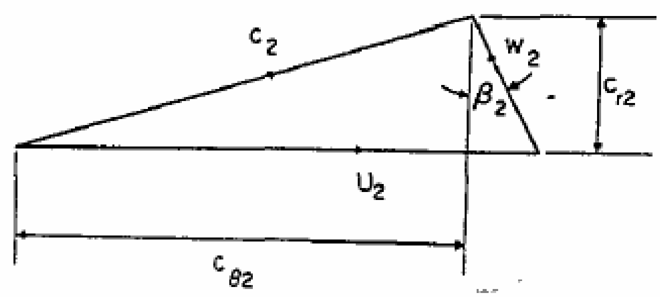
Ex. 7.2

Solution. The specific work required to compress the air is $\Delta W = h_{o3} - h_{o1} = U_2 c_{\theta 2}$, the flow being without swirl at impeller entry. It follows from the velocity triangle at the impeller exit (see sketch) that

$$c_{\theta 2} = U_2 - c_{r2} \tan \beta_2$$
$$= 500 - 120 \times \tan 26.6^\circ$$
$$= 440 \text{ m/s}$$
$$\therefore \Delta W = 440 \times 500 = 220 \text{ kJ/kg}$$



The theoretical power needed (i.e. ignoring mechanical losses) is

$$\dot{W}_c = \dot{m}\,\Delta W = 2.5 \times 220$$
$$= 550 \text{ kW}$$

Hence, the actual power needed is

$$P = \dot{W}_c/\eta_m = 550/0.95$$
$$= \underline{578.9 \text{ kW}}$$

The equation of continuity, with $c_{x1} = c_1$, is

$$\dot{m} = \rho_1 A_1 c_1 = \pi \rho_1 (r_{s1}^2 - r_{h1}^2) c_1$$
$$= \pi \rho_1 c_1 r_{s1}^2 \left[1 - (r_{h1}/r_{s1})^2\right]$$

With the assumption that the flow at entry is incompressible, the density ρ_1 car determined from the stagnation pressure and temperature,

$$\rho_1 = \rho_{o1} = p_{o1}/(RT_{o1}) = 1.013 \times 10^5/(287 \times 288)$$
$$= 1.226 \text{ kg/m}^3$$
$$\therefore r_{s1}^2 = \dot{m}/\{\pi \rho_1 c_1 \left[1 - (r_{h1}/r_{s1})^2\right]\}$$
$$= 2.5/(\pi \times 1.226 \times 100 \times 0.91)$$
$$= 0.7135 \times 10^{-2}$$
$$\therefore \underline{d_{s1} = 169 \text{ mm}}$$

The total to total efficiency of the compressor ~~(eqn. 7.20)~~ is

$$\eta_c = \frac{h_{o3ss} - h_{o1}}{h_{o3} - h_{o1}} = C_p T_{o1} \left[(p_{o3}/p_{o1})^{(\gamma-1)/\gamma} - 1\right] / \Delta W$$

$$\therefore \frac{p_{o3}}{p_{o1}} = \left(1 + \frac{\eta_c \,\Delta W}{C_p T_{o1}}\right)^{\gamma/(\gamma-1)} = \left(1 + \frac{0.8 \times 220 \times 10^3}{1005 \times 288}\right)^{3.5}$$
$$= 1.608^{3.5}$$
$$= \underline{5.273}$$

Ex. 7.3

Solution. The absolute Mach number at impeller exit (eqn. ~~7.24~~) is

$$M_2 = c_2/a_2 = c_2/(\gamma R T_2)^{1/2}$$

so that c_2 and T_2 need to be determined. From the velocity triangle at impeller exit

$$c_2^2 = c_{\theta 2}^2 + c_{r2}^2 = (\sigma_s U_2)^2 + c_{r2}^2 = (0.9 \times 366)^2 + 30.5^2$$
$$= 1.094 \times 10^5$$

From the definitions of specific work (eqn. ~~7.1~~) and slip factor 7.1a

$$\Delta W = \sigma_s U_2^2 = h_{o2} - h_{o1} = C_p(T_{o2} - T_{o1})$$
$$= C_p(T_2 - T_{o1}) + \tfrac{1}{2} c_2^2$$
$$\therefore T_{o2} = T_{o1} + \sigma_s U_2^2/C_p = 288 + 0.9 \times 366^2/1005 = 408 \text{ K}$$
$$\therefore T_2 = T_{o2} - \tfrac{1}{2} c_2^2/C_p = 408 - \tfrac{1}{2} \times 1.094 \times 10^5/1005 = 353.5 \text{ K}$$

Thus,

$$M_2 = \left[1.094 \times 10^5/(1.4 \times 287 \times 353.5)\right]^{1/2} = 0.8778$$

The rate of mass flow is $\dot{m} = \rho_2 A_2 c_{r2}$ where the density is $\rho_2 = p_2/(RT_2)$. The static pressure p_2 must be determined by first solving for the impeller total pressure ratio p_{o2}/p_{o1} and then relating p_2 to p_{o2} by means of the isentropic temperature-pressure equation. Thus, the impeller total to total efficiency is

$$\eta_i = \frac{T_{o2s} - T_{o1}}{T_{o2} - T_{o1}} = \frac{(p_{o2}/p_{o1})^{(\gamma-1)/\gamma} - 1}{T_{o2}/T_{o1} - 1}$$

$$\therefore \frac{p_{o2}}{p_{o1}} = \left(1 + \eta_i \left(\frac{T_{o2}}{T_{o1}} - 1\right)\right)^{3.5} = \left(1 + 0.9 \times \left(\frac{408}{288} - 1\right)\right)^{3.5} = 3.047$$

$$\frac{p_2}{p_{o2}} = \left(\frac{T_2}{T_{o2}}\right)^{\gamma/(\gamma-1)} = \left(\frac{353.5}{408}\right)^{3.5} = 1/1.154^{3.5} = 1/1.651$$

$$\therefore p_2 = \frac{p_2}{p_{o2}} \cdot \frac{p_{o2}}{p_{o1}} \cdot p_{o1} = \frac{3.047}{1.651} \times 10^5 = \cancel{184.6 \text{ kPa}}$$ 1.013x/ 186.95

$$\dot{m} = p_2 A_2 c_{r2}/(RT_2) = \cancel{184.6} \times 10^3 \times 0.1 \times 30.5/(287 \times 353.5)$$ 186.95

$$= \cancel{5.550} \text{ kg/s}$$ 5.6202