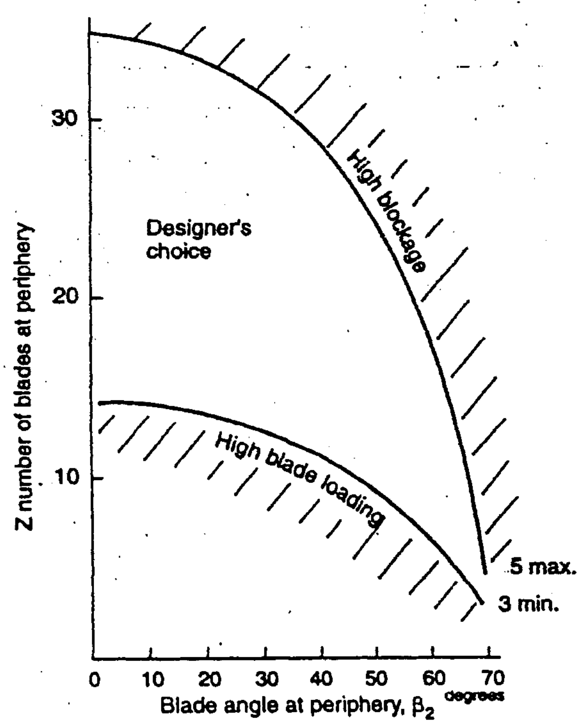
A radial flow compressor impeller is to be designed with no preswirl at the inlet, with $C_{2r} = C_1$ and with the following conditions to be met:

- the flow coefficient at the impeller exit $\Phi_2 = 0.4$
- the impeller tip velocity $U_2 = 300$ m/s
- the inlet hub to outlet diameter ratio $d_{hb,1}/d_2 = 0.3$
- the relative velocity ratio $W_2/W_{sh,2} = 0.75$
- the slip factor $\sigma = 0.78$
- the impeller outlet blade angle = 50°

The flowing medium is air which can be considered as a perfect gas with Cp=1005 J/(kg*K), $\gamma = 1.4$.

For the above conditions calculate the following:
a) the loading coefficient ψ
b) the ratio of the inlet shroud-to-hub diameter
c) the number of impeller blades corresponding to the prescribed slip factor. In reference to the figure below, is the calculated number of blades a reasonable choice?



We can add further design specification to the problem; namely:

- the impeller outlet speed Mach number = 0.7
- the inlet total pressure P01=100 kPa
- the impeller total pressure ratio P02/P01=1.3
- the mass flow rate through the impeller $\dot{m} = 20$ kg/s

For these additional operating conditions, calculate the following:
d) the total-to-total efficiency
e) the polytropic efficiency
f) the impeller outlet diameter and the inlet hub and shroud diameters.
g) the impeller rotational speed in RPM
h) the outlet impeller width b_2

Lecture – Thu 11 Aug

Radial compressor impeller
no preswirl at inlet $c_1 = c_x$

$c_{2r} = c_1$

$\Phi = 0.4 = \dfrac{c_{r2}}{u_2}$

$u_2 = 300\ m/s$

$\dfrac{d_{hb1}}{d_2} = 0.3$

$W_2 / W_{sh1} = 0.75$

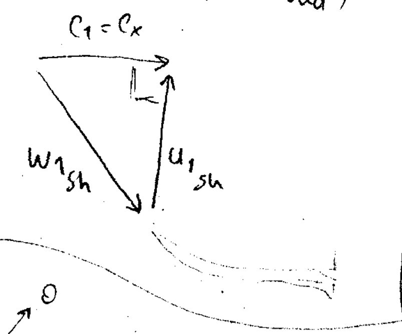
slip factor $\sigma = 0.78$

blade angle ← $\beta_2' = 50°$ backward

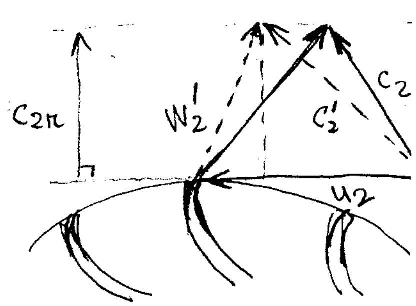
air
$c_p = 1005 \frac{J}{kg K}$
~~R = 287~~
$\gamma = 1.4$

Calculate a) ψ, b) $\dfrac{d_{sh1}}{d_{hb1}}$, c) Z number of blades

impeller inlet (@ shroud)



impeller outlet



a) $\psi = \dfrac{\Delta W_c}{u_2^2}$, $\Delta W_c = u_2 c_{\theta 2} - \cancel{u_1 c_{\theta 1}}$? 0

use the impeller outlet blade speed

slip factor $\sigma = 0.78 = \dfrac{c_{\theta 2}}{c_{\theta 2s}} \rightarrow c_{\theta 2} = \sigma\, c_{\theta 2s}$?

$\Phi = \dfrac{c_{r2}}{u_2} = 0.4 \rightarrow c_{r2} = \phi\, u_2 = 0.4 \times 300\ m/s = 120\ \frac{m}{s}$

$W_2' = \dfrac{c_{r2}}{\cos\beta_2'} = \dfrac{120}{\cos 50°} = 186.7\ m/s$

theoretical

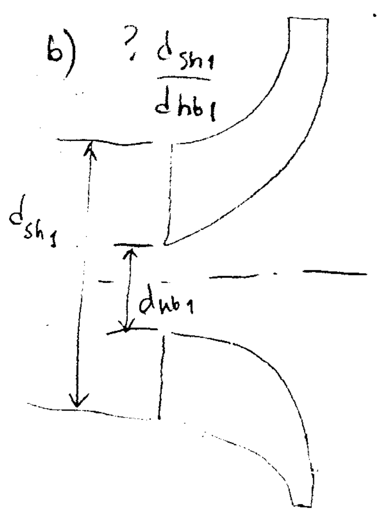
$c_{\theta_2}' = u_2 - W_2' \sin\beta_2' = 300 - 186.7 \sin 50° = 157\ m/s$

$c_{\theta_2} = \sigma\, c_{\theta_2}' = 0.78 \times 157 = 122.5\ m/s$

$\psi = \dfrac{c_{\theta 2}}{u_2} = \dfrac{122.5\ m/s}{300\ m/s} = 0.408$ ⇝ moderately loaded

($\psi > 0.5$ heavily loaded
$\psi < 0.3$ lightly loaded)

b) ? $\dfrac{d_{sh1}}{d_{hb1}}$



$\dfrac{d_{hb1}}{d_2} = 0.3 \rightarrow \dfrac{d_{sh1}}{d_{hb1}} = \dfrac{d_{sh1}}{d_2}\dfrac{d_2}{d_{hb1}} = \dfrac{d_{sh1}}{d_2}\dfrac{1}{0.3}$?

$\left.\begin{aligned} u_2 &= \omega r_2 = \omega \frac{d_2}{2} \\ u_{sh1} &= \omega \frac{d_{sh1}}{2} \end{aligned}\right\} \Rightarrow \dfrac{d_{sh1}}{d_2} = \dfrac{u_{sh1}}{u_2}$?

$u_{sh1} = \sqrt{W_{sh1}^2 - c_1^2}$, $c_1 = c_{1x} = c_{2r} = 120\ m/s$

find W_{sh_1} :

$$\frac{W_2}{W_{sh_1}} = 0.75 \quad , \text{find } W_2 : \quad W_2 = \sqrt{C_{2R}^2 + W_{2\theta}^2} = \sqrt{C_{2R}^2 + (u_2 - C_{2\theta})^2}$$

$$= \sqrt{120^2 + (300-122.5)^2} = 214.3 \text{ m/s}$$

$$W_{sh_1} = \frac{W_2}{0.75} = \frac{214.3}{0.75} = 285.7 \text{ m/s}$$

$$u_{sh_1} = \sqrt{W_{sh_1}^2 - C_1^2} = \sqrt{285.7^2 - 120^2} = 259.3 \text{ m/s}$$

$$\frac{d_{sh_1}}{d_2} = \frac{u_{sh_1}}{u_2} = \frac{259.3}{300} = 0.864$$

$$\boxed{\frac{d_{sh_1}}{d_{hb1}}} = \frac{d_{sh_1}}{d_2} \cdot \frac{1}{0.3} = \frac{0.864}{0.3} = 2.88$$

c) ? Z = number of blades

$$\sigma = 0.78 = \frac{C_{\theta 2}}{C'_{\theta 2}}$$

use for example Wiesner correl. $\sigma = 1 - \frac{u_2}{C'_{\theta_2}} \frac{\sqrt{\cos\beta_2'}}{Z^{0.7}}$

$$Z^{0.7} = \frac{1}{1-\sigma_W} \frac{U_2}{C'_{\theta_2}} \sqrt{\cos\beta_2'} = \frac{1}{1-0.78} \frac{300}{157} \cdot \sqrt{\cos 50^\circ} = 6.96$$

$Z = 16$ blades

in fig. for $\beta_2 = 50^\circ$: $Z_{min} = 9$ blades (high blade loading)

$Z_{max} = 24$ " (high blockage effect)

2nd part

$$M_{u2} = \frac{u_2}{\sqrt{\gamma R T_2}} = 0.7$$

Calculate: d) η_{tt}

e) $\eta_{p,tt}$

f) d_2, d_{sh1}, d_{hb1}

g) N [RPM]

h) b_2

$p_{01} = 100$ kPa

$$\frac{p_{02}}{p_{01}} = 1.3$$

$\dot{m} = 20$ kg/s

usually it's for the whole stage or whole compressor but in this case we know only about the impeller!

$$d)\quad \eta_{tt} = \frac{h_{02s} - h_{01}}{h_{02} - h_{01}} = \frac{T_{02s} - T_{01}}{T_{02} - T_{01}} = \frac{\frac{T_{02s}}{T_{01}} - 1}{\frac{T_{02}}{T_{01}} - 1}$$

where $\frac{T_{02s}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} = (1.3)^{1/3.5} = 1.0778$

$$T_{02} = T_2 + \frac{c_2^2}{2c_p}$$

where $c_2^2 = \sqrt{c_{2R}^2 + c_{2\theta}^2} = \sqrt{120^2 + 122.5^2} = 171$ m/s

& get T_2 from $M_{u2} = 0.7 \Rightarrow \gamma R T_2 = \left(\frac{u_2}{M_{u2}}\right)^2$

$$T_2 = \frac{1}{\gamma R}\left(\frac{u_2}{M_{u2}}\right)^2 = \frac{1}{1.4 \times 287 \frac{J}{kg K}} \left(\frac{300}{0.7}\right)^2 \frac{m^2}{s^2} = 457.1 \text{ K}$$

$$T_{02} = T_2 + \frac{c_2^2}{2c_p} = 457.1 + \frac{171^2}{2 \times 1005} = 471.7 \text{ m/s}$$

$$\Delta W_c = h_{02} - h_{01} = c_p(T_{02} - T_{01}) = u_2 c_{\theta 2} - u_1 \cancel{c_{\theta 1}}^{0} = 300 \times 122.5 = 36750 \frac{J}{kg}$$

$$\eta_{tt} = \frac{\frac{T_{02s}}{T_{01}} - 1}{\frac{T_{02}}{T_{01}} - 1} = \frac{1.0778 - 1}{\frac{471.7}{435.1} - 1} = 92.5\%$$

isentropic total-to-total efficiency

e) ? $\eta_{p,tt}$:

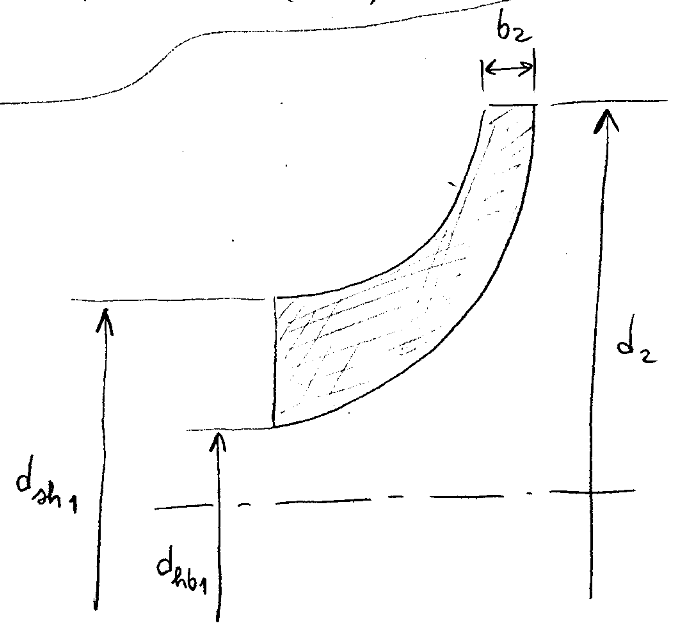
$$\frac{T_{02}}{T_{01}} = \left(\frac{p_{02}}{p_{01}}\right)^{\frac{R}{c_p \eta_{p,tt}}}$$

polytropic total-to-total efficiency :

$$\ln\frac{T_{02}}{T_{01}} = \frac{R}{c_p \eta_{p,tt}} \ln\frac{p_{02}}{p_{01}} \Rightarrow \boxed{\eta_{p,tt}} = \frac{\frac{R}{c_p}\ln\frac{p_{02}}{p_{01}}}{\ln\frac{T_{02}}{T_{01}}} = \frac{\frac{1}{3.5}\ln 1.3}{\ln\left(\frac{471.7}{435.1}\right)} = \boxed{92.8\%}$$

Note $\eta_{s,tt} < \eta_{p,tt}$ for a compressor

f) ? d_2, d_{sh_1}, d_{hb_1} :



$$\dot{m} = 20\ \frac{kg}{s} = \rho_1 A_1 c_{1x} = \rho_2 A_2 c_{2r}$$

where $A_1 = \frac{\pi}{4}\left(d_{sh_1}^2 - d_{hb_1}^2\right)$

and $A_2 = \pi d_2 b_2$

find ρ_1 : $\rho_1 = \frac{P_1}{RT_1}$

$c_1 = 120\ \frac{m}{s}$

$$T_1 = T_{01} - \frac{c_1^2}{2c_p} = 435.1 - \frac{120^2}{2\cdot 1005} = 427.9\ K$$

$$M_1 = \frac{c_1}{\sqrt{\gamma R T_1}} = \frac{120}{\sqrt{1.4 * 287 * 427.9}} = 0.29$$

$$P_1 = P_{01}\left[1 + \frac{\gamma-1}{2}M_1^2\right]^{-\frac{\gamma}{\gamma-1}} = 100\left[1 + \frac{1.4-1}{2}0.29^2\right]^{-\frac{1.4}{0.4}} = 94.33\ kPa$$

$$\rho_1 = \frac{P_1}{RT_1} = \frac{94.33 * 10^3}{287 * 427.9} = 0.768\ \frac{kg}{m^3}$$

$$\rightarrow A_1 = \frac{\dot{m}}{\rho_1 c_{1x}} = \frac{20\ \frac{kg}{s}}{0.768\ \frac{kg}{m^3}\ 120\ \frac{m}{s}} = 0.217\ m^2 = \frac{\pi}{4}\left(d_{sh_1}^2 - d_{hb_1}^2\right)$$

$$\begin{cases} d_{sh_1}^2 - d_{hb_1}^2 = \frac{4A_1}{\pi} = 0.27631\ m^2 \\ \frac{d_{sh_1}}{d_{hb_1}} = 2.88 \quad \text{(from part b)} \end{cases}$$

$$d_{sh_1} = 2.88\, d_{hb_1} \Rightarrow (2.88^2 - 1)\, d_{hb_1}^2 = 0.27631 \Rightarrow \boxed{d_{hb_1} = 0.1946\ m}$$

$$\boxed{d_{sh_1}} = 2.88 \times 0.1946 = \boxed{0.5605\ m}$$

$$\frac{d_{hb_1}}{d_2} = 0.3 \text{ (given)} \Rightarrow \boxed{d_2} = \frac{d_{hb_1}}{0.3} = \frac{0.1946}{0.3} = \boxed{0.6488\ m}$$

g) ? N [RPM]

$$u_2 = \omega r_2 = \frac{\cancel{2}\pi N}{60} \cdot \frac{d_2}{\cancel{2}}$$

$$\Rightarrow \boxed{N} = \frac{60\, u_2}{\pi d_2} = \frac{60 \cdot 300}{\pi \cdot 0.6488} = \boxed{8831 \text{ RPM}}$$

h) ? b_2 (outer impeller width):

$$\dot{m} = \rho_2 A_2 c_{2r} \qquad \text{where } A_2 = \pi d_2 b_2$$

find ρ_2 :

$$M_2 = \frac{c_2}{\sqrt{\gamma R T_2}} = \frac{171.5}{\sqrt{1.4 \times 287 \times 457.1}} = 0.4$$

$$p_{02} = 1.3\, p_{01} = 130 \text{ kPa}$$

$$p_2 = p_{02}\left[1 + \frac{\gamma-1}{2} M_2^2\right]^{-\frac{\gamma}{\gamma-1}} = 130\left[1 + \frac{1.4-1}{2} M_2^2\right]^{-\frac{1.4}{0.4}} = 116.4 \text{ kPa}$$

$$\rho_2 = \frac{p_2}{R T_2} = \frac{116.4 \times 10^3}{287 \times 457.1} = 0.887 \ \frac{kg}{m^3}$$

$$\text{hence } A_2 = \frac{\dot{m}}{\rho_2 c_{2r}} = \frac{20}{0.887 \cdot 120} = 0.1878 \text{ m}^2 = \pi d_2 b_2$$

$$\Rightarrow \boxed{b_2} = \frac{A_2}{\pi d_2} = \frac{0.1878}{\pi \times 0.6488} = \boxed{0.0921 \text{ m}}$$

---

i) $$N_{s_1} = \frac{2\pi N \sqrt{\dot{V}_1}}{60\,(h_{02}-h_{01})^{3/4}} \qquad \text{where } h_{02} - h_{01} = c_p (T_{02} - T_{01})$$

$$\dot{V}_1 = A_1 c_{1x} = \frac{\dot{m}}{\rho_1} = \frac{20}{0.768} = 26.04$$

$$\boxed{N_{s1}} = \frac{2\pi \cdot 8831 \sqrt{26.04}}{60\left[1005\,(471.7 - 435.1)\right]^{3/4}} = \boxed{1.78}$$

~~Also if we use $\dot{V}_2$~~

Fig. 5.20: for $\beta_2 = 50^\circ$, $M_{u_2} = 0.7$, $\phi = \frac{c_{x1}}{u_2} = 0.4$

$\eta_{P,tt} \simeq 93\%$ corresponds to $N_{s_1} \simeq 0.8$

while in the present case $\eta_{P,tt} = 93\%$ corresponds to $N_{s_1} = 1.78$,

Thus, the performance characteristics in fig. 5.20 apply for a different impeller.