volatile areas such as food and energy were removed, to create the Core CPI.

6. http://inflationdata.com/inflation/Consumer_Price_Historical CPI
7. www.bls.gov/cpi/cpifaq.htm

The CPI in 2007 was 250, compared with 100 in 1984. This means that it would take \$250 in 2007 dollars to purchase the same goods and services as in 1984. For design purposes the CPI is less important than the Producer Price Index (PPI), which is discussed in Sec. 16.9.1.

EXAMPLE 18.12 The CPI in 1984 was 100.0, and in 2007 it was 250. 1 Find the rate of inflation over this period of 23 years.

$$F = P(F/P, i, n) \qquad 250.1 = 100(F/P, i, 23) \qquad (F/P, i, 23) = 2.501$$

and on interpolation we find $f = 0.041$.

Another way to find the inflation rate is to use the equation for *annualized return*.

$$\text{Annualized return} = \left(\frac{\text{current value}}{\text{orginal value}}\right)^{\frac{1}{n}} - 1$$
$$= \left(\frac{250.1}{100.0}\right)^{\frac{1}{23}} = 1.0406 - 1.000 = 0.041 = 4.1\% \qquad (18.31)$$

Money in one time period t_1 can be brought to the same value as money in another time period t_2 by the equation

$$\text{Dollars in period } t_1 = \frac{\text{Dollars in period } t_2}{\text{Inflation rate between } t_1 \text{ and } t_2} \qquad (18.32)$$

It is useful to define two situations: then-current money and constant-value money. Let the dollars in period t_1 be constant-value dollars. Constant-value represents equal purchasing power at any future time. Current money, in time period t_2, represents ordinary money units that decline in purchasing power with time. For example, if an item cost \$10 in 1998 and inflation was 3 percent during the previous year, in constant 1997 dollars, the cost is equal to \$10/1.03 = \$9.71.

There are three different rates to be considered when dealing with inflation.

Ordinary or inflation-free interest rate i: This is the rate at which interest is earned when effects of inflation have been ignored. This is the interest rate we have used up until now in this chapter.

Market interest rate i_f: This is the interest rate that is quoted on the business news every day. It is a combination of the real interest rate i and the inflation rate f. This is also called the *inflated interest rate*.

Inflation rate f: This is a measure of the rate of change in the value of the currency.

Consider the equation for the present worth of a future sum F in current dollars. F must be first discounted for the real interest rate and then for the inflation rate.

$$P \frac{F}{(1+i)^n} \frac{1}{(1+f)^n} = F \frac{1}{(1+i+f+if)^n} = \frac{F}{(1+i_f)^n} \qquad (18.33)$$

where $i_f = i + f + if$ is the market interest rate, also called the inflated interest rate.

EXAMPLE 18.13 A project requires an investment of $10,000 and is expected to return, in future, or "then current," dollars, $2500 at the end of year 1, $3000 at the end of year 2, and $7000 at the end of year 3. The monetary (ordinary) interest rate is 10 percent, and the inflation rate is 6 percent per year. Find the net present worth of this investment opportunity.

Solution

The inflated interest rate is 0.10 + 0.06 + (0.10)(0.06) = 0.166; for simplicity we shall use $i_f = 0.17$.

Current-dollars approach

Year	Cash Flow	(P/F, 17, n)	Present Worth
0	−10,000	1.00	−10,000
1	2,500	0.8547	2,137
2	3,000	0.7305	2,191
3	7,000	0.6244	4,971
		NPW =	−711

Constant-Value-Dollars Approach

Year	Cash Flow*	(P/F,10,n)	Present Worth
0	−10,000	1.00	−10,000
1	2,358	0.9091	2,144
2	2,670	0.8264	2,206
3	5,877	0.7513	4,415
		NPW =	−1,235

*Adjusted for $f = 0.06\ (1.06)^n$.

In the current-dollars approach the inflated interest rate is used to discount the cash flows to the present time. For the constant-dollars approach the cash flow is adjusted by [constant (real) $] = [current (actual) $] $(1+f)^{-n}$.

The difference in the NPWs found by the two treatments is due to using an approximate combined discount rate instead of the more accurate value of $i_f = 0.166$. However, the approximation is justified in view of the uncertainty in predicting the rate of inflation. It should be noted that, for this example, the NPW is +$10 if inflation is ignored. That emphasizes the fact that neglecting the influence of inflation overemphasizes the profitability.

When profitability is measured by the internal rate of return i, the inclusion of the inflation rate f results in an effective rate of return i' based on constant-value money.[8]

$$1+i=(1+i')(1+f)$$
$$i'=i-f-i'f\approx i-f \qquad (18.34)$$

8. F. A. Holland and F. A. Watson, *Chem. Eng.,* pp. 87–91, Feb. 14, 1977.

To a first approximation, the internal rate of return is reduced by an amount equivalent to the average inflation rate.

Interest rates are quoted to investors in current money i, but investors generally expect to cover any inflationary trends and still receive an acceptable return. In other words, investors hope to obtain a constant-value interest rate i'. Therefore, the current interest rate tends to fluctuate with the inflation rate. If the calculation is to be made with constant-value money, discounting should be done with the normal interest rate i. If calculations are in terms of current money, then the discount rate should be $i_f = i + f$.

Note that tax allowance for depreciation has a reduced benefit when constant money is used for profitability evaluations. By law, depreciation is defined in terms of current money. Therefore, under high inflation when constant-money conditions are appropriate, a full tax credit for depreciation is not achieved.

Another effect of inflation is that it increases the cash flow because the prices received for goods and services rise as the value of money falls. Even when constant-value money is used, the yearly cash flows should display the current money situation.

Detailed relations for capitalized cost that include an inflation factor have been developed by Jelen.[9] They can also be used to introduce inflation calculation into annual cost calculations.

18.9 SENSITIVITY AND BREAK-EVEN ANALYSIS

A *sensitivity analysis* determines the influence of each factor on the final result, and therefore it determines which factors are most critical in the economic decision. Since there is a considerable degree of uncertainty in predicting future events like sales volume, salvage value, and rate of inflation, it is important to see how much the economic analysis depends on the magnitude of the estimates. One factor is varied over a reasonable range and the others are held at their mean (expected) value. The amount of computation involved in a sensitivity analysis of an engineering economy problem can be considerable, but the use of computers has made sensitivity analysis a much more practical endeavor.

A *break-even analysis* often is used when there is particular uncertainty about one of the factors in an economic study. The break-even point is the value for the factor at which the project is just marginally justified.

EXAMPLE 18.14 Consider a $20,000 investment with a 5-year life. The salvage value is $4000, and the minimal acceptable return is 8 percent. The investment produces annual benefits of $10,000 at an operating cost of $3000. Suppose there is considerable uncertainty as to whether the new machinery will survive 5 years of continuous use. Find the break-even point, in terms of life, at which the project just becomes economically viable.

9. F.C. Jelen, *Chem. Eng.,* pp. 123–128, Jan. 27, 1958.

Solution

Using the annual cost method,

$$\$10{,}000 - 3000 - (20{,}000 - 4000)(A/P, 8, n) - 4000(0.08) = 0$$

$$(A/P, 8, n) = \frac{6680}{16{,}000} = 0.417$$

and interpolating in the interest tables gives us $n = 2.8$ years. Thus, if the machine does not last 2.8 years, the investment cannot be justified.

Break-even analysis frequently is used in problems dealing with staged construction. The usual problem is to decide whether to invest more money initially in unused capacity or to add the needed capacity at a later date when needed, but at higher unit costs.

EXAMPLE 18.14 A new plant will cost $100 million for the first stage and $120 million for the second stage at *n* years in the future. If it is built to full capacity now, it will cost $140 million. All facilities are expected to last 40 years. Salvage value is neglected. Find the preferable course of action.

Solution

The annual cost of operation and maintenance is assumed the same for a two-stage construction and full-capacity construction. We shall use a present worth (PW) calculation with a 10 percent interest rate. For full-capacity construction now, PW = $140 million ($140M). For two-stage construction

$$\text{PW} = \$100\text{M} + \$120\text{M}(P/F, 10, n)$$

$$n = 5 \text{ years: PW} = 100 + 120(0.6201) = \$174\text{M}$$

$$n = 10 \text{ years: PW} = 100 + 120(0.3855) = \$146\text{M}$$

$$n = 20 \text{ years: PW} = 100 + 120(0.1486) = \$118\text{M}$$

$$n = 30 \text{ years: PW} = 100 + 120(0.0573) = \$107\text{M}$$

These results are plotted in Fig. 18.8. The break-even point (12 years) is the point at which the two alternatives have equivalent cost. If the full capacity will be needed before 12 years, then full capacity built now would be the preferred course of action.

18.10 UNCERTAINTY IN ECONOMIC ANALYSIS

In the preceding sections we discussed the fact that engineering economy deals chiefly with decisions based on future estimates of costs and benefits. Since none of us has a completely clear crystal ball, such estimates are likely to contain considerable uncertainty. In all of the examples presented so far in this chapter we have used a single value that was the implied best estimate of the future.

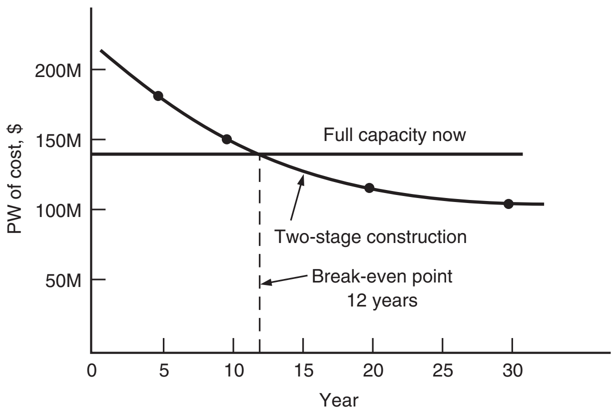

FIGURE 18.8
Break-even plot for Example 18.15.

Now that we are willing to recognize that estimates of the future may not be very precise, there are some ways by which we can guard against the imprecision. The simplest procedure is to supplement your estimated most likely value with an optimistic value and a pessimistic value. The three estimates are combined into a mean value by

$$\text{Mean value} = \frac{\text{optimistic value} + 4\,(\text{most likely value}) + \text{pessimistic value}}{6} \tag{18.35}$$

In Eq. (18.35) the distribution of values is assumed to be represented by a beta frequency distribution. The mean value determined from the equation is used in the economic analysis.

The next level of advance would be to associate a probability with certain factors in the economic analysis. In a sense, by this approach we are transferring the uncertainty from the value itself to the selection of the probability.

EXAMPLE 18.16 The expected life of a piece of mining equipment is highly uncertain. The machine costs \$40,000 and is expected to have \$5000 salvage value. The new machinery will save \$10,000 per year, but it will cost \$3000 annually for operations and maintenance. The service life is estimated to be:

3 years, with probability = 0.3
4 years, with probability = 0.4
5 years, with probability = 0.5

Solution

For 3-year life:

Net annual cost $= (10{,}000 - 3000) - (40{,}000 - 5000)(A/P, 10, 3) - 5000(0.10)$
(based on capital recovery) $= 7000 - 35{,}000(0.4021) - 500 = -8573$

For 4-year life:

$$\text{Net annual cost} = 7000 - 35{,}000(0.3155) - 500 = -4542$$

For 5-year life:

$$\text{Net annual cost} = 7000 - 35{,}000(0.2638) - 500 = -2733$$

Expected value of net annual cost = $E(\text{AC}) = \sum \text{AC} \times P(\text{AC}) =$

$$-8573(0.3) + [-4542(0.4)] + [-2733(0.3)] = -5207$$

18.11 BENEFIT-COST ANALYSIS

An important class of engineering decisions involves the selection of the preferred system design, material, purchased subsystem, etc., when economic resources are constrained. The methods of making cost comparisons and profitability analysis described in Secs. 18.3 and 18.6 are important decision-making tools in this type of situation.

Frequently, comparisons are based on a *benefit-cost ratio,* which relates the desired benefits to the capital investment required to produce the benefits. This method of selecting alternatives is most commonly used by governmental agencies for determining the desirability of public works projects. A project is considered viable when the net benefits associated with its implementation exceed its associated costs. *Benefits* are advantages to the public (or owner), expressed in terms of dollars. If the project involves disadvantages to the owner, these *disbenefits* must be subtracted from the benefits. The costs to be considered include the expenditures for construction, operation, and maintenance, less salvage. Both benefits, disbenefits, and costs must be expressed in common monetary terms by using the present worth or annual cost concept.

$$\text{Benefit-cost ratio (BCR)} = \frac{\text{benefits} - \text{disbenefits}}{\text{costs}} \tag{18.36}$$

A design or project for which BCR < 1 does not cover the cost of capital to create the design. Generally, only projects for which BCR > 1 are acceptable. The benefits used in the BCR would be factors like improved component performance, increased payload through reduced weight, and increased availability of equipment. Benefits are defined as the advantages minus any disadvantages, that is, the net benefits. Likewise, the costs are the total costs minus any savings. The costs should represent the initial capital cost as well as costs of operation and maintenance; see Chap. 16.

In problems of choosing between several alternatives, the incremental or marginal benefits and costs associated with changes beyond a base level or reference design should be used. The alternatives are ranked with respect to cost, and the lowest-cost situation is taken as the initial reference. This is compared with the next higher-cost alternative by computing the incremental benefit and incremental cost. If $\Delta B/\Delta C < 1$, then alternative 2 is rejected because the first alternative is superior. Alternative 1 now is compared with alternative 3. If $\Delta B/\Delta C > 1$, then alternative 1 is rejected and alternative 3 becomes the current best solution. Alternative 3 is compared with number 4, and if $\Delta B/\Delta C < 1$, then alternative 3 is the best choice. We should note that this may not be the alternative with the largest overall benefit-cost ratio.

EXAMPLE 18.17 You are asked to recommend a site for a small dam to generate hydroelectric power. The construction cost at various sites is given in the following table. These vary with topography and soil conditions. Each estimate includes $3M for the turbines and generators. The annual benefits from the sale of electricity vary between sites because of stream velocity.

We require an annual return of 10 percent. The life of the dam is infinite for purposes of calculation. The hydroelectric machinery has a 40-year life.

Site	Construction Cost, $M	Cost of Machinery, $M	Cost of Dam, $M	Annual Income, $M
A	9	3	6	1.0
B	8	3	5	0.9
C	12	3	9	1.25
D	6	3	3	0.5

Since the benefit (income) is on an annual basis, we have to convert the cost to an annual basis. Also, we are going to make our decision on an incremental basis. We construct Table 18.11, placing the alternatives in order of increasing cost (from left to right).

For example, the annual cost of capital recovery for $A = P_D i + P_{H\text{-}E}(A/p, 10, 40)$

$$\text{Annual cost of capital recovery} = 6{,}000{,}000(0.10) + 3{,}000{,}000(0.1023)$$
$$= 600{,}000 + 306{,}900 = \$907{,}000$$

We note that when compared to not building a dam, $\Delta B/\Delta C$ for site D is less than 1.0. The next-lowest-cost dam site is greater than 1.0, so it is selected in comparison to not building a dam. The benefit-cost ratio for sites A and C also is greater than 1.0, but now,

TABLE 18.11
Site Benefit-Cost Analysis, Example 18.17

	D	B	A	C
Annual cost of capital recovery ($1000)	607	807	907	1207
Annual benefits ($1000)	500	900	1000	1250
Comparison	D to do nothing	B to do nothing	A to B	C to B
Δ capital recovery	607	807	100	400
Δ annual benefits	500	900	100	350
$\Delta B/\Delta C$	0.82	1.11	1.00	0.87
Selection	Do nothing	B	B	B

having found a low-cost qualifying site (B), we need to determine whether the increment in benefits and costs is better than B. We see that on a ΔB/ΔC basis, A and C are not better choices than B. Therefore, we select site B.

When used in a strictly engineering context to aid in the selection of alternative materials, the benefit-cost ratio is a useful decision-making tool. However, it often is used with regard to public projects financed with tax monies and intended to serve the overall public good. There is a psychological advantage to the BCR concept over the internal rate of return in that it avoids the connotation that the government is profiting from public monies. Here questions that go beyond economic efficiency become part of the decision process. Many of the broader issues are difficult to quantify in monetary terms. Of even greater difficulty is the problem of relating monetary cost to the real values of society.

Consider the case of a hydroelectric facility. The dam produces electricity, but it also will provide flood control and an area for recreational boating. The value of each of the outputs should be included in the benefits. The costs include the expenditures for construction, operation, and maintenance. However, there may be social costs like the loss of virgin timberland or a scenic vista. Great controversy surrounds the assignment of costs to environmental and aesthetic issues.

Although benefit-cost analysis is a widely used methodology, it is not without problems. The assumption is that costs and benefits are relatively independent. Basically, it is a deterministic method that does not deal with uncertainty in a major way. As with most techniques, it is best not to try to push it too far. Although the quantitative ratios provided by Eq. (18.36) should be used to the greatest extent possible, they should not preempt the utilization of common sense and good judgment.

18.12 SUMMARY

Engineering economy is the methodology that promotes rational decision making about the allocation of amounts of money at various points in time and in various manners—for example, as a uniform series over time or a single payment in the future. As such, engineering economy accounts for the time value of money.

The basic relationship is the compound interest formula that relates the future sum F to the present sum P over n years at an interest rate i.

$$F = P(1+i)^n$$

If P is solved for in this equation, we are discounting the future sum F back to the present time. If the money occurs as equal end-of-the-period amounts A, then

$$F = A\frac{(1+i)^n - 1}{i}$$

If this equation is solved for A, it gives the annual payment to provide a sinking fund to replace worn-out equipment. More important is the annual payment to return the initial capital investment *plus* paying interest on the principal P tied up in the investment, where CRF is the capital recovery factor.

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1} = P(CRF)$$

Engineering economy allows rational decisions to be made about alternative courses of action involving money. To do this, each alternative must be placed on an equivalent basis. There are four common ways of doing this.

- *Present-worth analysis:* All costs or receipts are discounted to the present time to calculate the net present worth. This method works best when the alternatives have a common time period.
- *Annual cost analysis:* The cash flow over time is converted to an equivalent annual cost or benefit. This method works well when the alternatives have different time periods.
- *Capitalized cost analysis:* This is a special case of present worth analysis for a project that exists in perpetuity ($n = \infty$).
- *Benefit-cost ratio:* This method analyzes the costs and benefits of a project on one of the above three bases, and then decides to fund the project if the ratio of benefits to costs is greater than 1.0.

Realistic economic analysis requires consideration of *taxes,* chiefly federal income tax. Accurate determination of the taxable income requires allowance for *depreciation,* the reduction in value of owned assets due to wear and tear or obsolescence. Realistic economic analysis also requires allowance for *inflation,* the decrease in the value of currency over time.

An important use of engineering economy is in determining the profitability of proposed projects or investments. This usually starts with estimating the cash flow to be generated by the project.

$$\text{Cash flow} = \text{net annual cash income} + \text{depreciation}$$

Two common methods of estimating profitability are rate of *return on the investment* (ROI) and *payback period.*

$$\text{ROI} = \frac{\text{average annual net profit}}{\text{capital investment} + \text{working capital}}$$

Payback period is the period of time for the cumulative cash flow to fully recover the initial total capital investment. Both of these methods suffer from not considering the time value of money. A better method to measure profitability is *net present worth.*

$$\text{Net present worth} = \text{present worth of benefits} - \text{present worth of costs}$$

With this method the expected cash flows (both + and −) through the life of the project are discounted to time zero at an interest rate representing the minimum acceptable return on capital. The *internal rate of return* (IRR) is the interest rate for which the net present worth equals zero.

$$\text{Net PW} = \text{PW}\left(\text{benefits}\right) - \text{PW}\left(\text{costs}\right) = 0$$

Since there is considerable uncertainty in estimating future income streams and costs, engineering economic studies often estimate a range of values and utilize a mean value. Another approach is to place probabilities on the values and use an expected value in the analysis.

NEW TERMS AND CONCEPTS

Annual cost analysis	Discounting to the present	Net present worth
Benefit-cost analysis	Effective interest rate	Nominal interest rate
Capitalized cost	Future value	Payback period
Capital recovery factor	Inflation rate	Present value
Cash flow	Internal rate of return	Present worth analysis
Current dollars	MACRS	Sinking fund factor
Declining balance depreciation	Marginal incremental return	Time value of money
Depreciation	MARR	Uniform annual series

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PROBLEMS AND EXERCISES

The interest tables at the end of this chapter are available to help you solve these problems. Also, note that computer spreadsheet software provides most of the financial functions discussed in this chapter. It is recommended that you use a spreadsheet to solve the problems.

18.1. (*a*) Calculate the amount realized at the end of 7 years through annual deposits of \$1000 at 10 percent compound interest.
(*b*) What would the amount be if interest were compounded semiannually?

18.2. A young woman purchases a new car. After down payment and allowances, the amount to be paid is \$8000. If money is available at 10 percent, what is the monthly payment to pay off the loan in 4 years? What would it be at 4 percent interest?

18.3. A new machine tool costs \$15,000 and has a \$5000 salvage value at the end of 5 years. The interest rate is 10 percent. The annual cost of capital recovery is the annual depreciation charge (use straight-line depreciation) plus the equivalent annual interest charge. Work this out on a year-by-year basis and show that it equals the number obtained quickly by using the capital recovery factor.

18.4. A father desires to establish a fund for his new child's college education. He estimates that the current cost of a year of college education is \$12,000 and that the cost will escalate at an annual rate of 4 percent.
(*a*) What amount is needed on the child's eighteenth, nineteenth, twentieth, and twenty-first birthdays to provide for a 4-year college education?
(*b*) If a rich aunt gives \$5000 on the day the child is born, how much must be set aside at 10 percent on each of the first through seventeenth birthdays to build up the college fund?

18.5. A major industrialized nation manages its finances in such a way that it runs an annual trade deficit with other countries of \$100 billion. If the cost of borrowing is 10 percent, how long will it be before the debt (accumulated deficit) is one trillion dollars (\$1000B)? If nothing is done, how long will it take to accumulate the second \$1000B debt?

18.6. Machine A costs \$8500 and has annual operating costs of \$4500. Machine B costs \$7000 and has an annual operating cost of \$4800. Each machine has an economic life of 10 years. If the minimum required rate of return is 10 percent, compare the advantages of machine A by
(*a*) present worth method,
(*b*) annual cost method, and
(*c*) rate of return on investment.

18.7. Make a cost comparison between two conveyor systems for transporting raw materials.

	System A	**System B**
Installed cost	\$25,000	\$15,000
Annual operating cost	6,000	11,000

The service life of each system is 5 years and the write-off period is 5 years. Use straight-line depreciation and assume no salvage value for either system. At what rate of return *after taxes* would B be more attractive than A?

18.8. A resurfaced floor costs \$5000 and will last 2 years. If money is worth 10 percent after taxes, how long must a new floor costing \$19,000 last to be economically justified? The tax rate is 52 percent. For tax purposes a new floor can be written off in 1 year. Use sum-of-the-years-digits depreciation. Use the capitalized cost method for your analysis.

18.9. You are concerned with the purchase of a heat-treating furnace for the gas carburizing of steel parts. Furnace A will cost \$325,000 and will last 10 years; furnace B will cost \$400,000 and will also last 10 years. However, furnace B will provide closer control on case depth, which means that the heat treater can shoot for the low side of the specification range on case depth. That will mean that the production rate for furnace B will be 2740 lb/h compared with 2300 lb/h for furnace A. Total yearly production is required to be 15,400,000 lb. The cycle time for furnace A is 16.5 h, and that for furnace B is 13.8 h. The hourly operating cost is \$64.50 per h.

Justify the purchase of furnace B on the basis of

(*a*) payout time and
(*b*) discounted cash flow rate of return after taxes.
(*c*) Assume money is worth 10 percent and the tax rate is 50 percent.

18.10. The cost of capital has a strong influence on the willingness of management to invest in long-term projects. If the cost of capital in America is 10 percent and in Japan 4 percent, what must the return be after 2 years on a 2-year investment of \$1 million for each of the situations to provide an acceptable return on the investment? Repeat the analysis for a 20-year period.

18.11. In order to justify investment in a new plating facility, it is necessary to determine the present worth of the costs.

Calculate the present worth given the following information:

Cost of equipment	\$350,000
Planning period	5 years
Fixed charges	20 percent of investment each year
Variable charges	40,000 first year, escalating at 6 percent each year with inflation starting at $t = 0$
Rate of return	$i = 10\%$

18.12. Determine the net present worth of the costs for a major construction project under the following set of conditions:

(*a*) Estimated cost \$300 million over 3 years (baseline case).
(*b*) Project is delayed by 3 years with rate of inflation 10 percent and interest cost 16 percent.
(*c*) Project is delayed 6 years with rate of inflation 10 percent and interest costs 16 percent.

18.13. Whether a maintenance operation is classified as a repair (expense charged against revenues in current year) or improvement (capitalized expense) can have a big influence on taxes. Determine the net savings for a \$10,000 operation using the two different approaches if

(*a*) you are in a business that is in the 50 percent tax bracket and
(*b*) you are in a small business in the 20 percent tax bracket.

Use a 10 percent interest rate and a 10 percent investment tax credit.

18.14. As a new professional employee you need to worry about your retirement many years in the future. Construct a table showing how much you need to have invested, at 4 percent, 8 percent, and 12 percent annual rate of return, to provide each \$100 of monthly income. Assume that inflation will increase at 3 percent annually, so the numbers you calculate will be in inflation-adjusted dollars. Calculate the monthly amount needed for a retirement period of 25, 30, 35, and 40 years. Assume that the investments are made in tax-sheltered accounts.

18.15. At what annual mileage is it cheaper to provide your field representatives with cars than to pay them $0.32 per mile for the use of their own cars? The costs of furnishing a car are as follows:

Purchase price	$9000
Life	4 years
Salvage	$1500
Storage	$150 per year
Maintenance	$0.08 per mile

(*a*) Assume $i = 10$ percent.
(*b*) Assume $i = 16$ percent.

18.16. To *levelize expenditures* means to create a uniform end-of-year payment that will have the same present worth as a series of irregular end-of-year payments. To illustrate, consider the estimated 5-year maintenance budget for a pilot plant. Develop a levelized cost assuming that $i = 0.10$ and the annual inflation escalation will be 8 percent.

Year	Maintenance Budget Estimate
1	25,000
2	150,000
3	60,000
4	70,000
5	300,000

18.17. The marketing department made the following estimates about four different product designs. Use benefit-cost analysis to determine which design to pursue.

Design	Unit Manufacturing Cost	Sales Price	Est. Annual Sales
A	12.50	25.00	250,00
B	22.00	40.00	200,00
C	15.00	25.00	250,00
D	15.00	20.00	300,00

18.18. You buy 100 shares of stock in QBC Corp. at $40 per share. It is a good buy, for 4 years, 3 months later you sell these shares of stock for $114 7/8. What is the annual rate of return on this fortunate investment?