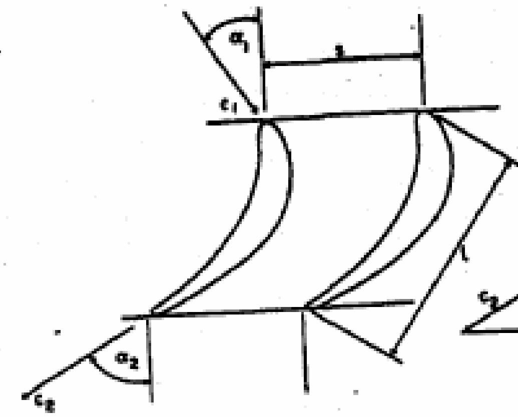
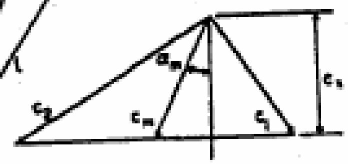
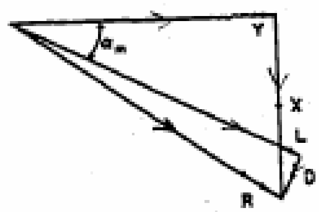
Ex. 3.2 Dixon

Solution. The figure shows part of a turbine blade cascade, the velocity triangle assuming c_x is constant and the force diagram.





From the velocity triangle the mean flow direction α_m is defined by $\tan \alpha_m = \frac{1}{2}(\tan \alpha_2 - \tan \alpha_1)$ so that



$c_m = c_x / \cos \alpha_m$. Referring to unit depth (span) of blade, the lift force L acting on the blade is perpendicular to c_m and the drag force D acting on the blade is parallel to c_m. The resultant force R has components X and Y in the axial and 'tangential' directions respectively. Resolving forces,

$$L = Y \cos \alpha_m + X \sin \alpha_m \tag{i}$$

$$D = X \cos \alpha_m - Y \sin \alpha_m \tag{ii}$$

With constant c_x the axial force acting on one blade is

$$X = (p_1 - p_2)s \tag{iii}$$

The tangential force acting on one blade is, from the momentum equation,

$$Y = \rho s c_x (c_{y2} + c_{y1}) = \rho s c_x^2 (\tan \alpha_2 + \tan \alpha_1) \tag{iv}$$

where ρ is a mean density through the cascade. With the 'incompressible' flow approximation (for simplicity), $p_o = p + \frac{1}{2}\rho c^2$, then the total pressure loss across the cascade is,

$$\Delta p_o = p_{o1} - p_{o2} = p_1 - p_2 + \frac{1}{2}\rho(c_1^2 - c_2^2)$$

$$\therefore p_1 - p_2 = \Delta p_o - \frac{1}{2}\rho(c_1^2 - c_2^2) \tag{v}$$

$$= \Delta p_o + \frac{1}{2}\rho(c_2^2 - c_1^2) =$$

$$= \Delta p_o + \frac{1}{2}\rho\left[\left(\cancel{c_x^2} + c_{2y}^2\right) - \left(\cancel{c_x^2} + c_{1y}^2\right)\right] =$$

$$= \Delta p_o + \frac{1}{2}\rho(c_{2y} + c_{1y})(c_{2y} - c_{1y})$$

where $c_y = c_1 \sin\alpha = c_x \, tg\alpha$

$$= \Delta p_o + \frac{1}{2}\rho c_x^2 (tg\alpha_1 + tg\alpha_2)(tg\alpha_2 - tg\alpha_1) =$$

$$= \Delta p_o + \rho c_x^2 \, tg\alpha_m (tg\alpha_2 + tg\alpha_1)$$

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After using eqns. (iv) and (vi) in eqns. (i) and (ii) it follows that

$$D = s\,\Delta p_o \cos\alpha_m$$

$$L = \rho s c_x^2 \sec\alpha_m (\tan\alpha_2 + \tan\alpha_1) + s\,\Delta p_o \sin\alpha_m$$

With the definitions $C_L = L/(\frac{1}{2}\rho c_m^2 \ell)$ and $C_D = D/(\frac{1}{2}\rho c_m^2 \ell)$,

$$C_L = 2(s/\ell)\cos\alpha_m(\tan\alpha_2 + \tan\alpha_1) + C_D \tan\alpha_m \qquad \text{(vii)}$$

$$C_D = 2(s/\ell)\cos\alpha_m\left[\Delta p_o/(\rho c_m^2)\right] \qquad \text{(viii)}$$

The blade load ratio, eqn. ~~(3.51)~~, is (3.55)

λ

$$\psi_T = 2(s/b)\cos^2\alpha_2(\tan\alpha_2 + \tan\alpha_1)$$

At cascade exit the flow angle α_2 is less than the blade outlet angle α_2' by the amount of the deviation.

$$\alpha_2 = \alpha_2' - \delta = 65.5 - 1.5 = 64 \text{ deg}$$

At cascade inlet the blade angle α_1' is zero, the flow incidence is zero so that the flow angle $\alpha_1 = 0$. Thus, with $\alpha_1 = 0$, the space/chord ratio is

$$s/\ell = (b/\ell)(s/b) = (b/\ell)\psi_T/\sin 2\alpha_2$$

$$= (32/45)0.85/\sin(2 \times 64^\circ) = \underline{0.767}$$

From the velocity triangles, $c_x = c_2\cos\alpha_2 = c_m\cos\alpha_m$, then $c_m = c_2\cos\alpha_2/\cos\alpha_m$ and $\tan\alpha_m = \frac{1}{2}\tan\alpha_2$. Thus, $\alpha_m = 45.71^\circ$. Using this expression in eqn. (viii) the drag coefficient becomes

$$C_D = (s/\ell)\cos\alpha_m\left(\frac{\Delta p_o}{\frac{1}{2}\rho c_2^2}\right)\left(\frac{\cos\alpha_m}{\cos\alpha_2}\right)^2$$

$$= (s/\ell)\,\lambda\cos^3\alpha_m/\cos^2\alpha_2$$

$$= 0.767 \times 0.035 \times \cos^3 45.71^\circ/\cos^2 64^\circ$$

$$= \underline{0.0476}$$

From eqn. (vii) the lift coefficient can now be calculated

$$C_L = 2 \times 0.767 \times \cos 45.71^\circ \times \tan 64^\circ + 0.0476 \times \tan 45.71^\circ$$

$$= 2.196 + 0.049 = \underline{2.245}$$

<u>N.B.</u> In a turbine cascade with $\alpha_m > 0$, the drag slightly increases the lift which is the converse of what occurs in a compressor cascade.

Ex. 3.6 Dixon

Solution. (a) The loss in total pressure across a compressor cascade due to irreversible processes is, for an incompressible flow,

$$\Delta p_o = p_{o1} - p_{o2} = (p_1 - p_2) + \frac{1}{2}\rho(c_1^2 - c_2^2)$$

$$= -\Delta p + \frac{1}{2}\rho c_1^2\left[1 - (c_2/c_1)^2\right]$$

where $\Delta p = p_2 - p_1$, is the static pressure rise across the cascade. With $c_1 \cos\alpha_1 = c_2 \cos\alpha_2 = c_x =$ constant

$$\Delta p_o/(\tfrac{1}{2}\rho c_1^2) = -\Delta p/(\tfrac{1}{2}\rho c_1^2) + (1 - \cos^2\alpha_1/\cos^2\alpha_2)$$

$$\therefore C_p = 1 - \Delta p_o/(\tfrac{1}{2}\rho c_1^2) - \cos^2\alpha_1/\cos^2\alpha_2$$

$$= 1 - \zeta\cos^2\alpha_1 - \cos^2\alpha_1/\cos^2\alpha_2$$

$$= 1 - (\zeta + \sec^2\alpha_2)/\sec^2\alpha_1 \qquad \text{(i)}$$

From the definition of diffuser efficiency

$$\Delta p = \frac{1}{2}\rho(c_1^2 - c_2^2)\eta_D$$

$$\therefore C_p = \eta_D(1 - c_2^2/c_1^2) = \eta_D(1 - \sec^2\alpha_2/\sec^2\alpha_1) \qquad \text{(ii)}$$

(b) For a compressor cascade of specified geometry the diffusion factor D_F increases rapidly with increasing inlet flow angle as the positive stall "point" is approached. With $\alpha_2 = 30$ deg, $s/\ell = 0.8$ and $D_F = 0.6$ substituted in the Lieblein formula:-

$$0.6 = 1 - \cos\alpha_1/0.866 + 0.4(\sin\alpha_1 - 0.5774\cos\alpha_1)$$

Putting $x = \cos\alpha_1$, $(1 - x^2)^{1/2} = \sin\alpha_1$ and rearranging,

$$x(1/0.866 + 0.4 \times 0.5774) = 0.4\left[1 + (1 - x^2)^{1/2}\right]$$

$$\therefore (3.464x - 1)^2 = 1 - x^2$$

$$\therefore 13x^2 - 6.928x \cancel{+1} = \cancel{1}$$

$$\therefore x = \cos\alpha_1 = 6.928/13 = 0.5329$$

Thus, the <u>maximum</u> inlet flow angle (i.e. for positive stall) to give a diffusion factor $D_F = 0.6$ is

$$\underline{\alpha_1 = 57.8 \text{ deg}}$$

(c) With $c_x = c_1\cos\alpha_1 = 100 \times \cos 57.8^\circ = 53.29$ m/s, the total pressure loss coefficient is immediately found, i.e.

$$\zeta = \Delta p_o/(\tfrac{1}{2}\rho c_x^2) = 149/(\tfrac{1}{2} \times 1.2 \times 53.29^2) = 0.0875$$

Using eqn. (i),

$$\begin{aligned} C_p &= 1 - (0.0875 + \sec^2 30^\circ)/\sec^2 57.8^\circ \\ &= 1 - (0.0875 + 1.3333) \times 0.5329^2 \\ &= 0.5965 \end{aligned}$$

The pressure rise is,

$$\begin{aligned} \Delta p &= p_2 - p_1 = \tfrac{1}{2} C_p \rho c_1^2 = \tfrac{1}{2} \times 0.5965 \times 1.2 \times 10^4 \\ &= \underline{3.579 \text{ kPa}} \end{aligned}$$

From eqn. (ii) the diffuser efficiency is,

$$\begin{aligned} \eta_D &= C_p/(1 - \cos^2 \alpha_1/\cos^2 \alpha_2) \\ &= 0.5965/(1 - 0.5329^2/0.866^2) = 0.5965/0.6213 \\ &= \underline{0.96} \end{aligned}$$

The drag coefficient is defined, eqns. (3.16b) and (3.17), as

$$\begin{aligned} C_D &= D/(\tfrac{1}{2}\rho c_m^2 \ell) = s \Delta p_o \cos \alpha_m/(\tfrac{1}{2}\rho c_m^2 \ell) \\ &= \zeta (s/\ell) \cos^3 \alpha_m \end{aligned}$$

where $\tan \alpha_m = \tfrac{1}{2}(\tan \alpha_1 + \tan \alpha_2) = \tfrac{1}{2}(\tan 57.8^\circ + \tan 30^\circ) = 1.0827$

$$\begin{aligned} \therefore \alpha_m &= 47.27 \text{ deg} \\ \therefore C_D &= 0.0875 \times 0.8 \times \cos^3 47.27^\circ \\ &= \underline{0.0219} \end{aligned}$$

The lift coefficient is defined for a compressor cascade, eqn. (3.18), as

$$\begin{aligned} C_L &= 2(s/\ell)\cos \alpha_m(\tan \alpha_1 - \tan \alpha_2) - C_D \tan \alpha_m \\ &= 2 \times 0.8 \times \cos 47.27^\circ(\tan 57.8^\circ - \tan 30^\circ) - 0.0219 \times 1.0827 \\ &= 1.0972 - 0.0237 \\ &= \underline{1.074} \end{aligned}$$