HW 5

Dixon Ch. 5: Ex. 1

<u>Solution</u>. The number of stages is determined from the stagnation temperature rise per stage ΔT_o, obtained from the *specific work done equation* and velocity diagram, and the overall stagnation temperature rise through the compressor, $T_{oB} - T_{oA}$, obtained from the overall stagnation pressure ratio, p_{oB}/p_{oA}, together with the polytropic efficiency, η_p. The number of identical compressor stages, n, is obtained to the nearest integer from

$$n = (T_{oB} - T_{oA})/\Delta T_o \qquad \text{(i)}$$

The specific work done by the rotor on the air, eqn. (5.1), is

$$\Delta W = h_{o2} - h_{o1} = C_p \Delta T_o = U(c_{y2} - c_{y1}) \qquad \text{(ii)}$$

Referring to the mean radius velocity diagram and noticing the velocity triangles are symmetrical for a reaction of 0.5 (i.e. $\beta_2 = \alpha_1$),

$$c_{y2} - c_{y1} = U - 2c_x \tan \alpha_1$$

and, from eqn. (ii)

$$\Delta T_o = U(U - 2c_x \tan \alpha_1)/C_p \qquad \text{(iii)}$$

The <u>average</u> axial velocity $\bar{c}_x$ is obtained from equation of continuity, $\dot{m} = \rho A \bar{c}_x$, the density being determined with the incompressible flow approximation $\rho = \rho_{o1} = p_{o1}/(RT_{o1})$. Thus,

$$\rho_{o1} = p_{o1}/(RT_{o1}) = 10^5/(287 \times 296) = 1.177 \text{ kg/m}^3$$

$$\bar{c}_x = 4\dot{m}/[\rho_{o1} \pi (d_{t1}^2 - d_{h1}^2)]$$

$$= 4 \times 50/[\pi \times 1.177 (0.728^2 - 0.436^2)]$$

$$= 159.1 \text{ m/s}$$

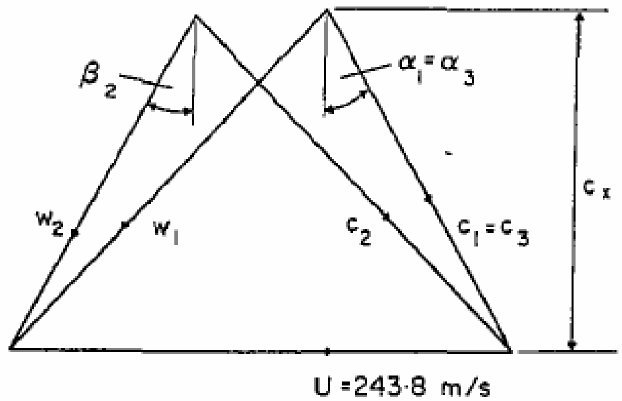
The axial velocity at the mean radius is

$$c_x = 1.05 \times \bar{c}_x = 167.1 \text{ m/s}$$

The mean blade speed is

$$U = \pi N d_m/60 = \pi N(d_{h1} + d_{t1})/120$$

$$= \pi \times 8000(0.436 + 0.728)/120 = 243.8 \text{ m/s}$$



Mean radius velocity diagram

Polytropic efficiency for a small compressor stage is defined, eqn. (2.31), as

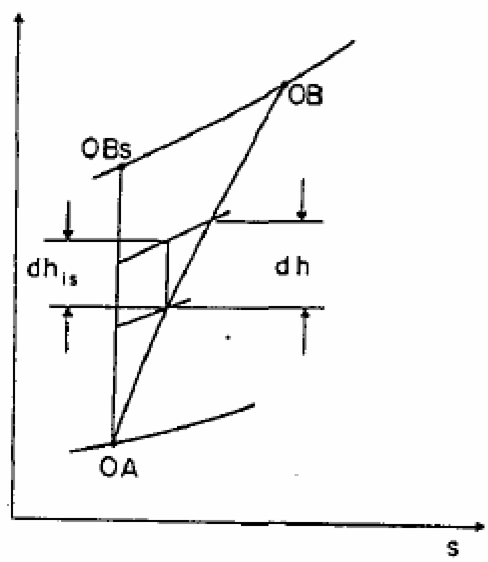
$$\eta_p = dh_{is}/dh = v\,dp/C_p\,dT = (\gamma - 1)T\,d_p/(\gamma p\,dT)$$

after using the perfect gas relations, $pv = RT$ and $C_p = \gamma R/(\gamma - 1)$.

$$\therefore T = \text{constant} \times p^{(\gamma-1)/\gamma\eta_p} \qquad \text{(iv)}$$

As the stages are similar with identical velocities, stagnation conditions can be used in eqn. (iv). Thus, across the whole compressor,

$$T_{oB}/T_{oA} = (p_{oB}/p_{oA})^{(\gamma-1)/\gamma\eta_p}$$
$$= 5^{1/(3.5 \times 0.89)}$$
$$= 5^{1/3.115}$$
$$= 1.6764$$
$$\therefore T_{oB} - T_{oA} = 0.6764 \times 296$$
$$= 200.2^\circ C$$



From eqn. (iii),

$$\Delta T_o = 243.8(243.8 - 2 \times 167.1 \times \tan 28.8^\circ)$$
$$= 14.57^\circ C$$

Using eqn. (i)

$$n = 200.2/14.57 = 13.74$$

$\therefore$ The number of stages required is 14.

Solution. The degree of reaction of an axial flow compressor stage is defined as the static enthalpy rise in the rotor divided by the static enthalpy rise in the stage, i.e.

$$R = (h_2 - h_1)/(h_3 - h_1) \qquad (i)$$

As the relative stagnation enthalpy is constant in the rotor, then

$$h_2 - h_1 = \frac{1}{2}(w_1^2 - w_2^2)$$

Assuming a normal stage (i.e. $c_1 = c_3$), then

$$h_3 - h_1 = h_{o3} - h_{o1} = \Delta W = U(c_{y2} - c_{y1})$$

Substituting into eqn. (i)

$$R = (w_1^2 - w_2^2)/\left[2U(c_{y2} - c_{y1})\right]$$

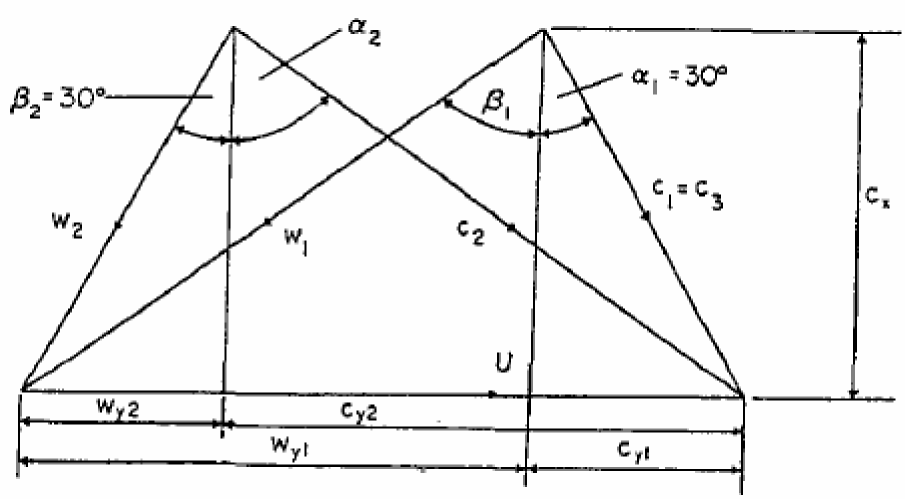
$$= (w_{y1} + w_{y2})(w_{y1} - w_{y2})/\left[2U(c_{y2} - c_{y1})\right] \qquad (ii)$$

where it is assumed that c_x is constant across the stage. From the velocity triangles for the compressor stage, $c_{y2} = U - w_{y2}$ and $c_{y1} = U - w_{y1}$ so that $c_{y2} - c_{y1} = w_{y1} - w_{y2}$. Simplifying eqn. (ii),

$$R = (w_{y1} + w_{y2})/(2U) = \phi(\tan\beta_1 + \tan\beta_2)/2 \qquad (iii)$$

where the flow coefficient $\phi = c_x/U$.

The data given in the problem enables the velocity diagram shape to be drawn



immediately. The magnitudes of the velocity vectors must be calculated from the information concerning maximum relative Mach number. From the velocity diagram the maximum relative velocity is w_1 and the corresponding relative Mach number

$$M_{r1} = w_1/(\gamma R T_1)^{1/2} \qquad \text{(iv)}$$

where the static temperature $T_1 = T_{o1} - c_1^2/(2C_p)$. It is most convenient to solve in terms of the axial velocity c_x. Writing $w_1 = c_x/\cos\beta_1$ and $c_1 = c_x/\cos\alpha_1 = c_x/0.866$, eqn. (iv) gives,

$$w_1^2 = \gamma R M_{r1}^2 T_1$$

$$= \gamma R M_{r1}^2 \left[T_{o1} - c_1^2/(2C_p)\right]$$

$$c_x^2 = \gamma R M_{r1}^2 \left[T_{o1} - c_x^2/(1.5\,C_p)\right] \cos^2\beta_1 \qquad \text{(v)}$$

Using the equation (iii), β_1 can be determined as follows,

$$\tan\beta_1 = 2R/\phi - \tan\beta_2$$

$$= 2 - \tan 30^o = 1.4227$$

$$\therefore \beta_1 = 54.9 \text{ deg}$$

Substituting values into eqn. (v),

$$c_x^2 = 1.4 \times 287 \times 0.49\left[289 - c_x^2/(1.5 \times 1005)\right] \times 0.5751^2$$

$$= 1.882 \times 10^4 - 0.0432\, c_x^2$$

$$\therefore c_x = 134.3 \text{ m/s}$$

The stagnation temperature rise in the stage ΔT_o can now be immediately determined using the equation for the specific work,

$$\Delta W = C_p \Delta T_o = U(c_{y2} - c_{y1}) = U(w_{y1} - w_{y2})$$

$$= c_x^2(\tan\beta_1 - \tan\beta_2)/\phi$$

$$\therefore \Delta T_o = c_x^2(\tan\beta_1 - \tan\beta_2)/(\phi C_p)$$

$$= 134.3^2(\tan 54.9^o - \tan 30^o)/(0.5 \times 1005)$$

$$= \underline{30.35^oC}$$

Solution. It is tactitly assumed that the flow preceding the first stage is deflected by inlet guide vanes to give an absolute flow angle α_1 of 30 deg, the same as all the other stages. The absolute inlet flow velocity c_1 is determined from the stagnation enthalpy definition

$$h_{o1} = h_1 + \frac{1}{2}c_1^2$$

$$\therefore c_1^2 = 2C_p(T_{o1} - T_1)$$

where $C_p = \gamma R/(\gamma - 1)$, and the isentropic temperature-pressure relationship,

$$T_1/T_{o1} = (p_1/p_{o1})^{(\gamma-1)/\gamma}$$

$$= (87.3/101.3)^{1/3.5} = 0.9584$$

$$\therefore c_1^2 = 2C_pT_{o1}(1 - T_1/T_{o1}) = 2 \times 1005 \times 278(1 - 0.9584)$$

$$= 2.325 \times 10^4$$

$$c_1 = 152.5 \text{ m/s}$$

Thus, the axial velocity is

$$c_x = c_1 \cos\alpha_1 = 152.5 \cos 30^\circ$$

$$= \underline{132.1 \text{ m/s}}$$

Using the equation of continuity, the mass flow rate is

$$\dot{m} = \rho_1 A_1 c_x$$

where

$$\rho_1 = p_1/(RT_1) = 87.3 \times 10^3/(287 \times 0.9584 \times 278)$$

$$= 1.1417 \text{ kg/m}^3$$

$$\therefore \dot{m} = 1.1417 \times 0.372 \times 132.1$$

$$= \underline{56.1 \text{ kg/s}}$$

The specific work done on the gas per stage is

$$\Delta W = U(c_{y2} - c_{y1}) = U(U - 2c_x \tan \alpha_1)$$
$$= U^2(1 - 2\phi \tan \alpha_1)$$

as the velocity triangles are similar for a reaction of 0.5.

$$\therefore \Delta W = (2 \times 132.1)^2 (1 - \tan 30^\circ)$$
$$= 29.5 \text{ kJ/kg}$$

The shaft power needed to drive the compressor (including mechanical losses) is

$$\dot{W}_c = n\dot{m}\,\Delta W/\eta_m$$

where n is the number of stages and η_m the mechanical efficiency. Thus,

$$\dot{W}_c = 6 \times 56.1 \times 29.5 \times 10^3/0.99$$

$$= \underline{10.03 \text{ MW}}$$