2.6

\( T_{\text{on}} = 1500 \text{K} \)
\( P_{\text{on}} = 86 \text{au} \)
\( \Delta h_{p_{12}} = 2u_m^2 = 2 \times 500^2 \times 500 \text{ kJ/kg} \)
\( u_m = 500 \text{ m/s} \)
\( x_2 = 0 \) to axial \( c_2 = c_2 \text{ax} \)
\( c_2 = 0.728 \text{ } u_m = 364 \text{ m/s} \)
\( c_3 = 0.5 \text{ } c_2 = 182 \text{ m/s} \)
\( p_3 - p_2 = 0.75 (p_3 \text{ abs} - p_2) \)

? \( W_{\text{ex}} \) and \( \gamma_{\text{a}}, \gamma_{\text{r}}, T_{\text{a}}, T_{\text{r}}, p_{\text{a}}, p_{\text{r}} \) at plane 2

Same geometry as ex. 2.5
but different operating conditions

Semi-perfect gas \( c_p = c_p(T) \)
\( \gamma_{\text{a}}, T_{\text{a}} = 0.95 \) (isentropic, total-to-total)

Note:
For a turbine:
\( \gamma_{\text{a}}, T_{\text{a}} > \gamma_{\text{r}}, T_{\text{r}} \)
\( \gamma_{\text{a}}, T_{\text{a}} > \gamma_{\text{a}}, T_{\text{a}} \)

Note: since the gas is not a perfect gas \( (c_p = c_p(T)) \)
the average \( c_p \) is calculated
an iteration is needed

\[ W_{\text{ex}} = m \Delta h_{p_{12}} = 2u_m^2 \times 500 \text{ kJ/kg} = 1000 \text{ kW} \]

\[ \gamma_{\text{a}}, T_{\text{a}} = \frac{h_{\text{on}} - h_{\text{p2}}}{h_{\text{on}} - h_{\text{p3}}} = \frac{T_{\text{on}} - T_{\text{p2}}}{T_{\text{on}} - T_{\text{p3}}} \]

Find \( T_{\text{p2}} \) and \( T_{\text{p3}} \) first:

\( T_{\text{p1}} = T_{\text{on}} \)

Find \( T_{\text{p2}} : \Delta h_{p_{12}} = c_p(T_{\text{on}} - T_{\text{p2}}) \rightarrow T_{\text{p2}} = T_{\text{on}} - \frac{\Delta h_{p_{12}}}{c_p} = 1500 - \frac{500 \text{ kJ/kg}}{c_p} (\text{eq. 1}) \)

where \( c_p = c_p\left(1500 K + T_{\text{p2}}\right) \):

\[
\begin{array}{c|c|c|c|c|}
\text{eqn} & \text{calc} & \gamma_{\text{a}} & \text{calculated} & \gamma_{\text{a}} \\
\hline
\text{1000} & 1.1761 & 1.1761 & 1.1761 \\
\text{1075} & 1.1833 & 1.1833 & 1.1833 \\
\text{1077.5} & 1.1835 & 1.1835 & 1.1835 \\
\end{array}
\]
Find $T_{2,0}$ from:

\[
\frac{T_{2,0}}{T_{0,2,0}} = \left( \frac{R}{\rho_{0,2}} \right)^{\frac{1}{2}}
\]

\[
\frac{T_2}{T_{02}} = \left( \frac{R}{\rho_{02}} \right)^{\frac{1}{2}}
\]

\[
\frac{T_{2,0}}{T_{0,2,0}} \cdot \frac{C_{p,2,0}}{R} \cdot \frac{\bar{C}_{p,2,0}}{R}
\]

\[
\frac{T_2}{T_{02}} = \left( \frac{T_{2,0}}{T_{0,2,0}} \right)
\]

Since $C_p(0,2,2) \approx C_p(0,2,2)$

\[
T_{2,0} = T_{0,2,0} \cdot \frac{T_2}{T_{02}}
\]

Find $T_{2,0}$:

\[
\frac{\rho_{2,0} - \rho_{0,2}}{\rho_{0,2} - \rho_{2,0}} = \frac{\bar{C}_{p,01-02}}{\bar{C}_{p,01-02}} \left( \frac{T_{01}-T_{02}}{T_{01}-T_{02}} \right) = 0.95
\]

\[
T_{2,0} = T_{02} \cdot \frac{\rho_{2,0} - \rho_{0,2}}{\rho_{0,2} - \rho_{2,0}} = 1500 \cdot 0.95 = 1455.31
\]

Find $T_2$:

\[
T_2 : T_{02} = \frac{C_p}{\bar{C}_{p,2,02}} = 1097.5 - \frac{364}{2} = 1017
\]

\[
\bar{C}_p \left( \frac{1077.5 + T_2}{2} \right) \right| T_2
\]

Guess $T_0 = 1000 K$ + $1017$

From table -> 1145.2

\[
T_{2,0} = T_{0,2,0} \cdot \frac{T_2}{T_{02}} = 1455.31 \cdot 1020 
\]

\[
\gamma_{0,2,0} = \frac{T_{01} - T_{02}}{T_{01} - T_{2,0}} = \frac{1500 - 1077.5}{1500 - 999} = 88.1
\]

\[
\gamma_{0,2,0} = \frac{\rho_{0,2} - \rho_{0,2}}{\rho_{0,2} - \rho_{0,2}} = \frac{\Delta H_{0,2}}{C_{p,0,2}} = \frac{\Delta H_{0,2}}{2} = \frac{500 \cdot 10^3}{0.95} \approx 84.4\
\]

\[
\gamma_{0,2,0} = \frac{\rho_{0,2} - \rho_{0,2}}{\rho_{0,2} - \rho_{0,2}} = \frac{\Delta H_{0,2}}{C_{p,0,2}} = \frac{\Delta H_{0,2}}{2} = \frac{500 \cdot 10^3}{0.95} \approx 84.4\
\]
\[
\frac{T_{22}}{T_{10}} = \ln \left( \frac{P_{22}}{P_{10}} \right) \quad \text{M}^{\text{Pitt}} = \frac{\ln T_{22}}{\ln P_{22}}
\]

\[
\left( \frac{P_{22}}{P_{10}} \right)^{\frac{d_{22}}{d_{10}}} = \frac{T_{22}}{T_{10}}
\]

\[
M^{\text{Pitt}, 2} = \frac{\ln \left( \frac{T_{22}}{T_{10}} \right)}{\ln \left( \frac{T_{22}}{T_{10}} \right)} = \frac{\ln (1077.5)}{1500} = 94.1\%
\]

\[
M^{\text{Pitt}, 2} = \frac{\ln \left( \frac{P_{22}}{P_{10}} \right)}{\ln \left( \frac{P_{22}}{P_{10}} \right)} = \frac{\ln (1055.3)}{1500} = 93.7\%
\]

\[
M^{\text{Pitt}, 2} = \frac{\ln \left( \frac{T_{22}}{T_{10}} \right)}{\ln \left( \frac{T_{22}}{T_{10}} \right)} = \frac{\ln (999)}{1500} = 93.4\%
\]