Ex. 2 (40 pt)

A centrifugal compressor has an impeller with blades which are 25° backward- swept and no inlet guide vane. At inlet the stagnation pressure is 100 kPa and stagnation temperature is 300K. The mass flow rate is 2.3 kg/s, the impeller speed at exit is 400 m/s and the radial velocity component at the impeller exit is 160 m/s.

The slip factor is given as: $\sigma = \dfrac{C_{2\vartheta}}{C'_{2\vartheta}} = 0.884$

a) Clearly draw the velocity triangles at the impeller exit for theoretical and actual flow.
b) Determine the driving power on the shaft given a mechanical efficiency of 96%.
c) Estimate the maximum permissible diameter ratio ($d_{sh,1}/d_2$) to avoid separation. Use the figure in the next page. You will need to calculate the actual flow angles, first.

Take the following fluid properties for air:

R=287 J/(kg*K)
Cp=1005 J/(kg*K)
γ=1.4

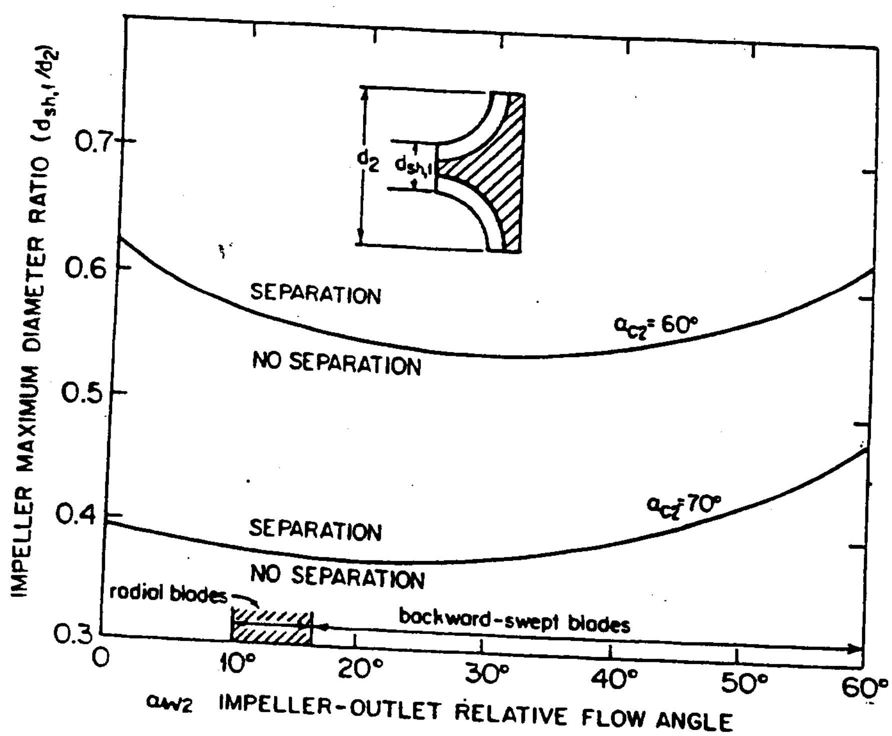


Figure 5.26. Rotor design to avoid separation

EX. 2

centrifugal compressor

blade angle: $\beta_2' = 25°$ backward

no IGV $\Rightarrow c_1 = c_{1x}$

$P_{01} = 100\ kPa$

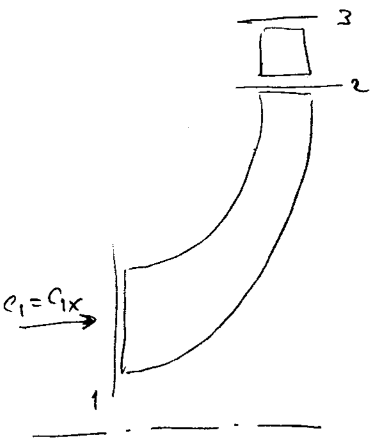
$T_{01} = 300\ K$

$\dot{m} = 2.3\ kg/s$

$U_2 = 400\ m/s$

$C_{2r} = 160\ m/s$

$\sigma = \dfrac{C_{2\theta}}{C_{2\theta}'} = 0.884$

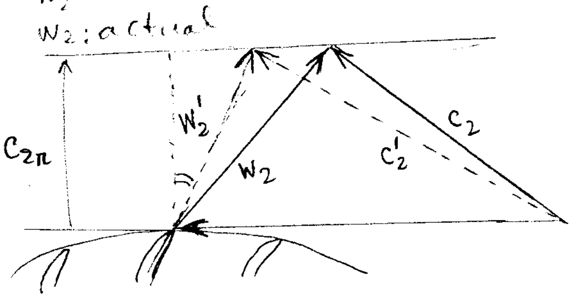


at impeller exit

W_2': theoretical

W_2: actual

a)



b) $\text{work} = U_2 C_{\theta 2} - U_1 \cancel{C_{\theta 1}}^{\,0}$

$C_{\theta 1} = 0$ (axial inlet flow) in absolute frame

$C_{\theta 2} = \sigma\, C_{\theta 2}'$

$C_{\theta 2}' = U_2 - C_{2r}\, tg\, \beta_2' =$

$= 400 - 160\ tg\, 25° = 325.4\ m/s$

$C_{\theta 2} = \sigma\, C_{\theta 2}' = 0.884 \times 325.4 = 287.6\ m/s$

$\text{Work} = U_2 C_{\theta 2} = 400 \times 287.6 = 115.1\ kJ/kg$

$\eta_{mech} = 0.96$

driving power on the shaft $\quad P = \dfrac{\dot{m} \times \text{Work}}{\eta_{mech}} = \dfrac{2.3 \times 115.1}{0.96} = \boxed{275.7\ kW}$

c) find α_2, β_2 :

$\dfrac{C_{\theta 2}}{C_{r2}} = tg\, \alpha_2 = \dfrac{287.6}{160} \Rightarrow \alpha_2 = 61°$

$C_{\theta 2} = U_2 - C_{r2}\, tg\, \beta_2 \Rightarrow tg\, \beta_2 = \dfrac{U_2 - C_{\theta 2}}{C_{r2}} = \dfrac{400 - 287.6}{160}$

$\Rightarrow \beta_2 = 35°$

from figure for $\alpha_2 \simeq 60°$ & $\beta_2 = 35°$

$\dfrac{d_{sh1}}{d_2} \leq 0.54$ to avoid separation