Solution. Sufficient data are given to solve the mean radius velocity triangles from which the flow coefficient and stage loading factors are obtained.

At $r = r_m = 0.36$ m, $\alpha_2 = 68$ deg, $p_2 = 207$ kPa, $R = 0.2$, $T_{o1} = T_{o2} = 838$ K and $p_{o1} = p_{o2} = 350$ kPa, assuming adiabatic frictionless nozzle flow. Since $h_{o2} = h_2 + \frac{1}{2}c_2^2$,

$$c_2^2 = 2C_p(T_{o2} - T_2)$$
$$= 2C_p T_{o2}\left[1 - (p_2/p_{o2})^{(\gamma-1)/\gamma}\right]$$
$$= 2 \times 1148 \times 838\left[1 - (207/350)^{0.248}\right] = 23.51 \times 10^4$$
$$\therefore c_2 = 484.9 \text{ m/s}$$

The mean blade speed is

$$U_m = (2\pi N/60)r_m = (2\pi \times 8000/60)0.36$$
$$= 301.6 \text{ m/s}$$

Hence, the mean flow coefficient is

$$\phi_m = c_x/U_m = c_2 \cos\alpha_2/U_m$$
$$= 484.9 \times \cos 68^\circ/301.6$$
$$\therefore \phi_m = 0.6023$$

The stage reaction is defined as

$$R = \frac{h_2 - h_3}{h_1 - h_3} = \frac{h_2 - h_3}{h_{o1} - h_{o3}} \qquad (\text{if } c_1 = c_3)$$

$$\therefore\ 1 - R = \frac{h_{o1} - h_{o3} - h_2 + h_3}{h_{o1} - h_{o3}} = \frac{c_2^2 - c_3^2}{2U(c_{\theta 2} + c_{\theta 3})} = \frac{c_{\theta 2} - c_{\theta 3}}{2U} \quad \text{(i)}$$

At the mean radius,

$$c_{\theta 2} - c_{\theta 3} = 2U_m(1 - R_m) = 2 \times 301.6 \times 0.8 = 482.6 \text{ m/s},$$

$$c_{\theta 2} = c_2 \sin \alpha_2 = 484.9 \times \sin 68^{\circ} = 449.6 \text{ m/s}$$

$$\therefore c_{\theta 3} = -33.0 \text{ m/s}$$

The stage loading factor at the mean radius is

$$\psi_m = \Delta W / U_m^2 = (c_{\theta 2} + c_{\theta 3})/U_m = (449.6 - 33)/301.6$$

$$\therefore \underline{\psi_m = 1.381}$$

From eqn. (i) above, the reaction at any radius is

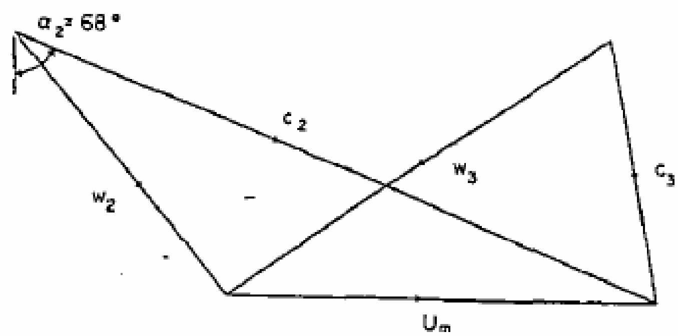
$$R = 1 - (c_{\theta 2} - c_{\theta 3})/(2U)$$

where, for a free-vortex, $c_{\theta 2} = K_2/r$, $c_{\theta 3} = K_3/r$ and the blade speed $U = \Omega r$.

Substituting for $c_{\theta 2}$, $c_{\theta 3}$ and U

$$R = 1 - k/(r/r_m)^2$$

where $k = (K_2 - K_3)/(2\,\Omega\, r_m^2)$



Velocity triangles at mean radius

Solving for k with $R = 0.2$ at $r = r_m$

$$R = 1 - 0.8/(r/r_m)^2$$

The reaction at the hub, $r = r_h = 0.31$ m, is

$$R_h = 1 - 0.8/0.861^2$$

$$= \underline{-0.079}$$

The negative reaction would imply that diffusion of the flow occurs in the rotor row (i.e. $w_3 < w_2$) at the root. For a turbine blade row, flow diffusion results in poor efficiency caused by large total pressure losses. A poor flow distribution will result and this can adversely affect the performance of any subsequent stages. Turbine designers always aim for a positive root reaction to avoid this problem.