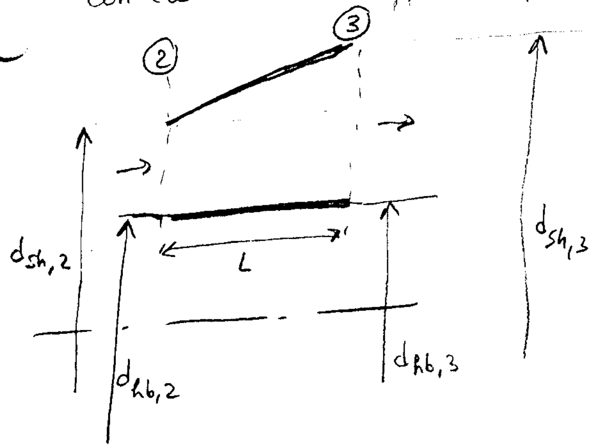
* hw 2 solution *

problem 1

Conical-annular diffuser for axial compressor :



ideal pressure rise coeff. $C_{p_i} = 0.55$

$\dot{m} = 30\ kg/s$

$p_{02} = 500\ kPa$

$T_{02} = 450\ K$

$C_2 = 200\ m/s$

$p_{02} - p_{03} = 0.1 (p_{02} - p_2)$

$\frac{d_{hb,2}}{d_{sh,2}} = 0.9$, $d_{hb,2} = d_{hb,3}$, ~~$\frac{2L}{d_{sh,2} - d_{hb,2}} = 5$~~

Find : i) real pressure rise coeff $C_p = \frac{p_3 - p_2}{p_{02} - p_2}$

ii) $d_{hb,2}$, $d_{sh,2}$, $d_{sh,3}$

Assume air is perfect gas with $\gamma = \frac{c_p}{c_v} = 1.4$

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i) $C_{p_i} = 1 - \left(\frac{C_3}{C_2}\right)^2 \rightarrow C_3 = C_2\sqrt{1 - C_{p_i}} = 200\sqrt{1 - 0.55} = 134.2\ m/s$

$$T_2 = T_{02} - \frac{C_2^2}{2c_p} = 450 - \frac{200^2\ m^2/s^2}{2 \times 1005\ \frac{J}{kg\,K}} = 430.1\ K$$

Note : $M_2 = \frac{C_2}{\sqrt{\gamma R T_2}} = 0.48 > 0.3 \rightarrow$ compressible flow

$$p_2 = p_{02}\left(\frac{T_2}{T_{02}}\right)^{\gamma/\gamma-1} = 500\ kPa \left(\frac{430.1}{450}\right)^{\frac{1.4}{0.4}} = 427\ kPa$$

since no work & adiabatic : $h_{02} = h_{03}$, since perfect gas : $T_{02} = T_{03}$

$$T_3 = T_{03} - \frac{C_3^2}{2c_p} = 450 - \frac{134.2^2\ m^2/s^2}{2 \times 1005\ J/(kg\,K)} = 441\ K$$

$$p_{03} = p_{02} - 0.1(p_{02} - p_2) = 500 - 0.1(500 - 427) = 492.7\ kPa$$

$$p_3 = p_{03}\left(\frac{T_3}{T_{03}}\right)^{\frac{\gamma}{\gamma-1}} = 492.7\left(\frac{441}{450}\right)^{\frac{1.4}{0.4}} = 459.2\ k$$

$$C_p = \frac{p_3 - p_2}{p_{02} - p_2} = \frac{459.2 - 427}{500 - 427} = 0.441 \leftarrow \text{ANS.}$$

ii) $$\rho_2 = \frac{p_2}{RT_2} = \frac{427\ kPa}{0.287\ \frac{kPa\,m^3}{kg\,K}\ 430.1\ K} = 3.46\ kg/m^3$$

annulus area @ 2 : $$A_2 = \frac{\dot{m}}{\rho_2 C_2} = \frac{30\ kg/s}{3.46\ \frac{kg}{m^3}\ 200\ \frac{m}{s}} = 0.04336\ m^2$$

$$A_2 = \frac{\pi}{4}\left(d_{sh,2}^2 - d_{hb}^2\right) = \frac{\pi}{4} d_{hb}^2\left[\left(\frac{d_{sh,2}}{d_{hb}}\right)^2 - 1\right] \Rightarrow d_{hb} = \sqrt{\frac{4A_2}{\pi\left(\frac{1}{0.9^2} - 1\right)}} = \underline{\underline{0.485\ m}}$$

$$d_{sh_2} = \frac{d_{hb}}{0.9} = 0.539 \text{ m}$$

$$\rho_3 = \frac{P_3}{RT_3} = \frac{459.2 \text{ kPa}}{0.287 \frac{\text{kPa m}^3}{\text{kg s}} \times 441 \text{ K}} = 3.63 \frac{\text{kg}}{\text{s}}$$

$$A_3 = \frac{\dot{m}}{\rho_3 C_3} = \frac{30 \text{ kg/s}}{3.63 \frac{\text{kg}}{\text{m}^3} \cdot 134.2 \frac{\text{m}}{\text{s}}} = 0.06162 \text{ m}^2$$

$$A_3 = \frac{\pi}{4}\left(d_{sh_3}^2 - d_{hb}^2\right) \Rightarrow d_{sh_3} = \sqrt{\frac{4A_3}{\pi} + d_{hb}^2} = 0.56 \text{ m}$$

Problem 2

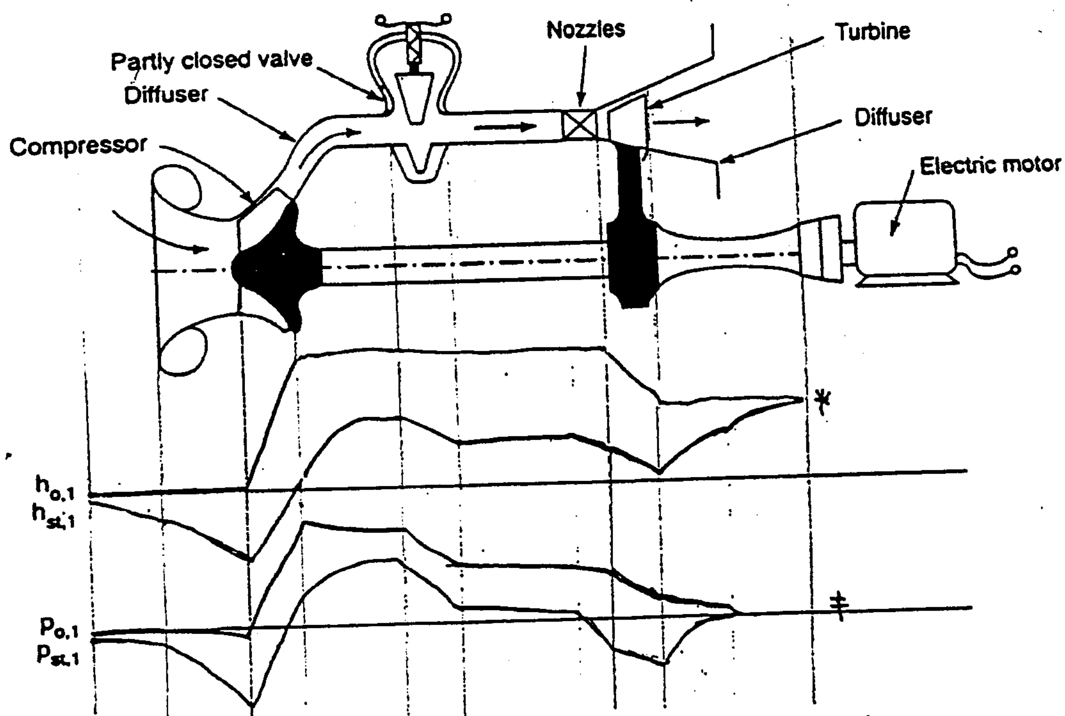


Figure P2.10. Stagnation and static pressure and enthalpy through a compressor test rig

* There is an increase of stagnation & static enthalpy across the unit

‡ The outlet static pressure equals the inlet stagnation pressure.

Problem 3

? N (RPM), T_1, P_1, M_1 and T-s diagram across turbine nozzle

$d_1 = 0{,}25\ m$

$\alpha_1 = 75°$ from radial

$l_1 = 0{,}1\, d_1 = 0.025\ m$

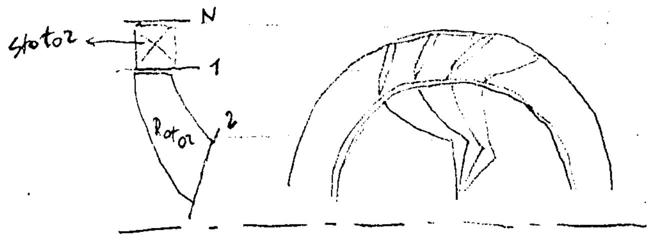
$\dot{m} = 2\ \frac{kg}{s}$

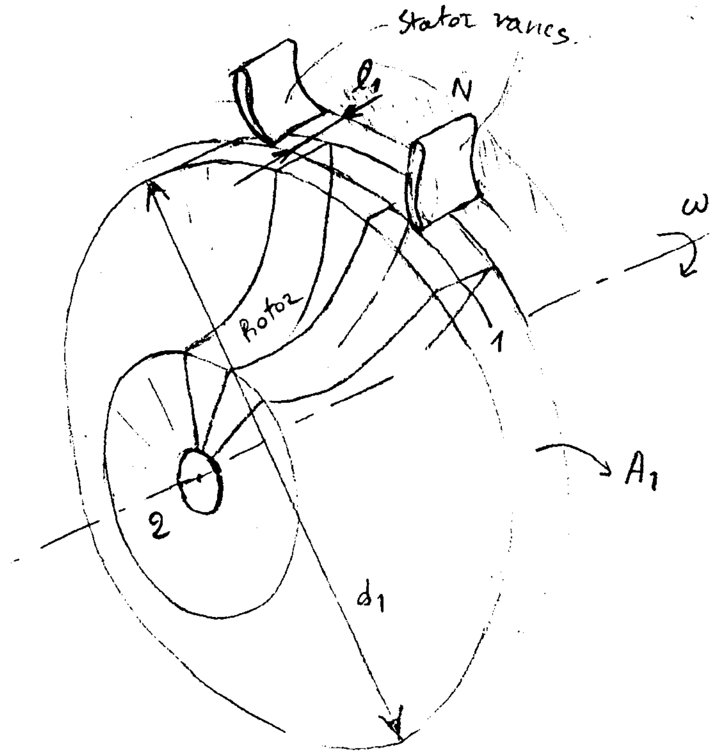
$P_{ON} = 2\ bar$

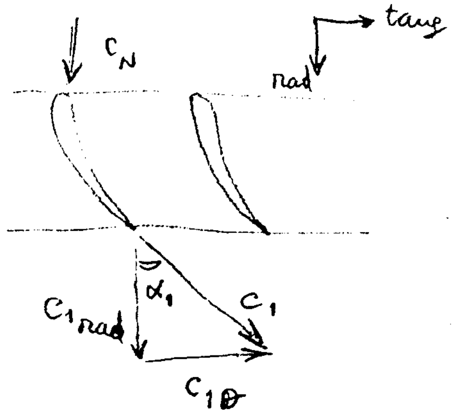
$T_{ON} = 125°C = 398\ K$

peripheral speed $u_1 = 0.9\, c_{1\theta}$

negligible losses $\Delta P_{O_{N-1}} \simeq 0$







Assume perfect gas (air) with $c_p = 1005\ J/KgK$ and $\gamma = 1.4$

Energy balance across the nozzle (stator):

$$\cancel{q_{in}}^{\,0} - \cancel{W_{out}}^{\,0} = h_{01} - h_{0N}$$

$$0 = c_p\,(T_{01} - T_{0N}) \rightarrow T_{01} = T_{0N} = 398\ K$$

Since the losses are negligible : $P_{01} = P_{0N} = 2 \cdot 10^5\ Pa$

$$\dot{m} = \rho_1 A_{1\perp} c_1 = \rho_1 A_1 c_1 \cos 75°$$

$$A_1 = \pi d_1 l_1 = \pi \cdot 0{,}25 \cdot 0{,}025 = 0{,}01963\ m^2$$

$$M_1 = \frac{\dot{m}}{P_{01} A_1 \cos 75^\circ} \sqrt{\frac{R T_{01}}{\gamma}} \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}$$ where $R = 287 \frac{J}{kg\,K}$

$$= \frac{2}{2 \cdot 10^5 \cdot 0{,}01963 \cos 75^\circ} \sqrt{\frac{287 \cdot 398}{1.4}} \left(1 + \frac{1.4-1}{2} M_1^2\right)^{\frac{1{,}4+1}{2(1{,}4-1)}} =$$

$$M_1 = 0{,}5623 \left(1 + 0{,}2 M_1^2\right)^3$$

iterate to find M_1 :

M_1
0
0.5623
0.672
⋮
0.822
0.823
0.823

$M_1 = 0.823$ ← ANS.

$$T_1 = T_{01} \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{-1} = 398 \left(1 + 0.2 \cdot 0.823^2\right)^{-1} = 350.5\,K$$ ← ANS.

$$P_1 = P_{01} \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{-1} = 2\,bar \left(1 + 0.2 \cdot 0.823^2\right)^{-\frac{1.4}{0.4}} = 1.282\,bar$$ ← ANS.

$$c_1 = M_1 \sqrt{\gamma R T_1} = 0.823 \sqrt{1.4 \cdot 287 \cdot 350.5} = 308.8\ \frac{m}{s}$$

$$u_1 = 0.9\, c_{\theta_1} = 0.9\, c_1 \sin 75^\circ = 0{,}9 \cdot 308.8 \sin 75^\circ = 268.5\ \frac{m}{s}$$

$$u_1 = \omega R_1 = \frac{2\pi N}{60} \frac{d_1}{2}$$

$$\hookrightarrow N = \frac{60\, u_1}{\pi d_1} = \frac{60 \cdot 268.5}{\pi\, 0.25} = 20'511 \text{ RPM}$$ ← ANS.

an alternative approach:

$$\rho_{01} = \frac{P_{01}}{RT_{01}} = \frac{2 \cdot 10^5 \ Pa}{287 \frac{J}{kg\,K} \cdot 398\,K} = 1.751 \frac{kg}{m^3}$$

guess ρ_1 $(\rho_1 < \rho_{01})$

$$\rightarrow \quad c_1 = \frac{\dot m}{\rho_1 A_1 \cos 75°} = \frac{2}{0{,}01963 \cdot \cos 75° \, \rho_1} = \frac{393.65}{\rho_1}$$

$$\rightarrow \quad T_1 = T_{01} - \frac{c_1^2}{2c_p} = 398 - \frac{c_1^2}{2 \cdot 1005} = 398 - \frac{c_1^2}{2010}$$

$$\rightarrow \quad P_1 = P_{01}\left(\frac{T_1}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = 2 \cdot \left(\frac{T_1}{398}\right)^{\frac{1.4}{0.4}} = 2\left(\frac{T_1}{398}\right)^{3.5}$$

$$\rightarrow \quad \rho_1 = \frac{P_1}{RT_1} = \frac{P_1}{287\,T_1}$$

	ρ_1 [kg/m³]	c_1 [m/s]	T_1 [K]	P_1 [bar]	
1st guess →	1.5	262.4	363.7	1.459	
	1.392				iterate
	...	...	...	...	
	1.274	308.8	350.5	1.282	← ANS.
	1.274				

then $M_1 = \frac{c_1}{\sqrt{\gamma R T_1}} = \frac{308.8}{\sqrt{1.4 \cdot 287 \cdot 350.5}} = \boxed{0.823}$ ← ANS.

get N (RPM) as in previous page

T-s diagram across turbine nozzle (stator) N−1 :