4.3 mixed flow turbine

$r_1 = 0.3\ m$

$r_2 = 0.1\ m$

$N = 20000\ RPM$

$\dot{W} = 430\ kW$

$c_1 = 700\ m/s$

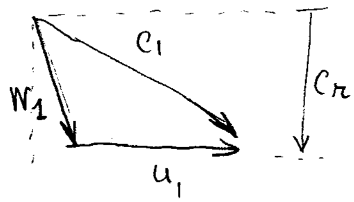
$\alpha_1 = 70°$

? $\alpha_2, \beta_1, \beta_2, c_2$

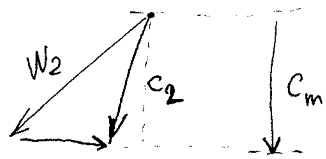
$\dot{m} = 1\ kg/s$

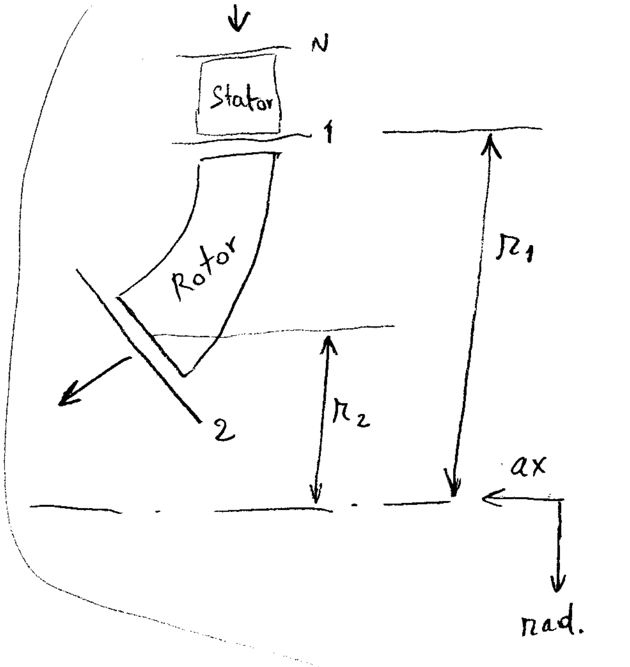
$c_r = c_m = const$

at station 1 :



at station 2 :





$$\dot{W} = \dot{m}\left(u_1 c_{1\theta} + u_2 c_{2\theta}\right)$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 20000}{60} = 2094.4\ \frac{rad}{s} \rightarrow \begin{cases} u_1 = \omega r_1 = 628.3\ m/s \\ u_2 = \omega r_2 = 209.4\ m/s < u_1 \end{cases}$$

$$c_{2\theta} = \left(\frac{\dot{W}}{\dot{m}} - u_1 c_{1\theta}\right)\frac{1}{u_2} = \left(\frac{430\times 10^3\ J/s}{1\ kg/s} - 628.3 \times 700 \times \sin 70°\right)\frac{1}{209.4} = 79.8\ \frac{m}{s}$$

$$c_m = c_r = c_1 \cos\alpha_1 = 700 \cdot \cos 70° = 239.4\ m/s$$

$$c_m = c_2 \cos\alpha_2 = c_{2\theta}/\tan\alpha_2 \rightarrow \alpha_2 = \tan^{-1}\left(\frac{c_{2\theta}}{c_m}\right) = \underline{18.4°}\ ✓$$

$$c_2 = \frac{c_m}{\cos\alpha_2} = \frac{239.4}{\cos 18.4°} = \underline{252.4\ m/s}\ ✓$$

$$\left.\begin{aligned} W_{1m} &= c_m \\ W_{1\theta} &= c_{1\theta} - u_1 = 700 \sin 70° - 628.3 = 29.5\ m/s \end{aligned}\right\} \rightarrow \tan\beta_1 = \frac{W_{1\theta}}{W_{1m}} = 0.123$$

$$\rightarrow \underline{\beta_1 = 7.02°}\ ✓$$

$$\left.\begin{aligned} W_{2m} &= c_m \\ W_{2\theta} &= u_2 + c_{2\theta} = 209.4 + 79.8 = 289.2\ m/s \end{aligned}\right\} \rightarrow \tan\beta_2 = \frac{W_{2\theta}}{c_m} = 0.83$$

$$\rightarrow \underline{\beta_2 = 50.3°}\ ✓$$

$$W_2 = \frac{W_{2\theta}}{\sin\beta_2} = 375.9\ m/s$$

$$W_1 = \frac{W_{1\theta}}{\sin\beta_1} = \frac{29.5}{\sin 7.02^\circ} = 241.4 \text{ m/s}$$

$$\Delta W_T = \frac{c_1^2 - c_2^2}{2} + \frac{u_1^2 - u_2^2}{2} + \frac{w_2^2 - w_1^2}{2} =$$

$$= 213.15 + 175.46 + 41.51 \ \frac{kJ}{kg} = 430.12 \ \frac{kJ}{kg}$$ (add up to ΔW_T ok!)

$$\Delta W_T = \frac{\dot{W}}{\dot{m}} = 430 \ \frac{kJ}{kg}$$

$$\left.\begin{array}{l} \dfrac{c_1^2 - c_2^2}{2\,\Delta W_T} \cdot 100\% = 49.6\% \\[2ex] \dfrac{u_1^2 - u_2^2}{2\,\Delta W_T} \cdot 100\% = 40.8\% \\[2ex] \dfrac{w_2^2 - w_1^2}{2\,\Delta W_T} \cdot 100\% = 9.6\% \end{array}\right\} \text{contribution to the work}$$

In this mixed flow turbine the flow experiences a significant change in radius across the rotor resulting in a larger work output than would be obtained from a turbine with a purely 2D flow.

EX. 4.4

Solution. The velocity diagram for the stage can be readily constructed from the data supplied and the specific work obtained from a scale drawing or, more accurately, by calculation. It will be noticed that as the efflux angle relative to each blade row is equal, i.e. $\alpha_2 = \beta_3 = 70$ deg, the velocity triangles are similar and the reaction is 50 per cent. The specific work per stage is

$$\Delta W = U(c_{y2} + c_{y3})$$

Solving for the unknown swirl velocities using the usual sign convention

$$c_{y2} = c_2 \sin \alpha_2 = 160 \sin 70^\circ$$
$$= 150.4 \text{ m/s}$$
$$c_{y3} = w_3 \sin \beta_3 - U = c_2 \sin \alpha_2 - U$$
$$= 150.4 - 152.5 = -2.1 \text{ m/s}$$
$$\therefore \Delta W = 152.5(150.4 - 2.1)$$
$$= \underline{22.62 \text{ kJ/kg}}$$

This stage is rather lightly loaded and the stage loading factor is

$$\psi = \Delta W/U^2 = (c_{y2} + c_{y3})/U = 148.3/152.5$$
$$= 0.9725$$

A turbine with ten similar stages to the one above will produce a specific work of 226.2 kJ/kg and this is equal to the change in stagnation enthalpy of the steam $h_{oA} - h_{oB}$ between turbine inlet (A) and turbine exhaust (B), i.e.

$$h_{oA} - h_{oB} = 226.2 \text{ kJ/kg}$$

It is implied that the "internal" efficiency is the total to total efficiency, defined as

$$\eta_{tt} = (h_{oA} - h_{oB})/(h_{oA} - h_{oBs})$$
$$\therefore h_{oA} - h_{oBs} = 226.2/0.8 = 282.8 \text{ kJ/kg}$$

From steam tables or Mollier chart at $p_{oA} = 1.5$ MPa (15 bar) and $T_{oA} = 300^\circ$C

$$h_{oA} = 3039 \text{ kJ/kg}$$
$$\therefore h_{oB} = 3039 - 226.2 = 2812.8 \text{ kJ/kg}$$

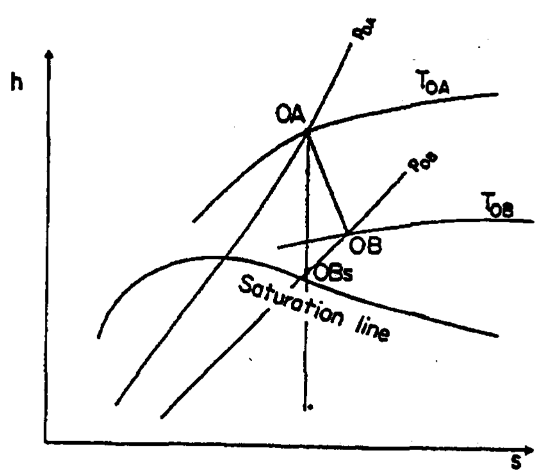
$$\therefore h_{oBs} = 2756.2 \text{ kJ/kg}$$

The less laborious method of determining the exhaust steam condition is by plotting these specific enthalpies on a large scale Mollier chart for steam, From such a plot the exhaust steam condition is

p_{oB} = <u>420 kPa</u> (4.2 bar);

T_{oB} = $\underline{177^\circ\text{C}}$

i.e. the steam is still <u>superheated</u> at exhaust.



(e.g. "Enthalpy-Entr Diagram for Steam" prepared by D.C. Hicks and F.R. Taylor)

4-6

4.6. In a certain axial flow turbine stage the axial velocity c_x is constant. The absolute velocities entering and leaving the stage are in the axial direction. If the flow coefficient c_x/U is 0.6 and the gas leaves the stator blades at 68.2 deg from the axial direction, calculate:

(i) the stage loading factor, $\Delta W/U^2$;

(ii) the flow angles relative to the rotor blades;

(iii) the degree of reaction;

(iv) the total to total and total to static efficiencies.

The Soderberg loss correlation, eqn. (4.12) should be used.

Solution. (i) The stage loading factor is

$$\psi = \Delta W/U^2 = c_{y2}/U, \text{ as } c_{y3} = 0$$
$$= (c_x/U)\tan\alpha_2$$
$$= 0.6 \times \tan 68.2^\circ = \underline{1.50}$$

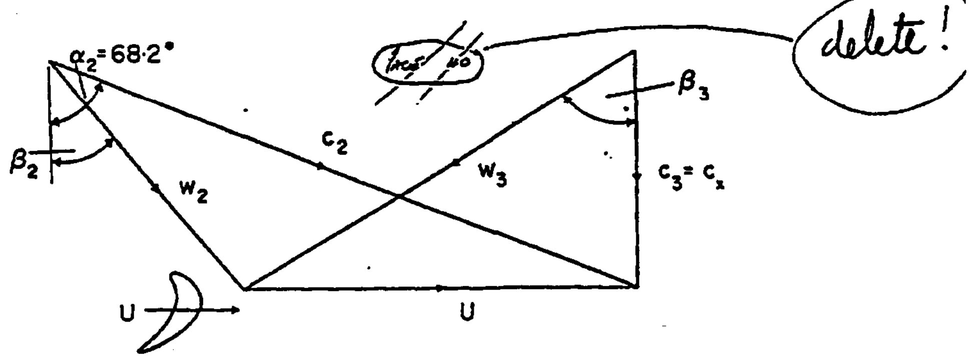
(ii) From the velocity diagram

$$\tan\beta_3 = U/c_x = 1/0.6 = 1.667$$
$$\therefore \beta_3 = \underline{59.04 \text{ deg}}$$

$$\tan\beta_2 = \tan\alpha_2 - U/c_x = 2.5 - 1.667 = 0.8335$$
$$\therefore \beta_2 = \underline{39.81 \text{ deg}}$$

(iii) The stage reaction, eqn. (4.22a), is

$$R = (\tan\beta_3 - \tan\beta_2)\, c_x/(2U)$$
$$= 0.3(1.667 - 0.8335) = \underline{0.25}$$



(iv) The total to total efficiency of a normal stage ($c_1 = c_3$) is,

$$\eta_{tt} = (h_{01} - h_{03})/(h_{01} - h_{03ss}) = (h_1 - h_3)/(h_1 - h_3 + h_3 - h_{3ss})$$
$$= 1/[1 + (h_3 - h_{3ss})/(h_1 - h_3)]$$

Referring to Fig. 4.2, the enthalpy difference $h_{3s} - h_{3ss}$, equal to $(h_2 - h_{2s})(T_3/T_2)$, is usually simplified to $h_2 - h_{2s}$ with only a small loss in accuracy in determining efficiency.

$$\therefore \eta_{tt} \simeq [1 + (h_3 - h_{3s} + h_2 - h_{2s})/(h_1 - h_3)]^{-1}$$

The enthalpy differences $h_2 - h_{2s}$ and $h_3 - h_{3s}$ representing the effects of irreversible flow in the nozzle and the rotor respectively, can be expressed in terms of loss coefficients ζ_N and ζ_R,

$$h_2 - h_{2s} = \frac{1}{2}c_2^2 \zeta_N$$

$$h_3 - h_{3s} = \frac{1}{2}w_3^2 \zeta_R$$

Thus, the total to total efficiency becomes, eqn. (4.9a),

$$\eta_{tt} = \left[1 + \frac{\zeta_R w_3^2 + \zeta_N c_2^2}{2(h_1 - h_3)}\right]^{-1} \qquad \text{(i)}$$

The total to static efficiency, defined as

$$\eta_{ts} = (h_{o1} - h_{o3})/(h_{o1} - h_{3ss})$$

$$= 1/\left[1 + (h_3 - h_{3ss} + \frac{1}{2}c_1^2)/(h_1 - h_3)\right]$$

is used when the exhaust kinetic energy $\frac{1}{2}c_3^2$ is wasted. This efficiency is most useful in the form,

$$\eta_{ts} = \left[1 + \frac{\zeta_R w_3^2 + \zeta_N c_2^2 + c_1^2}{2(h_1 - h_3)}\right]^{-1} \qquad \text{(ii)}$$

The enthalpy loss coefficients can be expressed, eqn. (4.12), in terms of the fluid deflection ε (deg) of each blade row, that is,

$$\zeta = 0.04\left[1 + 1.5(\varepsilon/100)^2\right]$$

where, for the nozzle, $\varepsilon = \varepsilon_N = \alpha_1 + \alpha_2 = 68.2$ deg (i.e. $\alpha_1 = 0$) and, for the rotor row, $\varepsilon = \varepsilon_R = \beta_2 + \beta_3 = 39.81 + 59.04 = 98.85$ deg. Thus, $\zeta_N = 0.06791$ and $\zeta_R = 0.09863$ after using the above equation.

From eqn. (i), with $w_3 = c_x \sec\beta_3$, $c_2 = c_x \sec\alpha_2$ and $h_1 - h_3 = Uc_x \tan\alpha_2$

$$\eta_{tt} = \left[1 + \frac{\zeta_R \sec^2\beta_3 + \zeta_N \sec^2\alpha_2}{(2\tan\alpha_2)/\phi}\right]^{-1}$$

$$= \left[1 + \frac{0.09863/0.5144^2 + 0.06791/0.3714^2}{2 \times 2.5/0.6}\right]^{-1}$$

$$= \left[1 + \frac{0.865}{8.334}\right]^{-1}$$

$$\therefore \underline{\eta_{tt} = 90.6\%}$$

From eqn. (ii), with $c_1 = c_x$

$$\eta_{ts} = \left[1 + (0.865 + 1)/8.334\right]^{-1} = \underline{81.7\%}$$