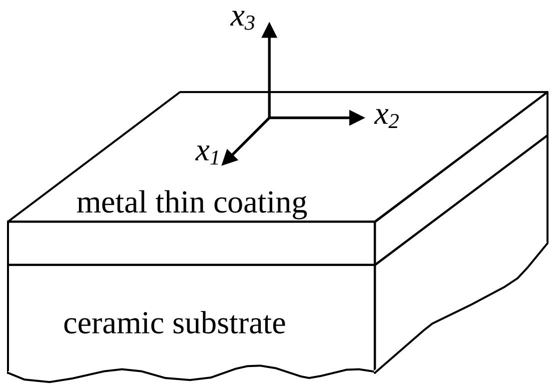
ME/MSE 485 Homework #4, due on Feb. 15, 2011

Consider a thin metal coatinγ on a thick ceramic substrate, which underwent a uniform temperature change ΔT.



CTE misfit strain

$$\varepsilon^* = (\alpha_c - \alpha_s)\Delta T \qquad (1)$$

$$\varepsilon^* = (\varepsilon^*_{11}, \varepsilon^*_{22}, 0)$$

where α_s and α_c are CTE of the substrate and coating, respectively. The elastic constants of the coating and substrate are E_c and ν_c, and E_s and ν_s, respectively.

Use the following material data:
E_c = 70 GPa, ν_c =0.34, E_s = 290 GPa, ν_s = 0.2, $\alpha_s = 6 \times 10^{-6}$/C, $\alpha_c = 16 \times 10^{-6}$/C.
Assume that the metal coating was deposited by evaporation at high temperature (220 C) and cooled down to room temperature (20C). It is assumed that the stress state is assumed to be zero at the processing temperature, 220 C. The length (width) of the coating and substrate is b = 20 mm.

Answer to the following questions:

1. Assume that the coating is much thinner than substrate and it is isotropic. Calculate the in-plane thermal stress in the metal coating, σ_c, at room temperature. Is this tensile or compressive stress ?

$$\sigma_c = \frac{E_c(\alpha_s - \alpha_c)\Delta T}{1 - \nu_c}$$

$$\sigma_c = 70 \times 10^9 \times (6 - 16) \times 10^{-6} \times (20 - 220) / (1 - 0.34) = 212 MPa (Tensile)$$

2. If the thickness of the coating (h_c) = 1 mm, and that of the substrate (h_s) = 5 mm, calculate the in-plane thermal stress in the coating, σ_c, and in the substrate, σ_s. How is this σ_c compared with σ_c of problem 1 above ?

$$\sigma_c = \frac{F_c}{h_c b} = \frac{E_c{}^0(\alpha_s - \alpha_c)\Delta T}{1 + \frac{E_c{}^0}{E_s{}^0}\frac{3h_c}{h_s}} = \frac{200(-10 \times 10^{-6}) \times 1.06 \times 10^4}{1 + \frac{1.06 \times 10^4}{3.625 \times 10^{11}}\frac{3}{5}} = -180 MPa$$

$$\sigma_s = \frac{Fs}{h_s b} = \frac{-Fc}{h_s b} = -36 MPa$$

3. If we assume there exist a thin interfacial compound(its thickness is 0.1 mm) made of 50% metal and 50% ceramic , thus, its thermomechanical properties (E, G, ν, α) are estimated by law of mixtures model, and G is the shear modulus that can be estimated by G=E/(2*(1+ν)). Calculate the maximum shear stress at the interface. Where is the location of such maximum shear stress?

$$\tau_{max} = \tau(l) = \frac{(\alpha_c - \alpha_s)\Delta T G_0 tanh(\beta l)}{\beta t_0} \text{ where } \beta^2 = \frac{G_0}{t_0}(\frac{1}{E_c^0 t_c} + \frac{1}{E_s^0 t_s})$$

and,

$$E_c^0 = \frac{E_c}{1-\nu_c} = \frac{70\times10^9}{1-0.34} = 106.06GPa$$

$$E_s^0 = \frac{E_s}{1-\nu_s} = \frac{290\times10^9}{1-0.2} = 362.5GPa$$

$$E_0 = E_c f_c + E_s f_s = 70\times10^9\times0.5 + 290\times10^9\times0.5 = 180GPa$$

$$G_0 = G_c f_c + G_s f_s = \frac{E_c f_c}{2(1+\nu_c)} + \frac{E_s f_s}{2(1+\nu_s)} = \frac{70\times10^9\times0.5}{2(1+0.34)} + \frac{290\times10^9\times0.5}{2(1+0.2)} = 73.48GPa$$

$$\beta^2 = \frac{G_0}{t_0}(\frac{1}{E_c^0 t_c} + \frac{1}{E_s^0 t_s}) = \frac{73.48\times10^9}{0.1\times10^{-3}}(\frac{1}{106.06\times10^9\times1\times10^{-3}} + \frac{1}{362.5\times10^9\times5\times10^{-3}})$$

$$\beta = 2708$$

So,

$$\tau_{max} = \tau(\frac{b}{2}) = \tau(0.01) = \frac{(16-6)\times10^{-6}\times(20-220)\times73.48\times10^9\times tanh(27.08)}{2708\times0.1\times10^{-3}} = -542.69MPa$$