Problem 1

ex. 2.4 Dixon

Note: if you assume that the given conditions are static then assume that the inlet and outlet turbine velocity are the same so that $\Delta h_0 = \Delta h$

Solution. From the definition of reheat factor, eqn. (2.39), the turbine total to total efficiency can be immediately determined:-

$$\eta_t = \eta_p R_H = 0.85 \times 1.04 = 0.884$$

Using the notation given in Q.2.1 the isentropic stagnation enthalpy drop, $h_{o1} - h_{o2s}$, can be determined using steam tables or, less accurately but more quickly, using a Mollier diagram for steam. From steam tables at $p_{o1} = 4$ MPa (40 bar) and $T_{o1} = 300^\circ$C the initial steam condition is superheated (about 50°C of superheat) with $h_{o1} = 2963$ kJ/kg and $s_{o1} = 6.364$ kJ/kg $^\circ$C). Inspection of the tables shows that at $p_{o2} = 0.35$ MPa (3.5 bar) the vapour saturation value of specific entropy $s_{go2} > s_{o1}$. This means the isentropic state point o2s is in the liquid-vapour phase. The dryness fraction q can be evaluated for point o2s,

$$q = (s_{o2} - s_{fo2})/(s_{go2} - s_{fo2})$$

$$= (6.364 - 1.727)/5.214 = 0.8893$$

Hence, the specific stagnation enthalpy at point o2s is

$$h_{o2s} = h_{fo2} + q(h_{go2} - h_{fo2})$$
$$= 584 + 0.8893 \times 2148 = 2494 \text{ kJ/kg}$$

Thus, the isentropic stagnation enthalpy drop is

$$h_{o1} - h_{o2s} = 2963 - 2494 = 469 \text{ kJ/kg}$$

As the total to total efficiency is known the actual stagnation enthalpy drop can be found, i.e.

$$h_{o1} - h_{o2} = \eta_t(h_{o1} - h_{o2s}) = 0.884 \times 469 = 414.6 \text{ kJ/kg}$$

and this is the specific work done by the steam, ΔW. The actual specific work delivered at the output shaft is less than this because of the mechanical losses. The shaft power delivered is

$$\dot{W}_t = \eta_m \dot{m} \Delta W$$

where η_m is the mechanical efficiency. Thus, the rate of mass flow

$$\dot{m} = \dot{W}_t/(\eta_m \Delta W)$$
$$= 20 \times 10^6/(0.98 \times 414.6 \times 10^3)$$
$$= \underline{49.22 \text{ kg/s}}$$

From the equation of continuity and assuming uniform flow at all radii,

$$\dot{m} = \rho A c_x = \rho \pi d_m h c \cos \alpha$$

Hence, the blade height is

$$h = \dot{m} v/(\pi d_m c \cos \alpha_1)$$
$$= 49.22 \times 0.0686/(\pi \times 0.762 \times 244 \times .2419) = 0.0239 \text{ m}$$
$$= \underline{23.9 \text{ mm}}$$

Problem 2

nozzle inlet ≡ N
nozzle exit ≡ 1

$d_{sh_1} = 0.283\ m$

$d_{hb_1/sh_1} = 0.85$

$\dot{m} = 8\ kg/s$

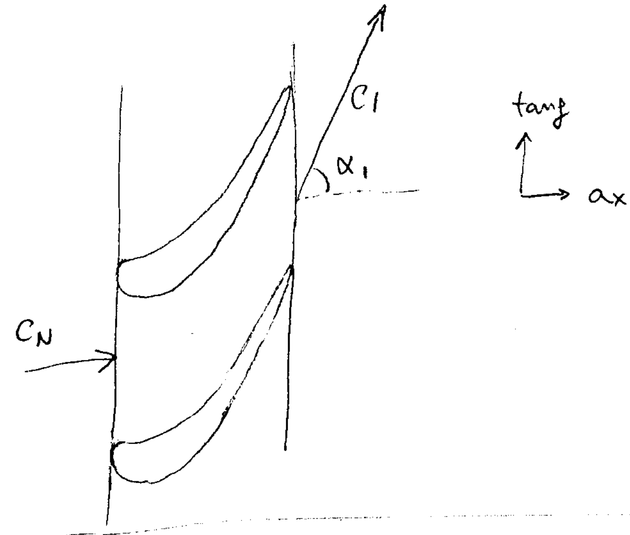
$T_{01} = 1550\ K$

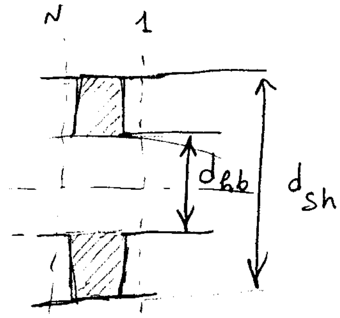
$P_{01} = 15\ bar$

$\gamma = 1.4$ (assume perfect gas)

flows enters nozzle w/o swirl

$R = 287 \frac{J}{kg\,K}$ (air)





? α_1 such that $M_1 = 1$

$$\dot{m} = P_{01} \underbrace{A_{1\perp}}_{?} \underset{1}{M_1} \sqrt{\frac{\gamma}{RT_{01}}} \left(1 + \frac{\gamma-1}{2} \underset{1}{M_1^2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}$$

$$A_1 = \frac{\pi}{4}\left(d_{sh_1}^2 - d_{hb_1}^2\right) = \frac{\pi}{4} d_{sh_1}^2 \left[1 - \left(\frac{d_{hb_1}}{d_{sh_1}}\right)^2\right] = \frac{\pi}{4} 0.283^2 \left(1 - 0.85^2\right) = 0.01745\ m^2$$

$$A_{1\perp} = A_1 \cos\alpha_1$$

$R = 287 \frac{J}{kg}K$ for air

$$A_{1\perp} = \frac{\dot{m}}{P_{01} M_1} \sqrt{\frac{RT_{01}}{\gamma}} \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma+1}{2(\gamma-1)}} = \frac{8}{15 \times 10^5 \times 1} \sqrt{\frac{287 \times 1550}{1.4}} \left(1 + 0.2 \times 1\right)^3 =$$

$$= 0.00519\ m^2$$

$$\Rightarrow \cos\alpha_1 = \frac{A_{1\perp}}{A_1} = \frac{0.00519}{0.01745} = 0.297 \longrightarrow \alpha_1 = 72.7^\circ$$

alternative way to find $A_{1\perp}$:

$$T_1 = T_{01}\left(1 + \frac{\gamma-1}{2} M_1^2\right)^{-1} = 1550\left(1 + 0.2\right)^{-1} = 1291.7\,K$$

$$P_1 = P_{01}\left(1 + \frac{\gamma-1}{2} M_1^2\right)^{-\frac{\gamma}{\gamma-1}} = 15\,(1.2)^{-3.5} = 7.924\ bar = 792.4\ kPa$$

$$\rightarrow \rho = \frac{P}{RT} = \frac{792.4\ kPa}{0.287 \frac{kPa\,m^3}{kg\,K} \times 1291.7} = 2.137\ kg/m^3$$

$$c_1 = M_1\sqrt{\gamma R T_1} = 1 \times \sqrt{1.4 \times 287 \times 1291.7} = 720.4\ m/s$$

$$\dot{m} = \rho_1 A_{1\perp} c_1 \Rightarrow A_{1\perp} = \frac{\dot{m}}{\rho_1 c_1} = \frac{8}{2.137 \times 720.4} = 5.19 \times 10^{-3}\ m^2$$

problem 3

compressor

$c_1 = 300$ m/s

$A_1 = 0.08\ m^2$

$T_{en} = 300$ K

$P_{en} = 100$ kPa

$\dot{W} = 3$ MW

air perfect gas $\gamma = 1.4$, $c_p = 1005$ J/kg K

Evaluate :

a) $T_{01}, P_{01}, \rho_{01}, T_1, P_1, \rho_1$ (inlet)

b) P_{02max} (outlet)

c) entropy generated if $p_{02} = 0.97\ p_{02max}$

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a) environmental conditions are equal to stagnation conditions at inlet

$$P_{01} = P_{en} = 100\ kPa \quad ✓$$

$$T_{01} = T_{en} = 300K \quad ✓$$

$$\rightarrow \rho_{01} = \frac{P_{01}}{RT_{01}} = \frac{100\ kPa}{0.287\ \frac{kPa \cdot m^3}{kg\ K}\ 300K} = 1.16\ \frac{kg}{m^3} \quad ✓$$

$$T_1 = T_{01} - \frac{c_1^2}{2c_p} = 300 - \frac{300^2}{2 \times 1005} = 255.2K \quad ✓$$

$$M_1 = \frac{c_1}{\sqrt{\gamma R T_1}} = \frac{300}{\sqrt{1.4 \times 287 \times 255.2}} = 0.937$$ Note: (flow is compressible!)

$$P_1 = p_{01}\left(1 + \frac{\gamma-1}{2} M_1^2\right)^{-\frac{\gamma}{\gamma-1}} = 100\ kPa \left(1 + 0.2 \times 0.937^2\right)^{-\frac{1.4}{0.4}} = 56.77\ kPa \quad ✓$$

$$\rho_1 = \frac{p_1}{RT_1} = \frac{57.77\ kPa}{0.287\ \frac{kPa\ m^3}{kg\ K}\ 255.2} = 0.775\ \frac{kg}{m^3} \quad ✓$$

b) $$\dot{m} = \rho_1 A_1 c_1 = 0.775\ \frac{kg}{m^3} \times 0.08\ m^2 \times 300\ \frac{m}{s} = 18.6\ \frac{kg}{s}$$

specific work : $$w = \frac{\dot{W}}{\dot{m}} = \frac{3 \cdot 10^6\ J/s}{18.6\ kg/s} = 161.3\ kJ/kg$$

$$w = h_{02} - h_{01} = c_p (T_{02} - T_{01}) \Rightarrow T_{02} = T_{01} + \frac{w}{c_p} = 300 + \frac{161.3 \times 10^3}{1005} = 460.5\ K$$

P_{02} is max when process $1 \to 2$ is isentropic : $$P_{02max} = p_{01}\left(\frac{T_{02}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = 100 kPa \left(\frac{460.5}{300}\right)^{\frac{1.4}{0.4}} = 448\ kPa$$

c) entropy generated if $P_{02} = 0.97\ P_{02max} = 0.97 \times 448.1 = 434.7\ kPa$

$$s_{gen} = \frac{\dot{S}_{gen}}{\dot{m}} = s_{02} - s_{01} = c_p \ln\frac{T_{02}}{T_{01}} - R \ln\frac{P_{02}}{P_{01}} = 1005 \ln\left(\frac{460.5}{300}\right) - 287 \ln\left(\frac{434.7}{100}\right) = 8.93\ \frac{J}{kg\,K}$$

$$\dot{S}_{gen} = \dot{m}(s_{02} - s_{01}) = 18.6 \times 8.93 = 166.2\ \frac{W}{K}$$