Ex. 1 (60 pt)
The first stage of an axial compressor is designed on free vortex principles, with no inlet guide vane (i.e. the inlet velocity is axial at all radii). The rotational speed is 6000 RPM and the stagnation temperature rise is 20K. The hub-tip ratio is 0.6 and the isentropic total-to-total efficiency of the stage is 0.89.
Assume an inlet velocity of 140 m/s and ambient conditions of 1.01 bar and 288 K (stagnation conditions).
The inlet relative Mach number is limited to 0.95.

a) As you solve the problem, clearly draw the velocity triangles at inlet, at outlet, at hub, at tip.

Calculate:

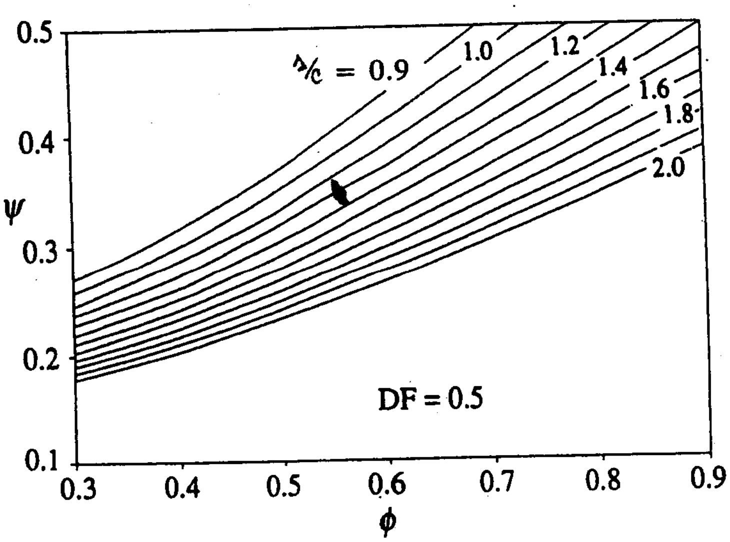
b) The tip radius and the corresponding inlet & outlet air angles relative to the rotor.
c) The mass flow entering the stage.
d) The stage stagnation pressure ratio and the power required.
e) The inlet & outlet air angles relative to the rotor at the root section.
f) The maximum outlet (absolute) Mach number

Take Cp=1005 J/(kg*K), R=287 J/(kg*K), γ=1.4

Make reasonable assumptions if needed.

Ex. 2 (20 pt)
An axial compressor rotates at 5000 RPM and has a mean radius of 0.5m.
The overall stagnation enthalpy rise is 97.72 kJ/kg.
- Calculate a suitable number of stages with a stage loading coefficient of 0.35 and a flow coefficient of 0.65.
- Estimate the appropriate pitch-to-chord ratio s/c for a diffusion factor of DF=0.5 (see figure).

Suppose we now decide to reduce the number of stages by 1 stage (for the same flow coefficient and diffusion factor).
- Is the number of blades increased or decreased? By how many?



ME 433 Final (2004) solution

EX. 1

axial compressor

free vortex

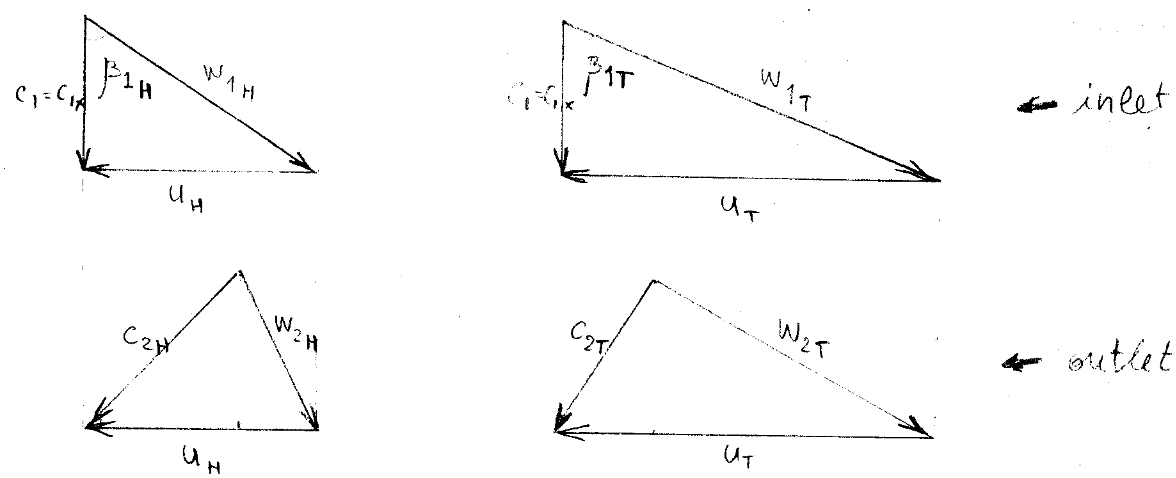
no IGV $\rightarrow c_1 = c_{1x} = 140\ \frac{m}{s}$ at all radii

$N = 6000$ rpm	a) velocity triangles
$T_{01} - T_{03} = 20\,K$	b) r_T, $\alpha_{w_{1T}}$ and $\alpha_{w_{2T}}$ (T = TIP)
$\frac{r_H}{r_T} = 0.6$	c) $\dot{m}$
$\eta_{0,H} = 0.89$	d) P_{03}/P_{01}, power $\mathbb{P}_{out}$
$T_{01} = 288\,K$	e) β_{1H} and β_{2H} (H = HUB)
$P_{01} = 1.01$ bar	f) $M_{2\,max}$
$M_{w_1} \leq 0.95$	

$c_p = 1005\ J/(kg\,K)$, $R = 287\ J/(kg\,K)$, $\gamma = 1.4$

a)

at hub   at tip



b) the maximum relative velocity at the inlet is at tip:

$$M_{w_1} = 0.95 = \frac{w_{1T}}{a_1} \quad \text{where } a_1 = \sqrt{\gamma R T_1} = \text{speed of sound}$$

$$T_1 = T_{01} - \frac{c_1^2}{2c_p} = 288 - \frac{140^2}{2 \times 1005} = 278.2\ K$$

$$a_1 = \sqrt{\gamma R T_1} = \sqrt{1.4 \times 287 \times 278.2} = 334.4\ m/s$$

$$w_{1T} = 0.95\, a_1 = 317.6\ m/s$$

$$u_T = \sqrt{w_{1T}^2 - c_1^2} = \sqrt{317.6^2 - 140^2} = 285.1\ m/s$$

$$u_T = \omega r_T$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi\ 6000}{60} = 628.3\ m/s$$

$$r_T = \frac{u_T}{\omega} = \frac{285.1}{628.3} = 0.454\ m$$

$$c_1 = W_{1T}\cos\beta_{1T} \Rightarrow \boxed{\beta_{1T}} = \cos^{-1}\left(\frac{140}{317.6}\right) = \boxed{63.8^\circ}$$

assume $c_{1x} = c_{3x}$

$$\Delta h_0 = c_p\,\Delta T_0 = U_T\left(c_{\theta 2T} - \cancel{c_{\theta 1T}}^{\,0}\right) = 1005 \cdot 20 = 20100\ \mathrm{J/kg}$$

$$\Rightarrow c_{\theta 2T} = \frac{\Delta h_0}{U_T} = \frac{20100}{285.1} = 70.5\ \mathrm{m/s}$$

$$\tan\beta_{2T} = \frac{U_T - c_{\theta 2T}}{c_x} = \frac{285.1 - 70.5}{140} \Rightarrow \boxed{\beta_{2T} = 56.9^\circ}$$

c) $$\dot m = p_{01} A_{n1} M_1 \sqrt{\frac{\gamma}{RT_{01}}}\left(1 + \frac{\gamma-1}{2}M_1^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}}, \quad \text{where } \frac{\gamma+1}{2(\gamma-1)} = \frac{2.4}{0.8} = 3$$

$$A_{n1} = A_1 = \pi\left(r_T^2 - r_H^2\right) = \pi\left(0.454^2 - 0.272^2\right) = 0.4144\ \mathrm{m}^2$$

$$r_H = 0.6\,r_T = 0.6 \times 0.454 = 0.272$$

$$M_1 = \frac{c_1}{a_1} = \frac{140}{334.4} = 0.4187$$

$$\dot m = 101300 \times 0.4144 \times 0.4187\sqrt{\frac{1.4}{287 \times 288}}\left(1 + \frac{0.4}{2}\,0.4187^2\right)^{-3} = 65\ \mathrm{kg/s}$$

d) $$\eta_{s,H} = \frac{T_{03s} - T_{01}}{T_{03} - T_{01}} = 0.89 \Rightarrow T_{03s} = T_{01} + 0.89\left(T_{03} - T_{01}\right) = 288 + 0.89 \times 20 = 305.8$$

$$\boxed{\frac{p_{03}}{p_{01}}} = \left(\frac{T_{03s}}{T_{01}}\right)^{c_p/R} = \left(\frac{305.8}{288}\right)^{3.5} = \boxed{1.234}$$

Power output $$\boxed{P_{out}} = \dot m\,c_p\left(T_{03} - T_{01}\right) = 65 \times 1005 \times 20 = \boxed{1.31\ \mathrm{MW}}$$

e) β_{1H}, β_{2H}:
$$\tan\beta_{1H} = \frac{U_H}{c_1} = \frac{171.1}{140} \Rightarrow \boxed{\beta_{1H} = 50.7^\circ}$$

$$U_H = 0.6\,U_T = 0.6 \times 285.1 = 171.1\ \mathrm{m/s}$$

free vortex: $r_H c_{\theta 2H} = r_T c_{\theta 2T}$

$$c_{\theta 2H} = \frac{r_T}{r_H}c_{\theta 2T} = \frac{70.5}{0.6} = 117.5\ \mathrm{m/s}$$

$$\tan\beta_{2H} = \frac{U_H - c_{\theta 2H}}{c_x} = \frac{171.1 - 117.5}{140} \Rightarrow \boxed{\beta_{2H} = 20.9^\circ}$$

f) $$M_{2\,max} = M_{2H} = \frac{c_{2H}}{\sqrt{\gamma R T_{2H}}} \quad \text{where } T_{2H} = T_{02} - \frac{c_{2H}^2}{2c_p} = 308 - \frac{182.8^2}{2 \times 1005} = 291.4\ \mathrm{K}$$

$$a_{2H} = \sqrt{\gamma R T_{2H}} = \sqrt{1.4 \times 287 \times 291.4} = 342.2$$

$$\boxed{M_{2\,max}} = \frac{182.8}{342.2} = \boxed{0.534}$$

note: T_{02} is assumed constant along the radius in a free vortex

$\boxed{\text{EX. 2}}$

$\left.\begin{array}{l} N = 5000\ RPM \\ r_m = 0.5\ m \end{array}\right\}$ $u_m = \frac{2\pi N}{60} \cdot r_m = \frac{2\pi\, 5000}{60} \times 0.5 = 261.8\ m/s$

$\Delta h_{0,overall} = 97.72\ \frac{kJ}{kg\,K}$

$\psi = 0.35 = \frac{\Delta h_{0,stage}}{u^2} \Rightarrow \Delta h_{0,stage} = \psi\, u_m^2 = 0.35 \times (261.8)^2 = 23.99\ \frac{kJ}{kg}$

\# of stages $= \frac{\Delta h_{0,overall}}{\Delta h_{0,stage}} = \frac{97.72}{23.99} = 4.07$

let's choose $\boxed{\text{4 stages}}$ and recalculate ψ :

$\Delta h_{0,stage} = \frac{97.72}{4} = 24.43\ \frac{kJ}{kg}$

$\psi = \frac{\Delta h_{0,stage}}{u_m^2} = \frac{24.43 \times 10^3}{261.8^2} = 0.356$

from the figure, for $\psi = 0.356$ and $\phi = 0.65 \Rightarrow \boxed{\frac{s}{c} = 1.45} = \frac{\text{pitch}}{\text{chord}}$

if the number of stages is decreased to 3 :

$\Delta h'_{0,stage} = \frac{\Delta h_{0,overall}}{3} = 32.57\ \frac{kJ}{kg}$

$\psi' = \frac{\Delta h'_{0,stage}}{u^2} = \frac{32.57 \times 10^3}{261.8} = 0.475$

for $\psi' = 0.475$ & $\phi = 0.65 \Rightarrow \left(\frac{s}{c}\right)' \simeq 0.95$

n° of blades $z = \frac{2\pi r}{s}$

$2\pi r = z \cdot s = z' s'$

assume the chord is not changed : $\frac{s/c}{s'/c'} = \frac{s}{s'}$

$z' = z \frac{s}{s'} = z \frac{1.45}{0.95} = 1.53\, z$

the number of blades increases by about 50%