Midterm Solution ME433

single-stage turbine $1 \xrightarrow{\text{stator}} 2 \xrightarrow{\text{rotor}} 3$

$c_1 = c_3 = c_x$

$\alpha_2 = 68°$

$T_{01} = 800°C = 1073K$

$P_{01} = 4$ bar

$c_x = 256$ m/s

$\eta_m = 0.318$

$N = 14400$ RPM

a) h-s diagram

b) work

c) $\eta_{ts}$

d) $M_3$

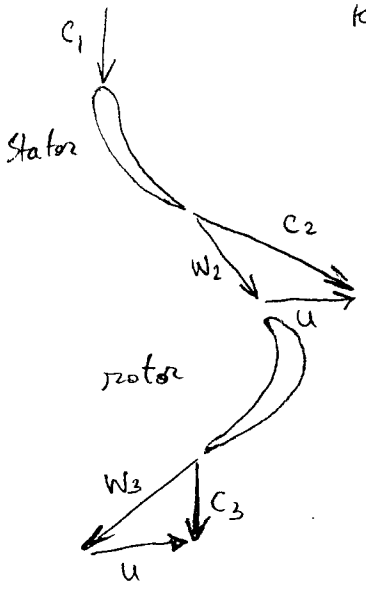
e) $\eta_{P,tt}$

perfect gas:

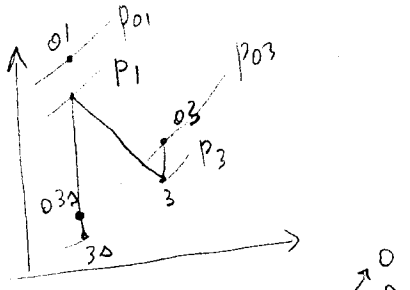
$c_p = 1147$ J/kgK

$\gamma = 1.33$

$R = 287 \frac{J}{kg\,K}$




a)




b) $\Delta W_T = u(c_{\theta 2} + c_{\theta 3})$

$\omega = \frac{2\pi N}{60} = \frac{2\pi \, 14400}{60} = 1508 \frac{rad}{s}$

$u = \omega r = 1508 \times 0.318 = 479.5$ m/s

$c_{\theta 2} = c_2 \sin\alpha_2 = c_x \tan\alpha_2 = 256 \times \tan 68° = 633.7$ m/s

$\Delta W_T = 479.5 \times 633.7 \frac{m^2}{s^2} = 303.9 \frac{kJ}{kg}$

c) $\eta_{ts} = \frac{h_{01} - h_{03}}{h_{01} - h_{3s}} = \frac{\Delta W_T}{c_p (T_{01} - T_{3s})}$

$T_{3s} = T_{01} \left(\frac{P_3}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 1073 \left(\frac{1}{4}\right)^{\frac{0.33}{1.33}} = 760.7$ K

$\eta_{ts} = \frac{303.9 \frac{kJ}{kg}}{1.147 \frac{kJ}{kg} (1073 - 760.7) K} = 84.8\%$

d) $M_3 = \dfrac{c_3}{\sqrt{\gamma R T_3}}$ $\qquad T_{03} = T_{01} - \dfrac{\Delta W_T}{c_p} = 1073 - \dfrac{303.9 \frac{kJ}{kg}}{1.147 \frac{kJ}{kg}} = 808\ K$

$c_3 = c_x = 256\ m/s$

$T_3 = T_{03} - \dfrac{c_3^2}{2 c_p} = 808 - \dfrac{256^2\ m^2/s^2}{2 \times 1147 \frac{J}{kg K}} = 779.4\ K$

$M_3 = \dfrac{256\ m/s}{\sqrt{1.33 \times 287 \frac{J}{kg K} \times 779.4\ K}} = 0.469$ $\quad (> 0.3 \Rightarrow$ compressible flow)

e) $\eta_{p,tt} \Rightarrow \ln \dfrac{T_{03}}{T_{01}} = \ln \left( \dfrac{p_{03}}{p_{01}} \right)^{\frac{\gamma - 1}{\gamma} \eta_{p,tt}} \Rightarrow \eta_{p,tt} = \dfrac{\ln \frac{T_{03}}{T_{01}}}{\frac{\gamma - 1}{\gamma} \ln \left( \frac{p_{03}}{p_{01}} \right)}$

$\dfrac{p_{03}}{p_3} = \left( 1 + \dfrac{\gamma - 1}{\gamma} M_3^2 \right)^{\frac{\gamma}{\gamma - 1}} = \left( 1 + \dfrac{0.33}{2} \times 0.469^2 \right)^{\frac{1.33}{0.33}} = 1.1547$

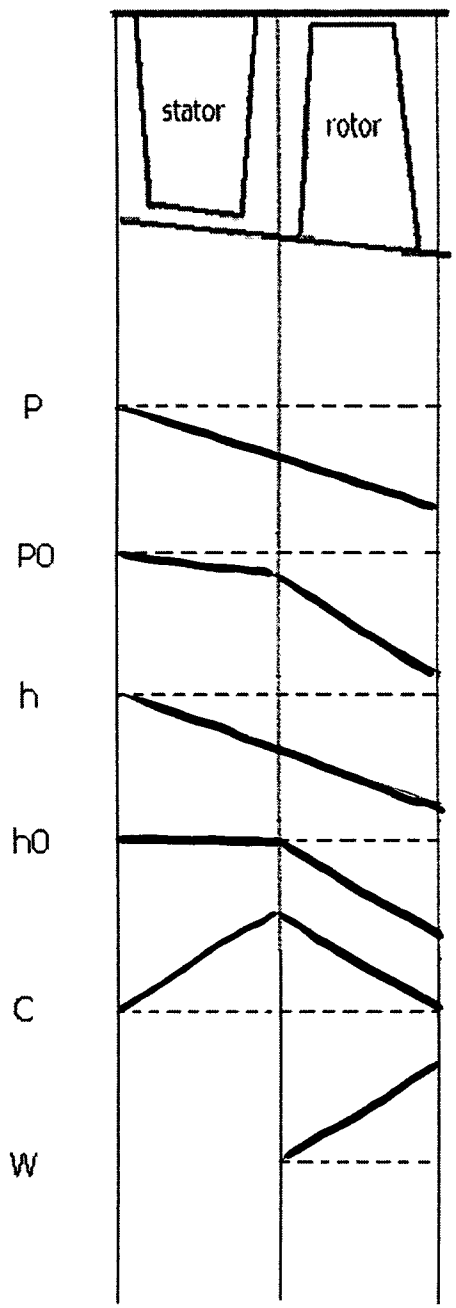
$p_{03} = 1.1547\ p_3 = 1.1547\ bar$

$\eta_{p,tt} = \dfrac{\ln \left( \frac{808}{1073} \right)}{\frac{0.33}{1.33} \ln \left( \frac{1.1547}{4} \right)} = 92\%$

EX. 2


A small pitch-to-chord ratio:

- provides better flow guidance (therefore less separation losses)
- has more friction losses
- has lower aerodynamic load (& lower tip clearance losses due to lower pressure difference between pressure & suction side)


stator
rotor
P
P0
h
h0
C
W