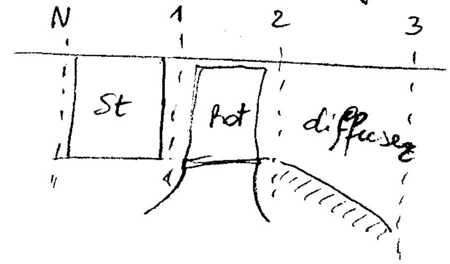
$\boxed{2.6}$

Same geometry as ~~ex.~~ 2.5
but different operating conditions



$T_{0N} = 1500\,K$

$P_{0N} = 8\,bar$

$\Delta h_{0_{12}} = 2u_m^2 = 2 \cdot 500^2 = 500\,\frac{kJ}{kg}$

$u_m = 500\,\frac{m}{s}$

$\alpha_2 = 0°$ to axial $\rightarrow c_2 = c_{2ax}$

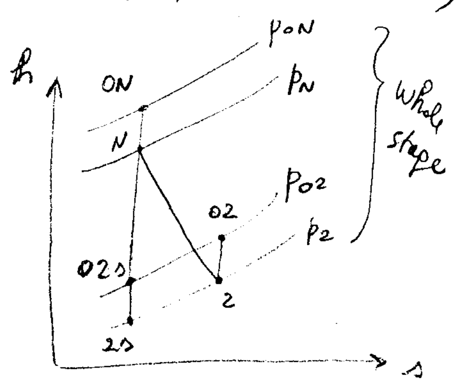
$c_2 = 0.728\,u_m = 364\,\frac{m}{s}$

$c_3 = 0.5\,c_2 = 182\,\frac{m}{s}$

$p_3 - p_2 = 0.75\,(p_{3iss} - p_2)$

semi-perfect gas $c_p = c_p(T)$

$\eta_{s,tt} = 0.95$ (isentropic, total-to-total)



? $\dot{W}_{ex}$ and $\eta_{s,ts}$, $\eta_{p,tt}$, $\eta_{p,ts}$ @ plane 2

isentropic

	outlet 2	~~plane 3~~
$\eta_{s,tt}$	95%	
$\eta_{s,ts}$	84.4%	
$\eta_{p,tt}$	94.1%	
$\eta_{p,ts}$	81.4%	

polytropic

note :
for a turbine :

$\eta_{s,tt} > \eta_{p,tt}$

$\eta_{s,ts} > \eta_{s,ts}$

Note : since the gas is not a perfect gas $(c_p = c_p(T))$
the average c_p is calculated
& iteration is needed

$\dot{W}_{ex} = \dot{m}\,\Delta h_{0_{12}} = 2\,\frac{kg}{s} \cdot 500\,\frac{kJ}{kg} = 1000\,kW$ ← ANS.

$$\eta_{s,ts,2} = \frac{h_{0N} - h_{02}}{h_{0N} - h_{2s}} = \frac{\cancel{\bar{c}_{p01-02}}\,(T_{0N} - T_{02})}{\cancel{\bar{c}_{p01-2s}}\,(T_{0N} - T_{2s})}$$

find T_{02}, T_{2s} first :

$T_{01} = T_{0N}$

find T_{02} : $\Delta h_{0_{12}} = \bar{c}_p\,(T_{0N} - T_{02}) \rightarrow T_{02} = T_{0N} - \frac{\Delta h_{0_{N-2}}}{\bar{c}_p} = 1500\,K - \frac{500}{\bar{c}_p}\frac{kJ}{kg\,K}$ (eq. 1)

where $\bar{c}_p = c_p\left(\frac{1500\,K + T_{02}}{2}\right)$:
↓
from table

	T_{02} [K]	$\bar{c}_p$ $\left[\frac{kJ}{kg\,K}\right]$
guess →	1000	1.1761
calculate from eq. 1 →	1075	1.1833
	1077.5	1.1835
	1077.5	

find T_{2s} from:

$$\frac{T_{2s}}{T_{02s}} = \left(\frac{p_2}{p_{02}}\right)^{R/\bar{c}_p} \qquad \frac{T_2}{T_{02}} = \left(\frac{p_2}{p_{02}}\right)^{R/\bar{c}_p} \quad \Bigg\} \rightarrow \left(\frac{T_{2s}}{T_{02s}}\right)^{\frac{\bar{c}_{p\,2s-02s}}{R}} = \left(\frac{T_2}{T_{02}}\right)^{\frac{\bar{c}_{p\,2-02}}{R}}$$

since $\bar{c}_p(02s, 2s) \simeq \bar{c}_p(02, 2)$

$$\rightarrow T_{2s} = T_{02s}^{\,?} \cdot \frac{T_2^{\,?}}{T_{02}}$$

find T_{02ss}:

$$\eta_{s,tt,2} = \frac{h_{0N} - h_{02}}{h_{0N} - h_{02s}} \simeq \frac{\cancel{\bar{c}_{p\,01-02}}(T_{0N} - T_{02})}{\cancel{\bar{c}_{p\,01-02ss}}(T_{0N} - T_{02s})} = 0.95$$

$$\rightarrow T_{02s} = T_{0N} - \frac{T_{0N} - T_{02}}{0.95} = 1500\,K - \frac{1500 - 1077.5}{0.95} = 1055.3\,K$$

(1059.9k book)

find T_2:

$$T_2 = T_{02} - \frac{c_2^2}{2\bar{c}_{p\,2,02}} = 1077.5\,K - \frac{364^2}{2 \cdot \bar{c}_p}$$

	$\bar{c}_p\left(\frac{1077.5 + T_2}{2}\right)$	T_2
guess →	1100 $\frac{J}{kgK}$	1017
from table →	1145.2	1020
	1145.5	

$$T_{2s} = T_{02s}\frac{T_2}{T_{02}} = 1055.3 \cdot \frac{1020}{1077.5} = 999\,K$$

$$\eta_{s,ts,2} = \frac{T_{01} - T_{02}}{T_{01} - T_{2s}} = \frac{1500 - 1077.5}{1500 - 999} = \boxed{84.3\,\%}$$ ← ANS.

ALTERNATIVE WAY TO GET $\eta_{s,ts,2}$ (anyway $T_{02}, T_2, \dots$ are needed for the polytropic η)

$$\eta_{s,ts,2} = \frac{h_{0N} - h_{02}}{h_{0N} - h_{2s}} = \frac{h_{0N} - h_{02}}{(h_{0N} - h_{02s}) + (h_{02s} - h_{2s})} = \frac{\Delta h_{0N\text{-}2}}{\frac{\Delta h_{0N\text{-}2}}{\eta_{s,tt}} + \frac{c_{2s}^2}{2}} = \frac{500 \cdot 10^3}{\frac{500 \cdot 10^3}{0.95} + \frac{364^2}{2}} = 84.4\,\%$$

$c_2 \simeq c_{2s}$

$$\ln \frac{T_{02}}{T_{0N}} = \ln\left(\frac{P_{02}}{P_{0N}}\right)^{\eta_{p,tt}\frac{R}{\bar{C}_p}} \qquad \eta_{p,tt} = \frac{\ln \frac{T_{02}}{T_{0N}}}{\ln\left(\frac{P_{02}}{P_{0N}}\right)^{R/\bar{C}_p}}$$

$$\left(\frac{P_{02}}{P_{0N}}\right)^{R/\bar{C}_p} = \frac{T_{02ss}}{T_{0N}}$$

$$\eta_{p,tt,2} = \frac{\ln \frac{T_{02}}{T_{0N}}}{\ln\left(\frac{T_{02s}}{T_{0N}}\right)} = \frac{\ln\left(\frac{1077.5}{1500}\right)}{\ln\left(\frac{1055.3}{1500}\right)} = 94.1\%$$

$$\eta_{p,ts,2} = \frac{\ln\left(\frac{T_{02}}{T_{0N}}\right)}{\ln\left(\frac{T_{2s}}{T_{0N}}\right)} = \frac{\ln\left(\frac{1077.5}{1500}\right)}{\ln\left(\frac{999}{1500}\right)} = 81.4\%$$