Bi-directional Velocity Estimation

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In lab #7 we estimate the motor velocity by computing the relative angular displacement during the sample period. An incremental encoder, connected to a 12-bit up/down counter is used to measure the position. We interpret the counter output as a 12-bit signed binary number in the range $[-2048, 2047]$, with each count corresponding to one state change of the encoder. The relative displacement is the difference between the current count c_n and the count of the previous sample c_{n-1}.

Because the subtraction will be performed using 16-bit signed integer arithmetic, each 12-bit signed binary encoder count should be converted to a 16-bit signed value by extending the sign from the 12^{th} through the 16^{th} bit before the subtraction.

Although the difference is valid, its interpretation is ambiguous because of the modulo-4096 nature of the counter. For example, suppose that $c_n = 2$ and $c_{n-1} = -2$. Has the encoder moved in the positive direction by 4 counts, or has it moved in the negative direction by 4092 counts? Or has it moved either of those amounts plus any integer multiple of 4096 counts?

To resolve this ambiguity we must make an assumption about the nature of the motion, and arrange our sampling such that the assumption is valid. We will assume that, during a single sample period, the counter has not processed more counts than the range of a 12-bit signed binary number $[-2048, 2047]$. Notice that this resolves the ambiguity in the above example: The relative displacement must be +4 counts.

To enforce this requirement, we compute the difference modulo-4096 . Because our number circle extends from −2048 to 2047, rather than from 0 to 4096 (as in lab #4) we need to use an *offset modulo* function defined as:

$$\text{mod}\ (m, n, d) = m - n \left\lfloor \frac{m-d}{n} \right\rfloor \tag{1}$$

where m is the value, n is the modulus, d is the offset, and $\lfloor x \rfloor$ is the "floor" function (i.e. the greatest integer less than or equal to x.) The result is modulo-n, and always in the range $[d, d+n-1]$.

Then, in our case, we estimate the relative displacement using modulo 4096, with offset $d = -2048$.

$$\begin{aligned} \Delta\theta &= \text{mod}\ (c_n - c_{n-1}, 4096, -2048) \\ &= c_n - c_{n-1} - 4096 \left\lfloor \frac{c_n - c_{n-1} - (-2048)}{4096} \right\rfloor \end{aligned} \tag{2}$$

As usual, we seek a numerically efficient method to implement this calculation.

Because the numerical value of $c_n - c_{n-1}$ must be in the range $[-4095, 4095]$, there are just *three cases* to be considered, according to the value of the floor function in Eq. (2):

1. $-4095 \le c_n - c_{n-1} < -2048$ Floor function is -1.

   $$\Rightarrow \Delta\theta = c_n - c_{n-1} - 4096 \times (-1)$$

2. $-2048 \le c_n - c_{n-1} < +2048$ Floor function is 0.

   $$\Rightarrow \Delta\theta = c_n - c_{n-1} - 4096 \times (0)$$

3. $+2048 \le c_n - c_{n-1} < +4096$ Floor function is 1.

   $$\Rightarrow \Delta\theta = c_n - c_{n-1} - 4096 \times (1)$$

In the <u>case 1</u>, all numerical values of $c_n - c_{n-1}$ are of the form

1111 0 − − − − − − − − − − −

When 4096 (0x1000) is added the results are

0000 0 − − − − − − − − − − −

In the <u>case 2</u>, the positive numerical values of $c_n - c_{n-1}$ are of the form

0000 0 − − − − − − − − − − −

while the negative value are of the form

1111 1 − − − − − − − − − − −

Because 0 is added in this case, the values are unchanged.

In the <u>case 3</u>, all numerical values of $c_n - c_{n-1}$ are of the form

0000 1 − − − − − − − − − − −

When -4096 (0xF000) is added the results are

1111 1 − − − − − − − − − − −

Examining these results, notice that the additions caused by the floor function in all three cases are equivalent to extending the 12^{th} bit of $c_n - c_{n-1}$ through the 16^{th} bit.

In summary, first we perform a 12^{th}–to–16^{th} bit sign extension on 12-bit counter reading to compute the signed 16-bit position c_n. Second, compute the difference: $c_n - c_{n-1}$. Third, perform an identical sign extension to on the difference to account for the modulo-4096 nature of the counter.