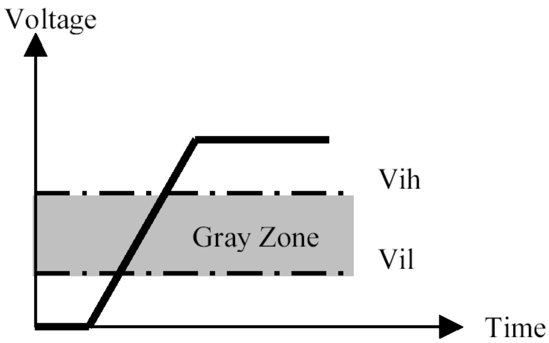
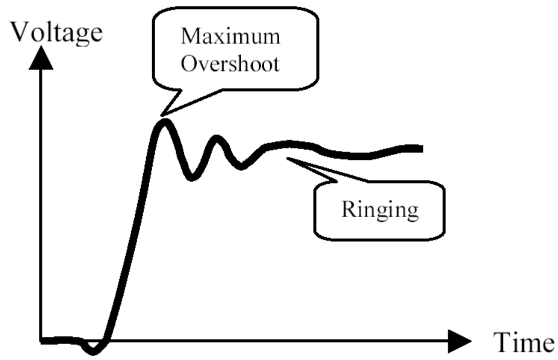
ME 458: Electronic Packaging and Materials
"Electromagnetic part" Lecture note

I. What is Signal Integrity (SI)

- Timing and quality of the signal



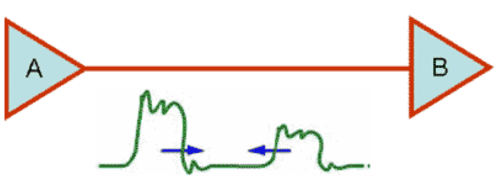


- Higher signal bandwidth and small size of packaging leads to increasingly difficulties in maintaining 'good' signal.

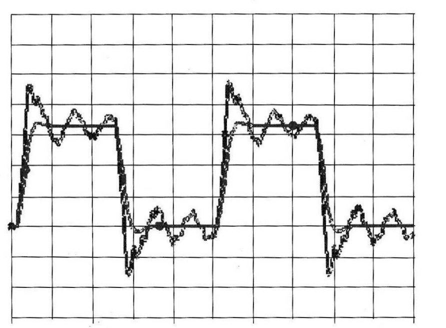
SI problems

1) Reflection Noise

Impedance discontinuity along the signal transmission path is the root cause of reflection noise. When a signal reaches the end of an improperly terminated transmission line, some of the energy within the signal will return along the same line. Reflections also occur when signals jump routing layers and impedance values are discontinuous at the boundary. This could be the result of manufacturing variations, design considerations, etc. When a trace is routed over planes with via holes, stubs, gaps at different locations or within the proximity of other traces, impedance discontinuity will occur and reflection can be observed. In high-speed systems, reflection noise increases time delay and produces overshoot, undershoot and ringing. Reflection noise can be minimized by controlling trace characteristic impedance, eliminating stubs and by using appropriate termination.

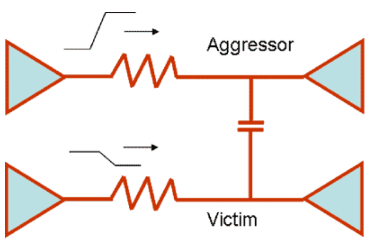


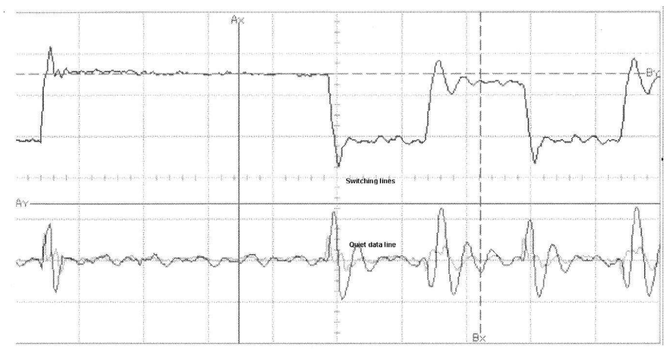
Reflections with impedance mismatch
Overshoot, undershoot, ringing



2) Crosstalk Noise

Crosstalk is caused by electromagnetic coupling between multiple parallel transmission lines. A quiet line (victim) when coupled to an active signal line (aggressor) can pick up noise causing false logic switching. In larger packages where multiple active lines switch simultaneously, crosstalk can induce power/ground noise. When two lines switch in the same direction, either from high to low or low to high, extra delay is introduced. This delay can significantly increase or decrease the sampling window. Crosstalk can be controlled by line spacing, placement of ground pins between signal pins and keeping reference planes close to signal pins (by reducing the loop inductance which is a function of area of the return current loop).

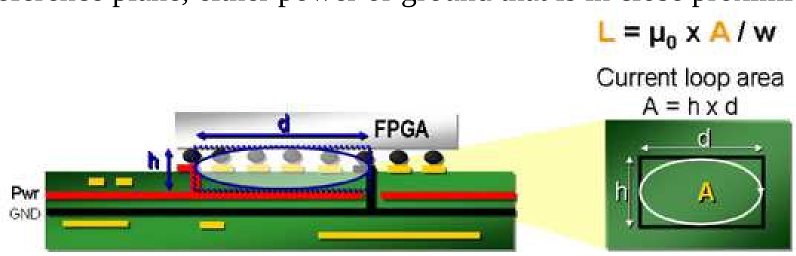




3) Power/Ground Noise

This noise is due to parasitics of the power/ground delivery system during drivers' simultaneous switching output (SSO). It is sometimes also called Ground Bounce, Delta-I Noise or Simultaneous Switching Noise (SSN). The dI/dt when the input to a gate is changed from low to high or vice versa multiplied by the package inductance (L) causes fluctuations, or simultaneous switching noise (SSN: $\Delta v = L.dI/dt$) between power and ground planes. The I/O supply current always travels in a loop, and in the breakout region of large packages, multiple I/Os can share a common return loop. The location of power/ground and I/O pins determine the size of the loop. Via crosstalk couples the loops formed by the I/O and the common return path exacerbating the problem.

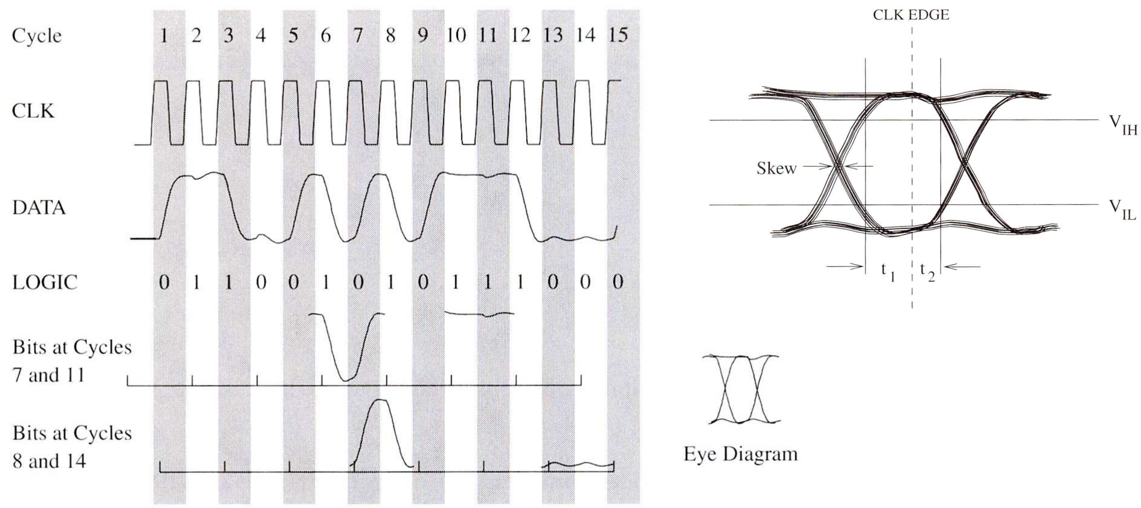
SSN can impact system timing, cause false logic switching, introduce noise in the power rail and create output jitter and increase radiation at resonant frequencies. Lowering core voltage to reduce SSN reduces noise margin and makes the package more susceptible to signal integrity problems. SSN can be reduced by providing proper return paths to signals. The return current will travel on a reference plane, either power or ground that is in close proximity to the signal.

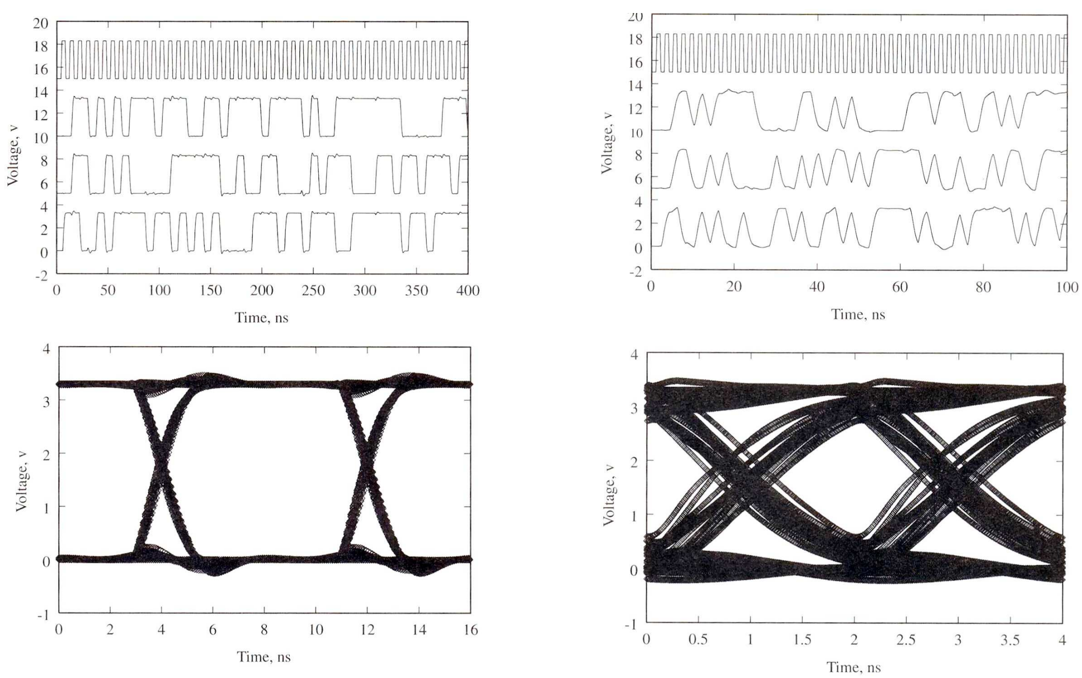


4) Electromagnetic interference (EMI)

EMI is the interference from electromagnetic radiation from nearby source. In general, EMI will get worse at higher frequencies.

5) SI Testing: eye diagram





SI Terms

Term	Definition
Bit Error Rate	A measurement of the number of errors detected at a receiver in a given length of time, sometimes specified as a percentage of received bits; sometimes specified in exponential form (10E-8 to indicate 1 bit error in 10E-8 bits).
Crosstalk	Undesirable signal coupling from noisy aggressor nets to victim nets. May be eliminated by increasing the spacing between the nets or reducing signal amplitude of the aggressor net.
Dispersion	"Smearing" of a signal or waveform as a result of transmission through a

	non-ideal transmission line. Through a non-ideal medium, signals travel at different velocities according to their frequency. Dispersion of the signal is the result. All cables and PCB transmission lines are non-ideal.
Equalization	Amplification or attenuation of certain frequency components of a signal. Used to counteract the effects of a non-ideal transmission medium.
Eye Diagram	An eye diagram of a signal overlays the signal waveform over many cycles. Each cycle waveform is aligned to a common timing reference, typically a clock. An eye diagram provides a visual indication of the voltage and timing uncertainty associated with the signal. It can be generated by synchronizing an oscilloscope to a timing reference.
Fall time	The time it takes for a waveform to transition from the high logic state to the low logic state. Falltime is usually measured from 90% of the total signal swing to 10% of the signal swing.
Impedance (Characteristic Impedance)	Electrical characteristic of a transmission line, derived from the capacitance and inductance per unit length.
Inter-Symbol Interference	A form of data corruption or noise due to the effect that data has on data-dependent channel characteristics.
Jitter	The jitter of a periodic signal is the delay between the expected transition of the signal and the actual transition. Jitter is a zero mean random variable. When worst case analysis is undertaken the maximum value of this random variable is used.
Overshoot	Phenomenon where a signal rises to a level greater than its steady-state voltage before settling to its steady-state voltage.
Ringing	Common name for the waveform that is seen when a transmission line ends at a high impedance discontinuity. The signal first overshoots, then dips down below the target value, and continues this with decreasing amplitude until it converges on the target voltage.
Risetime	The time it takes for a signal to rise from 10% of its total logic swing to 90% of its total logic swing.
Skin Effect/Loss	Electrical loss in a non-ideal medium due to skin effect. Skin effect is the tendancy for high-frequency signal components to travel close to the surface of the medium.
Electromagnetic Interference (EMI)	Electromagnetic interference (EMI) is any electromagnetic disturbance that degrades or limits the performance of the considered electronic system. It can be induced by the system being considered or its environment. The amount of interference an equipment can emit is regulated.
Mutual Capacitance	The capacitance between two conductors (one considered aggressor, the other victim) when all other conductors are connected together and then regarded as an ignored ground. It describes the amount of coupling due to the electric field. The mutual capacitance will inject an often undesired current into the victim line proportional to the rate of change of voltage on the aggressor line. Mutual Capacitance it a cause of crosstalk.
Mutual Inductance	The inductance between two conductors (one considered aggressor, the other victim) placed close enough that the magnetic field induced by a

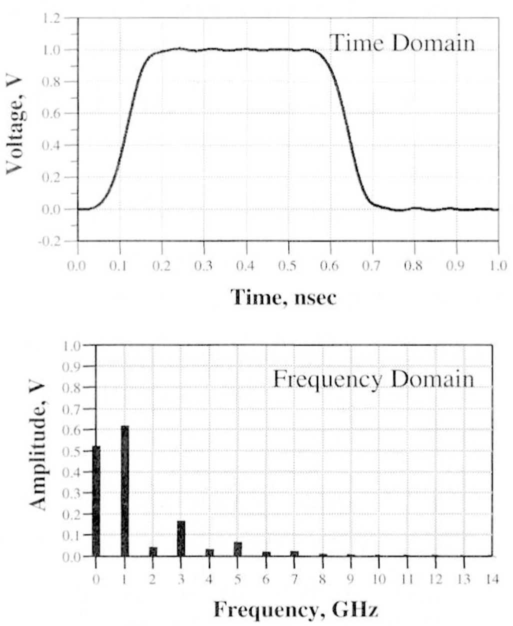
	current flowing into the aggressor line encompasses the victim. The mutual inductance will inject an often undesired voltage noise onto the victim proportional to the rate of change of the current on the aggressor line.
Reflection	A process that occurs when a propagating electromagnetic wave impinges upon a change in its supporting media properties. In the case of an abrupt change the incident wave will "bounce" off of the barrier in the opposite direction it came from. In other cases, some of the wave reflects while the rest continues on.
Skew	The difference in arrival time of bits transmitted at the same time. Bits can be elements of a parallel bus or members of a differential pair.
SSO (Simultaneous switching outputs)	Defines a moment in time when multiple device outputs switch to the same level. Such simultaneous switching may induce rapid current changes which combined with the inductance of ground pins, bond wires, and group metalization can be source of ground bounce.

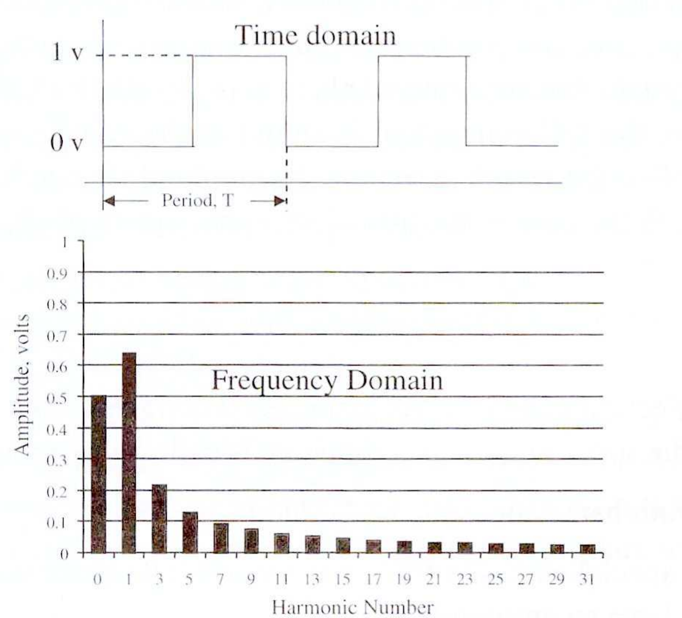
II. Important Mathematical Background

Fourier Transform

Frequency domain <=> Time domain

$$F(\omega)=\int_{-\infty}^{\infty} f(t)\exp(-j\omega t)\,dt \longleftrightarrow f(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\exp(j\omega t)\,d\omega$$





Example: ‘Square wave’ signal => odd harmonics

$$x(t)=\frac{4}{\pi}\left(\sin(2\pi ft)+\frac{1}{3}\sin(2\pi 3ft)+\frac{1}{5}\sin(2\pi 5ft)+....\right)$$

Phasor

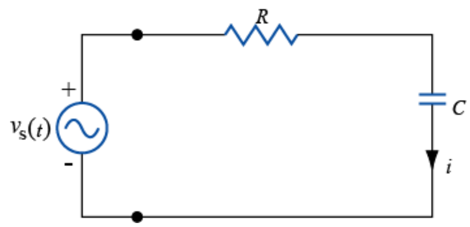
Phasor is a tool for help solving linear system. It relies on amplitude and phase function without the time dependent.

$z(t) = \mathrm{Re}[\tilde{Z}e^{j\omega t}]$ => $\tilde{Z}$ is the phasor of the instantaneous function $z(t)$

Kirchoff's voltage law: The directed sum of the electrical potential differences around a closed circuit must be zero.

Kirchoff's current law: At any point in an electrical circuit, the sum of currents flowing towards that point is equal to the sum of currents flowing away from that point.

RC circuit example: Given voltage source $v_s(t)$, find the current $i(t)$



The voltage source is $v_s(t) = V_o \sin(\omega t + \phi_o)$. Applying Kirchhoff's voltage law (KVL), we get

$$Ri(t) + \frac{1}{C}\int i(t)\, dt = v_s(t)$$

Step 1: Adopt a cosine reference

$$v_s(t) = V_o \sin(\omega t + \phi_o) = V_o \cos\left(\frac{\pi}{2} - \omega t - \phi_o\right) = V_o \cos\left(\omega t + \phi_o - \frac{\pi}{2}\right)$$

Step 2: Express time-dependent variables as phasors

$$v_s(t) = V_o \cos\left(\omega t + \phi_o - \frac{\pi}{2}\right) = \mathrm{Re}\left[V_o e^{j(\omega t + \phi_o - \pi/2)}\right] = \mathrm{Re}\left[\tilde{V}_s e^{j\omega t}\right] \text{ where } \tilde{V}_s = V_o e^{j(\phi_o - \pi/2)}$$

$$i(t) = \mathrm{Re}[\tilde{I}e^{j\omega t}]$$

$$\int i\, dt = \int \mathrm{Re}[\tilde{I}e^{j\omega t}]\, dt = \mathrm{Re}\left[\int \tilde{I}e^{j\omega t}\, dt\right] = \mathrm{Re}\left[\frac{\tilde{I}}{j\omega}e^{j\omega t}\right]$$

Step 3: Recast the equation in phasor form

$$Ri(t) + \frac{1}{C}\int i(t)\, dt = v_s(t) \quad \text{becomes} \quad R\,\mathrm{Re}[\tilde{I}e^{j\omega t}] + \frac{1}{C}\mathrm{Re}\left[\frac{\tilde{I}}{j\omega}e^{j\omega t}\right] = \mathrm{Re}[\tilde{V}_s e^{j\omega t}] \text{ or}$$

$$R\tilde{I} + \frac{1}{j\omega C}\tilde{I} = \tilde{V}_s$$

Step 4: Solve equation in phasor domain

$$R\tilde{I} + \frac{1}{j\omega C}\tilde{I} = \tilde{V}_s \rightarrow \left(R + \frac{1}{j\omega C}\right)\tilde{I} = \tilde{V}_s$$

$$\tilde{I} = \frac{\tilde{V}_s}{\left(R + \frac{1}{j\omega C}\right)} = \tilde{V}_s \frac{j\omega C}{1 + j\omega CR} = V_o e^{j(\phi_o - \pi/2)} \left[\frac{j\omega C}{1 + j\omega CR}\right]$$

Then, we can write it in magnitude and phase format

$$\tilde{I} = V_o e^{j(\phi_o - \pi/2)} \left[\frac{j\omega C}{1 + j\omega CR}\right] = V_o e^{j(\phi_o - \pi/2)} \left[\frac{\omega C e^{j\pi/2}}{\sqrt{1 + (\omega CR)^2}\, e^{j\phi_1}}\right] = \frac{V_o \omega C}{\sqrt{1 + (\omega CR)^2}} e^{j(\phi_o - \phi_1)}$$

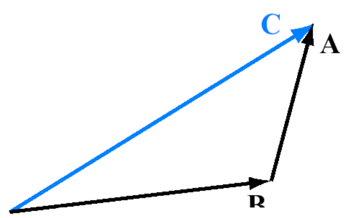
where $\phi_1 = \tan^{-1}(\omega RC)$

Step 5: Find the instantaneous value

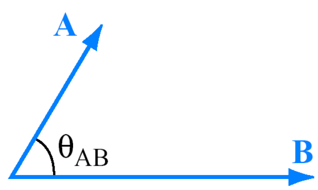
$$i(t) = \mathrm{Re}\left[\tilde{I} e^{j\omega t}\right] = \mathrm{Re}\left[\frac{V_o \omega C}{\sqrt{1 + (\omega CR)^2}} e^{j(\phi_o - \phi_1)} e^{j\omega t}\right] = \frac{V_o \omega C}{\sqrt{1 + (\omega CR)^2}} \cos(\omega t + \phi_o - \phi_1)$$

Vector analysis

- Vector addition and subtraction $\mathbf{C} = \mathbf{A} + \mathbf{B}$



- Dot product $\mathbf{A} \cdot \mathbf{B} = AB\cos\theta_{AB}$

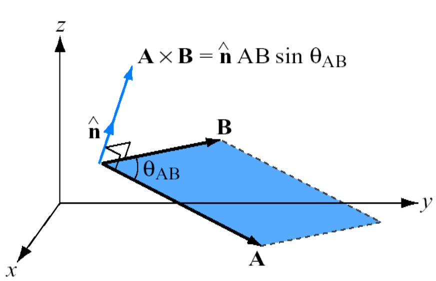


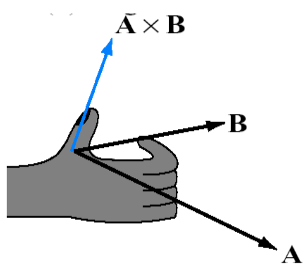
if

$\mathbf{A} = A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$ and $\mathbf{B} = B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}$

$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$

- Cross product $\mathbf{A} \times \mathbf{B} = \hat{\mathbf{n}} AB\sin\theta_{AB}$





$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y)\hat{\mathbf{x}} - (A_x B_z - A_z B_x)\hat{\mathbf{y}} + (A_x B_y - A_y B_x)\hat{\mathbf{z}}$$

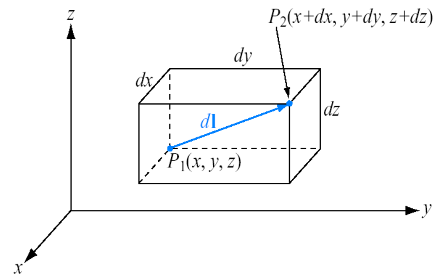
- Coordinate systems
    - Cartisian Coordinates
    - Cylindrical Coordinates
    - Spherical Coordinates

Gradient of a scalar field ∇ (del or gradient operator)

$$\nabla = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}$$

For example $\nabla T = \hat{\mathbf{x}}\frac{\partial T}{\partial x} + \hat{\mathbf{y}}\frac{\partial T}{\partial y} + \hat{\mathbf{z}}\frac{\partial T}{\partial z}$

=> directional derivative

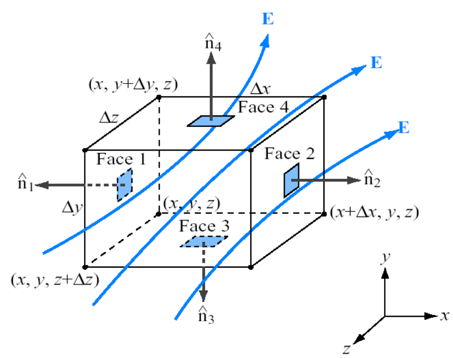


Divergence of a vector field

$$\nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

Divergence theorem

$$\int_v \nabla \cdot \mathbf{E} dv = \int_S \mathbf{E} \cdot d\mathbf{s}$$



'Outgoingness' of vector field

Curl of a vector field

$$\nabla \times \mathbf{B} = \hat{\mathbf{x}}\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) - \hat{\mathbf{y}}\left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) \quad \text{Or} \quad \nabla \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix}$$

Stoke's theorem $\int_s (\nabla \times \mathbf{B}) \cdot d\mathbf{s} = \int_c \mathbf{B} \cdot dl$

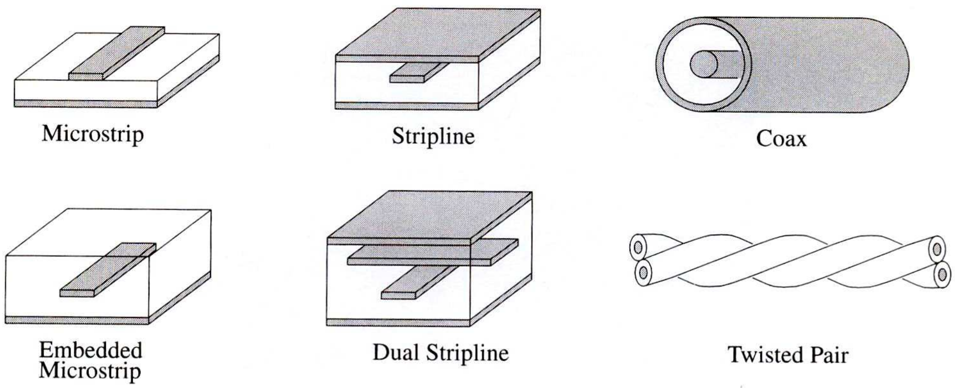
'Rotation' of the vector field

Laplacian Operator

scalar => $\nabla^2 V = \nabla \cdot (\nabla V) = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

vector => $\nabla^2 \mathbf{E} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\mathbf{E} = \hat{\mathbf{x}}\nabla^2 E_x + \hat{\mathbf{y}}\nabla^2 E_y + \hat{\mathbf{z}}\nabla^2 E_z$

III. Physical basic for transmission line

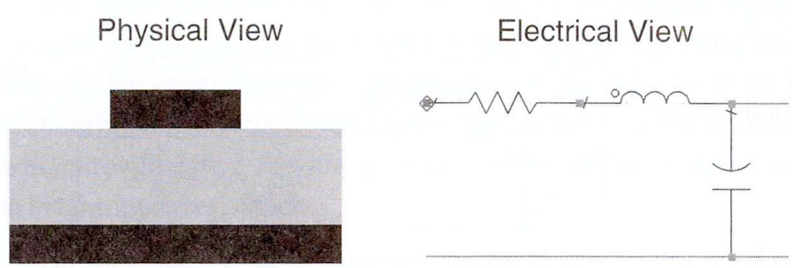


Transmission line is used to transport a signal from one point to another
Important parameters

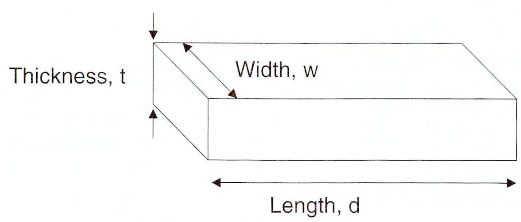
- characteristic impedance
- wave speed in transmission line
- capacitance
- inductance
- loss

Physical basic for resistance

- Approximation for the resistance of interconnects



$$R = \frac{\rho d}{A}$$



$R =$ the resistance (Ohm)

$A =$ the cross-section area (m^2)

$\rho =$ the bulk resisitivity of the conductor (Ohm-m)

$d =$ the distance between the ends of the interconnect (m)

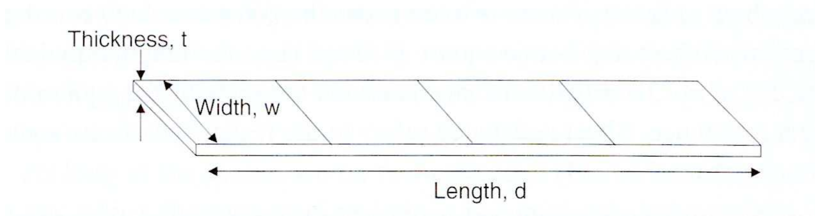
- Bulk resistivity

$\rho = \frac{1}{\sigma}$ σ is the conductivity (Siemens/meter)

It is intrinsic material property

- Resistance per length

$$R_L = \frac{R}{d} = \frac{\rho}{A}$$



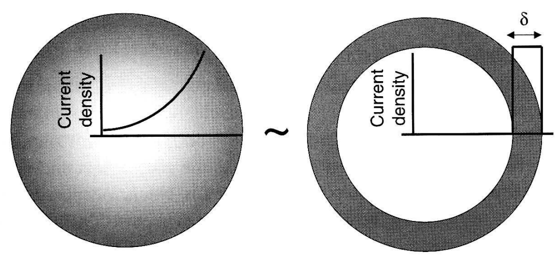
- Sheet resistance

$$R = \frac{\rho d}{tW}$$

Current distribution and skin depth

- skin depth

$$\delta = \sqrt{\frac{1}{\sigma \pi \mu_o \mu_r f}}$$

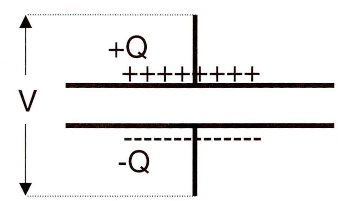


where
$\delta =$ skin depth (m)
$\sigma =$ conductivity of conductor (Siemens/m)
$\mu_o =$ permeability of free space
$\mu_r =$ relative permeability of conductor
$f =$ frequency of the wave (Hz)

=> Current crowding problem

Physical basic for capacitance

$$i = C\frac{dV}{dt}$$

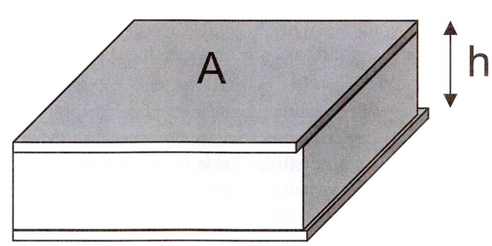


Complex permittivity $\varepsilon = \varepsilon' - j\varepsilon''$

Dielectric constant $\varepsilon_r = \frac{\varepsilon'}{\varepsilon_0}$, Dielectric loss ε'' => loss tangent $\frac{\varepsilon''}{\varepsilon'}$

- Parallel plate approximation

$$C = \varepsilon_0 \frac{A}{h}$$



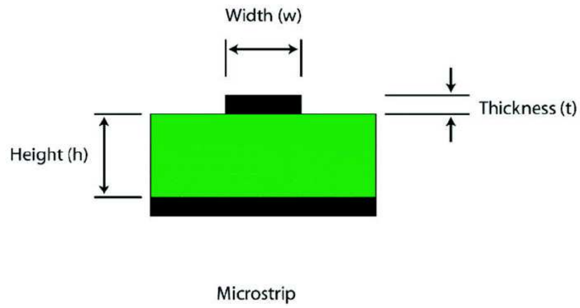
$C =$ capacitance (Farad)

$\varepsilon_0 =$ permittivity of free space (Farad/m)

$A =$ area of the plate

$h =$ separation between the plates

- Capacitance per length of microstrip line

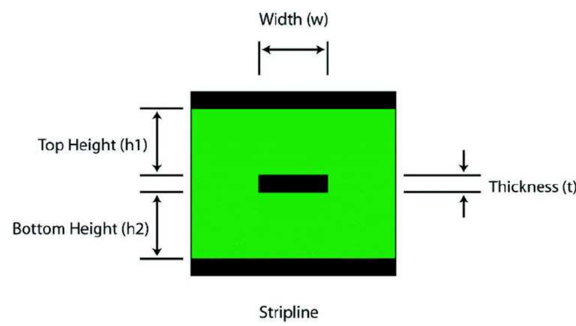


$$C_L = \frac{0.67(1.41+\varepsilon_r)}{\ln\left(\frac{5.98h}{0.8w+t}\right)}$$

where
C_L = capacitance per length (pF/inch)
ε_r = relative dielectric constant of the insulation
h = dielectric thickness (mils), w = line width (mils), t = thickness of conductor (mils)

1 mil = 1/1000 inches = 0.0254 mm

- Capacitance per length of strip line

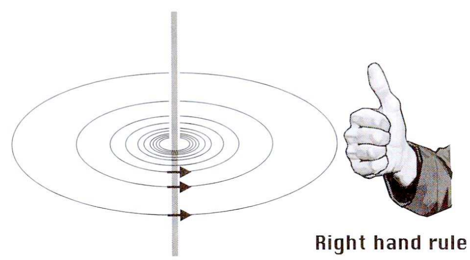


$$C_L = \frac{1.4\varepsilon_r}{\ln\left(\frac{1.9b}{0.8w+t}\right)}$$

where
C_L = capacitance per length (pF/inch)
ε_r = relative dielectric constant of the insulation
$b = h_1 + h_2 + t =$ total dielectric thickness (mils)
w = line width (mils), t = thickness of conductor (mils)

Physical basic for inductance

- What is inductance?

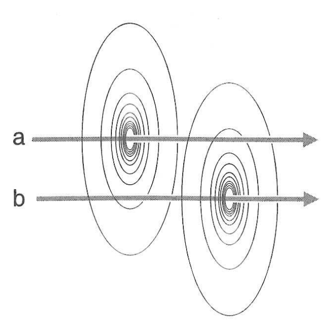


$$V = L\frac{di}{dt}$$

- There are circular magnetic-field line loops around all

- Inductance is the number of Webers of field line loops around a conductor per ampere of current through it.
$L = N/I$ L is inductance (Henry) N is the number of magnetic-field line loops (Webers)

- Self inductance and mutual inductance



$$V_{noise} = M\frac{dI}{dt}$$

$V_{noise} =$ the voltage noise induced to the quite wire
$M =$ the mutual inductance between the two wire
$I =$ the current in the second wire

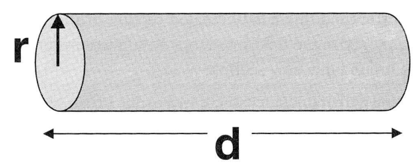
=> lead to *cross talk, ground bounce*

Partial inductance: consider only a section of wire

Approximation of partial self inductance of a round rod

$$L = 5d\left\{\ln\left(\frac{2d}{r}\right) - \frac{3}{4}\right\}$$

$L =$ partial self-inductance (nH)
$r =$ radius of the wire (inches)
$d =$ length of wire (inches)



Approximation of partial mutual inductance between two conductor segments

$$M = 5d\left\{\ln\left(\frac{2d}{s}\right) - 1 + \frac{s}{d} - \left(\frac{s}{d}\right)^2\right\}$$

$M =$ partial mutual inductance (nH)
$s =$ center-to-center separation (inches)
$d =$ length of the two wire (inches)

Electrical Impedance

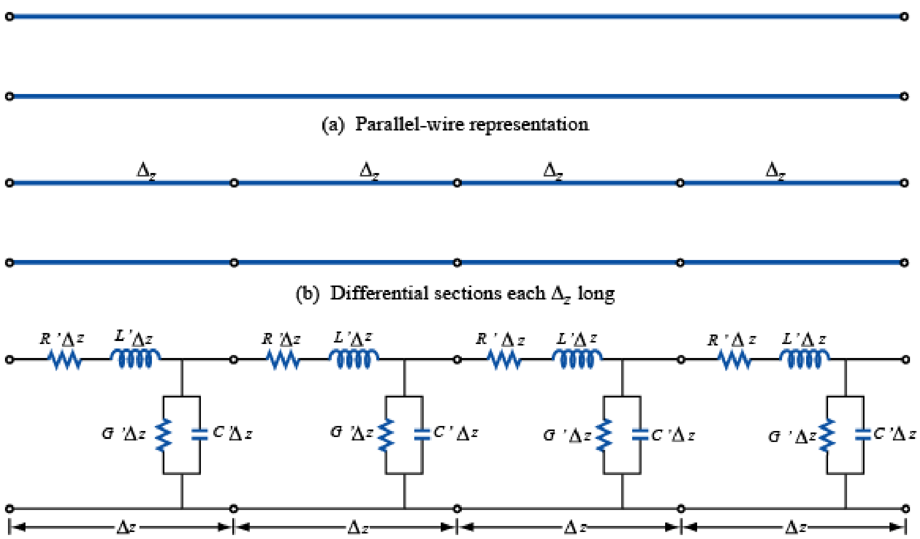
Electrical Impedance or simply **impedance**, describes a measure of opposition to a sinusoidal alternating current (AC). Electrical impedance extends the concept of resistance to AC circuits, describing not only the relative amplitudes of the voltage and current, but also the relative phases. In general impedance is a complex quantity and the term *complex impedance* may be used interchangeably; the polar form conveniently captures both magnitude and phase characteristics,

$Z = R + jX$ where $Z =$ impedance; $R =$ resistance; $X =$ reactance

Resistor => $Z = R$
Capacitor => $Z = \frac{1}{j\omega C}$
Inductor => $Z = j\omega L$

IV. Transmission line model

Lump element model



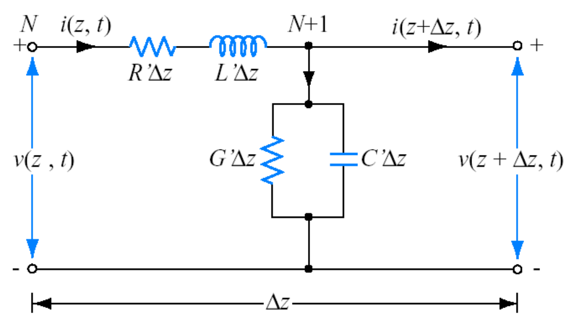
(a) Parallel-wire representation

(b) Differential sections each Δ_z long

(c) Each section is represented by an equivalent circuit

R': the combined resistance of both conductors per unit length (Ω/m)
L': the combined inductance of both conductor per unit length (H/m)
G': the conductance of the insulation medium per unit length (S/m)
C': the capacitance of the two conductors per unit length (F/m)

Transmission line equation



KVL: voltage law

$$v(z,t) - R'\Delta z\ i(z,t) - L'\Delta z\ \frac{\partial i(z,t)}{\partial t} - v(z+\Delta z,t) = 0$$

$$-\left[v(z+\Delta z,t) - v(z,t)\right] = R'\Delta z\ i(z,t) + L'\Delta z\ \frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\left[v(z+\Delta z,t) - v(z,t)\right]}{\Delta z} = R'\ i(z,t) + L'\ \frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\partial v(z,t)}{\partial z} = R'\ i(z,t) + L'\ \frac{\partial i(z,t)}{\partial t}$$

KCL: Current law

$$i(z,t) - G'\Delta z\ v(z+\Delta z,t) - C'\Delta z\ \frac{\partial v(z+\Delta z,t)}{\partial t} - i(z+\Delta z,t) = 0$$

$$\left[i(z+\Delta z,t) - i(z,t)\right] = G'\Delta z\ v(z+\Delta z,t) + C'\Delta z\ \frac{\partial v(z+\Delta z,t)}{\partial t}$$

$$-\frac{\left[i(z+\Delta z,t) - i(z,t)\right]}{\Delta z} = G'\ v(z+\Delta z,t) + C'\ \frac{\partial v(z+\Delta z,t)}{\partial t}$$

$$-\frac{\partial i(z,t)}{\partial z} = G' \, v(z,t) + C' \, \frac{\partial v(z,t)}{\partial t}$$

Time dependent form

$$-\frac{\partial v(z,t)}{\partial z} = R' \, i(z,t) + L' \, \frac{\partial i(z,t)}{\partial t}$$

$$-\frac{\partial i(z,t)}{\partial z} = G' \, v(z,t) + C' \, \frac{\partial v(z,t)}{\partial t}$$

Phasor form

$$-\frac{d\tilde{V}(z)}{dz} = \left(R' + j\omega L'\right) \tilde{I}(z)$$

$$-\frac{d\tilde{I}(z)}{dz} = \left(G' + j\omega C'\right) \tilde{V}(z)$$

$$-\frac{d^2\tilde{V}(z)}{dz^2} = \left(R' + j\omega L'\right) \frac{d\tilde{I}(z)}{dz}$$

$$-\frac{d^2\tilde{V}(z)}{dz^2} = \left(R' + j\omega L'\right)\left(G' + j\omega C'\right) \tilde{V}(z)$$

$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2 \tilde{V}(z) = 0$$

$$\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2 \tilde{I}(z) = 0$$

where $\gamma^2 = \left(R' + j\omega L'\right)\left(G' + j\omega C'\right)$

γ is the complex propagation constant. It can be written as $\gamma = \alpha + j\beta$

$\alpha = \text{Re}\left[\sqrt{\left(R' + j\omega L'\right)\left(G' + j\omega C'\right)}\right]; \quad \beta = \text{Im}\left[\sqrt{\left(R' + j\omega L'\right)\left(G' + j\omega C'\right)}\right]$

α is the attenuation constant, β is the phase constant

Wave propagation in transmission line

Wave equations

$$\frac{d^2\tilde{V}(z)}{dz^2} - \gamma^2 \tilde{V}(z) = 0$$

$$\frac{d^2\tilde{I}(z)}{dz^2} - \gamma^2 \tilde{I}(z) = 0$$

Solution to the wave equation

$$\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\,; \qquad \tilde{I}(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$e^{-\gamma z}$ corresponds to wave propagation in +z direction

$e^{+\gamma z}$ corresponds to wave propagation in –z direction

Because

$-\frac{d\tilde{V}(z)}{dz} = \left(R' + j\omega L'\right) \tilde{I}(z)$ and $\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$

$$\rightarrow -\frac{d}{dz}\left[V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}\right] = \left(R' + j\omega L'\right)\tilde{I}(z)$$

$$\rightarrow \gamma\left[V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right] = \left(R' + j\omega L'\right)\tilde{I}(z)$$

$$\rightarrow \tilde{I}(z) = \frac{\gamma}{\left(R' + j\omega L'\right)}\left[V_0^+ e^{-\gamma z} - V_0^- e^{\gamma z}\right]$$

Characteristic Impedance

$$\frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = Z_0 = \frac{R' + j\omega L'}{\gamma} = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \quad (\Omega)$$

Lossless transmission line $R' = G' = 0$ => $Z_0 = \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} = \sqrt{\frac{L'}{C'}}$ (Ω)

Wavelength and wave velocity in the lossless transmission line

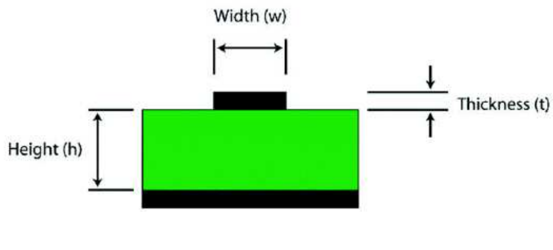
$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{L'C'}} \text{ (m)}; \quad u_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{L'C'}} \text{ (m/s)}$$

Characteristic impedance of microstrip line and strip line

Microstrip line

$$Z_0 = \frac{87}{\sqrt{\varepsilon_r + 1.41}} \ln\left(\frac{5.98h}{0.8w + t}\right)$$

$$0.1 < \frac{w}{h} < 3.0; \quad 1 < \varepsilon_r < 15$$



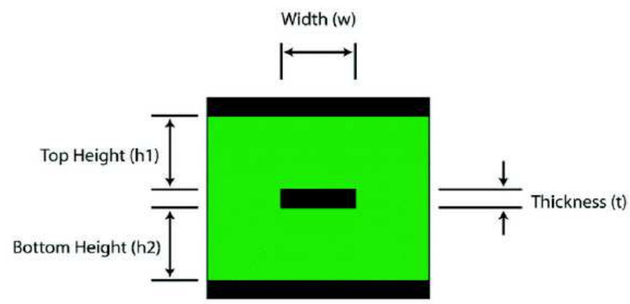
Microstrip

Strip line

$$Z_0 = \frac{87}{\sqrt{\varepsilon_r}} \ln\left(\frac{1.9(2h_1 + t)}{0.8w + t}\right)\left(1 - \frac{h_1}{4h_2}\right)$$

$$h_1 < h_2; \quad 0.1 < \frac{w}{h_1} < 2.0;$$

$$\frac{t}{h_1} < 0.25; \quad 1 < \varepsilon_r < 15$$

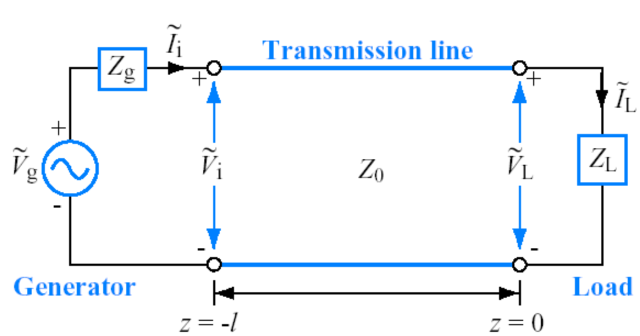


Stripline

Reflection

From $\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$



For lossless line $\gamma = j\beta$

$$\tilde{V}(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

$Z_L = \frac{\tilde{V}_L}{\tilde{I}_L}$ => from $\begin{aligned} \tilde{V}_L &= \tilde{V}(z=0) = V_0^+ + V_0^- \\ \tilde{I}_L &= \tilde{I}(z=0) = \frac{V_0^+}{Z_0} - \frac{V_0^-}{Z_0} \end{aligned}$, we have $Z_L = \left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-}\right) Z_0$

Solving for V_0^- gives

$$V_0^- = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right) V_0^+$$

Reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \left(\frac{Z_L - Z_0}{Z_L + Z_0}\right)$$

Voltage reflection coefficient Γ is the ratio of the amplitude of the reflected voltage wave to the amplitude of the incident voltage wave at the load

Γ is a complex number

$\Gamma = 0$ means perfectly matched line => no reflection

$|\Gamma| = 1$ means total reflection

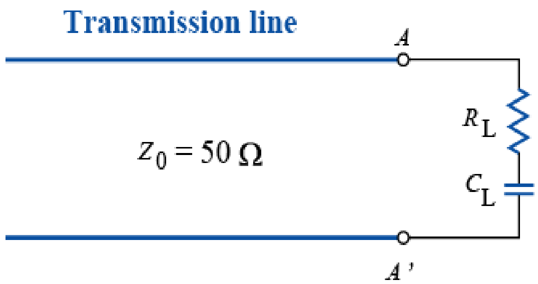
Example:

A 50 Ω transmission line is connected to a load as shown on the right. What is the reflection coefficient when we send a 100 MHz signal? Given that load is

a) $R_L = 50\ \Omega,\ \ C_L = 0$

b) $R_L = 50\ \Omega,\ \ C_L = 10$ pF

c) $R_L = 0\ \Omega,\ \ C_L = 10$ pF



Solutions:

a) $\Gamma = \left(\frac{Z_L - Z_o}{Z_L + Z_o}\right) = 0$ => matched line

b) $Z_L = R_L - \frac{j}{\omega C_L} = 50 - j\frac{1}{2\pi \times 10^8 \times 10^{-11}} = (50 - j159)$

$$\Gamma = \left(\frac{Z_L - Z_o}{Z_L + Z_o}\right) = \left(\frac{(50 - j159) - 50}{(50 - j159) + 50}\right) = \frac{-j159}{100 - j159}$$

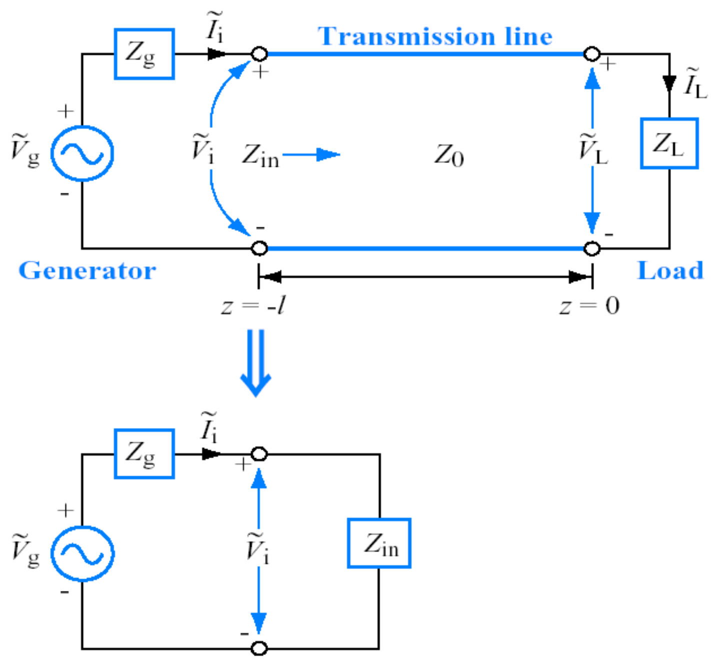
$$= \frac{159e^{-j90^\circ}}{187.83e^{-j57.83^\circ}} = 0.85e^{-j32.17^\circ} = 0.85\angle -32.17^\circ$$

c) $$Z_L = R_L - \frac{j}{\omega C_L} = 0 - j\frac{1}{2\pi \times 10^8 \times 10^{-11}} = -j159$$

$$\Gamma = \left(\frac{Z_L - Z_o}{Z_L + Z_o}\right) = \left(\frac{-j159 - 50}{-j159 + 50}\right) = \frac{166.68e^{-j107.46^\circ}}{166.68e^{-j72.54^\circ}}$$

$$= 1e^{-j34.92^\circ} = 1\angle -34.92^\circ$$

Input impedance of the lossless line



From $\tilde{V}(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

For lossless line $\gamma = j\beta$

and $V_0^- = \Gamma V_0^+$

$$\tilde{V}(z) = V_0^+ \left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)$$

$$\tilde{I}(z) = \frac{V_0^+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)$$

$$Z(z) = \frac{\tilde{V}(z)}{\tilde{I}(z)}$$

$$Z_{in}(-l) = Z_0\left(\frac{Z_L \cos\beta l + jZ_0 \sin\beta l}{Z_0 \cos\beta l + jZ_L \sin\beta l}\right)$$

$$= Z_0\left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right)$$

Derivation of the input impedance

$$Z(z) = \frac{\tilde{V}(z)}{\tilde{I}(z)} = \frac{V_0^+\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)}{\frac{V_0^+}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)} = Z_0 \frac{\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)}{\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)}$$

$$Z_{in}(-l) = Z_0 \frac{\left(e^{j\beta l} + \Gamma e^{-j\beta l}\right)}{\left(e^{j\beta z} - \Gamma e^{-j\beta l}\right)}$$

From

$e^{j\beta l} = \cos\beta l + j\sin\beta l$, and $e^{-j\beta l} = \cos\beta l - j\sin\beta l$

We get

$$Z_{in}(-l) = Z_0 \frac{\left(\cos\beta l + j\sin\beta l + \Gamma\left(\cos\beta l - j\sin\beta l\right)\right)}{\left(\cos\beta l + j\sin\beta l - \Gamma\left(\cos\beta l - j\sin\beta l\right)\right)}$$

$$= Z_0 \frac{\left((1+\Gamma)\cos\beta l + (1-\Gamma)\, j\sin\beta l\right)}{\left((1-\Gamma)\cos\beta l + (1+\Gamma)\, j\sin\beta l\right)}$$

From

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} \quad \rightarrow \quad 1+\Gamma = \frac{Z_L + Z_0 + Z_L - Z_0}{Z_L + Z_0} = \frac{2Z_L}{Z_L + Z_0}$$

$$\rightarrow \quad 1-\Gamma = \frac{Z_L + Z_0 - Z_L + Z_0}{Z_L + Z_0} = \frac{2Z_0}{Z_L + Z_0}$$

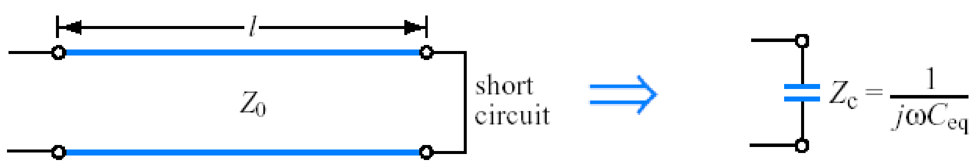
$$Z_{in}(-l) = Z_0 \frac{\left((1+\Gamma)\cos\beta l + (1-\Gamma)\, j\sin\beta l\right)}{\left((1-\Gamma)\cos\beta l + (1+\Gamma)\, j\sin\beta l\right)}$$

$$= Z_0 \frac{\left(\left(\frac{2Z_L}{Z_L + Z_0}\right)\cos\beta l + \left(\frac{2Z_0}{Z_L + Z_0}\right) j\sin\beta l\right)}{\left(\left(\frac{2Z_0}{Z_L + Z_0}\right)\cos\beta l + \left(\frac{2Z_L}{Z_L + Z_0}\right) j\sin\beta l\right)}$$

$$= Z_0 \frac{\left(Z_L\cos\beta l + Z_0\, j\sin\beta l\right)}{\left(Z_0\cos\beta l + Z_L\, j\sin\beta l\right)}$$

Example

Find the length l of the shorted $50\,\Omega$ lossless transmission line (see figure below) such that its input impedance at 2 GHz is equivalent to impedance of a capacitor with capacitance (C_{eq}) 5 pF. Assume that the phase velocity of the wave on the line is 2×10^8 m/s.



From $Z_{in} = Z_0\left(\frac{Z_l\cos\beta l + jZ_0\sin\beta l}{Z_0\cos\beta l + jZ_l\sin\beta l}\right)$, in this case, $Z_l = 0$. Then, we have

$$Z_{in} = Z_0\left(\frac{jZ_0 \sin\beta l}{Z_0 \cos\beta l}\right) = jZ_0 \tan\beta l$$

We want $Z_c = Z_{in} \rightarrow \dfrac{1}{j\omega C_{eq}} = jZ_o \tan\beta l$ or $\tan\beta l = -\dfrac{1}{Z_o \omega C_{eq}}$

$\beta = \dfrac{2\pi}{\lambda}$ and from $\lambda = \dfrac{v_p}{f} \rightarrow \quad \beta = \dfrac{2\pi f}{v_p} = \dfrac{2\pi\left(2\times10^9\right)}{2\times10^8} = 20\pi$

So we have to solve for

$$\tan\beta l = -\frac{1}{Z_o \omega C_{eq}} = -\frac{1}{50\left(2\pi\times2\times10^9\right)\left(5\times10^{-12}\right)} = -0.3183$$

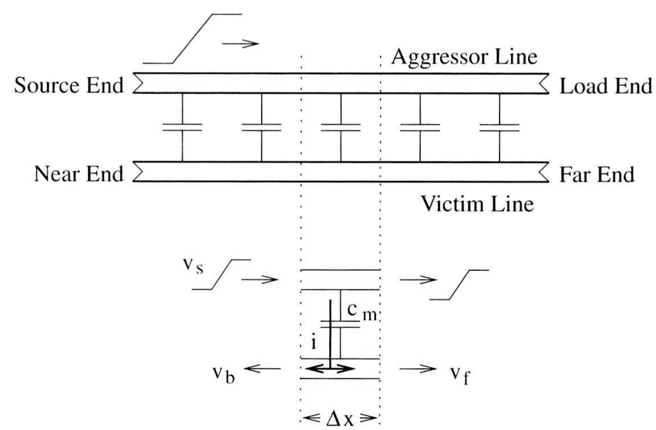
There are several solutions for $\tan\beta l = -0.3183$.

$$\beta l = 2.8334 \rightarrow l = \frac{2.8334}{\beta} = \frac{2.8334}{20\pi} = 0.0451 \text{ m or } 45 \text{ mm}$$

Other solutions satisfy $\beta l = 2.8334 + n\pi$ where n is any integer

V. Crosstalk in transmission line

Capacitive crosstalk



KCL:

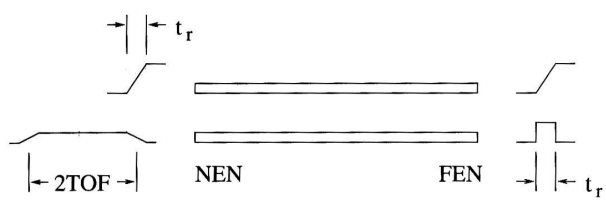
$$\frac{v_b}{Z_0} + \frac{v_f}{Z_0} = c_m \Delta x \frac{dv_s}{dt}$$

$$v_f = v_b = \frac{1}{2} Z_0 c_m \Delta x \frac{dv_s}{dt}$$

- Far-end noise

$$v_{FE} = \frac{1}{2} Z_0 c_m d \frac{dv_s}{dt}$$

for the line length d, c_m is the mutual capacitance per-unit-length



- Near-end noise

$$v_{NE} = \frac{1}{2} Z_0 c_m \Delta x \frac{dv_s}{dt}$$

v_o is the peak voltage => $\frac{dv_s}{dt} = \frac{v_o}{\Delta t}$ (assuming triangular edge)

c is the capacitance per unit length, $v_p = \frac{2\Delta x}{\Delta t}$ is the phase velocity , Δt is the rise time,

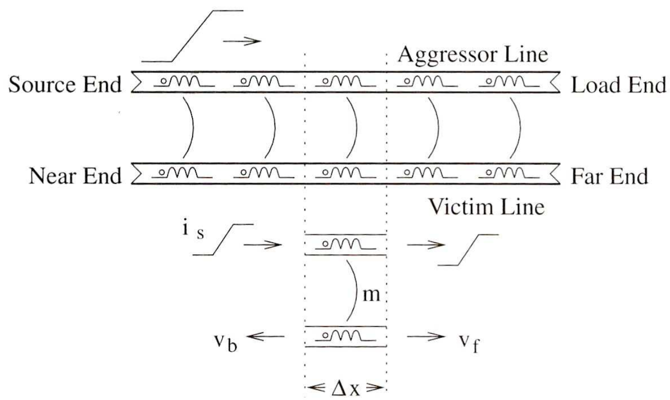
TOF= time of flight= distance/phase velocity

$$v_{NE} = \frac{1}{2} Z_0 c_m \frac{v_p \Delta t}{2} \frac{v_o}{\Delta t} = \frac{1}{4} Z_0 c_m v_p v_o$$

Because $Z_0 = \sqrt{\frac{L'}{C'}}$ and $v_p = \sqrt{\frac{1}{L'C'}}$ so $Z_0 v_p = \sqrt{\frac{L'}{L'C'^2}} = \frac{1}{C'}$

$$v_{NE} = \frac{1}{4} Z_0 c_m v_p v_o = \frac{1}{4} \frac{c_m}{C'} v_o$$

Inductive crosstalk



KVL:

$$v_b = m\Delta x \frac{di_s}{dt} + v_f$$

m is the mutual inductance

Current is continuous

$$\frac{v_b}{Z_0} = -\frac{v_f}{Z_0} \text{ and } i_s = \frac{v_s}{Z_0}$$

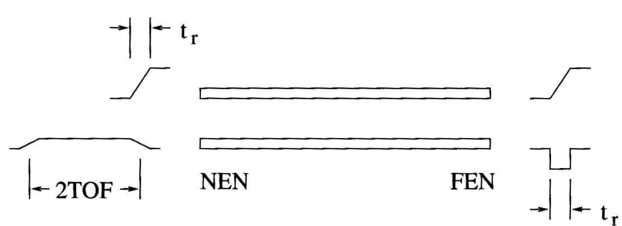
$$v_b = m\Delta x \frac{di_s}{dt} + v_f \rightarrow v_b = \frac{m\Delta x}{Z_0} \frac{dv_s}{dt} - v_b$$

$$v_b = \frac{m}{2Z_0} \Delta x \frac{dv_s}{dt}$$

$$v_f = -\frac{m}{2Z_0} \Delta x \frac{dv_s}{dt}$$

- Far-end noise

$$v_{FE} = -\frac{1}{2} \frac{m}{Z_0} d \frac{dv_s}{dt}$$



- Near-end noise

$$v_{NE} = \frac{m}{2Z_0}\Delta x \frac{dv_s}{dt}$$

v_o is the peak voltage => $\frac{dv_s}{dt} = \frac{v_o}{\Delta t}$ (assuming triangular edge)

c is the capacitance per unit length, $v_p = \frac{2\Delta x}{\Delta t}$ is the phase velocity , Δt is the rise time,

TOF= time of flight= distance/phase velocity

$$v_{NE} = \frac{1}{2}\frac{m}{Z_0}\frac{v_p \Delta t}{2}\frac{v_o}{\Delta t} = \frac{1}{4} m \frac{v_p}{Z_0} v_o$$

Because $Z_0 = \sqrt{\frac{L'}{C'}}$ and $v_p = \sqrt{\frac{1}{L'C'}}$ so $\frac{v_p}{Z_0} = \sqrt{\frac{C'}{L'^2 C'}} = \frac{1}{L'}$

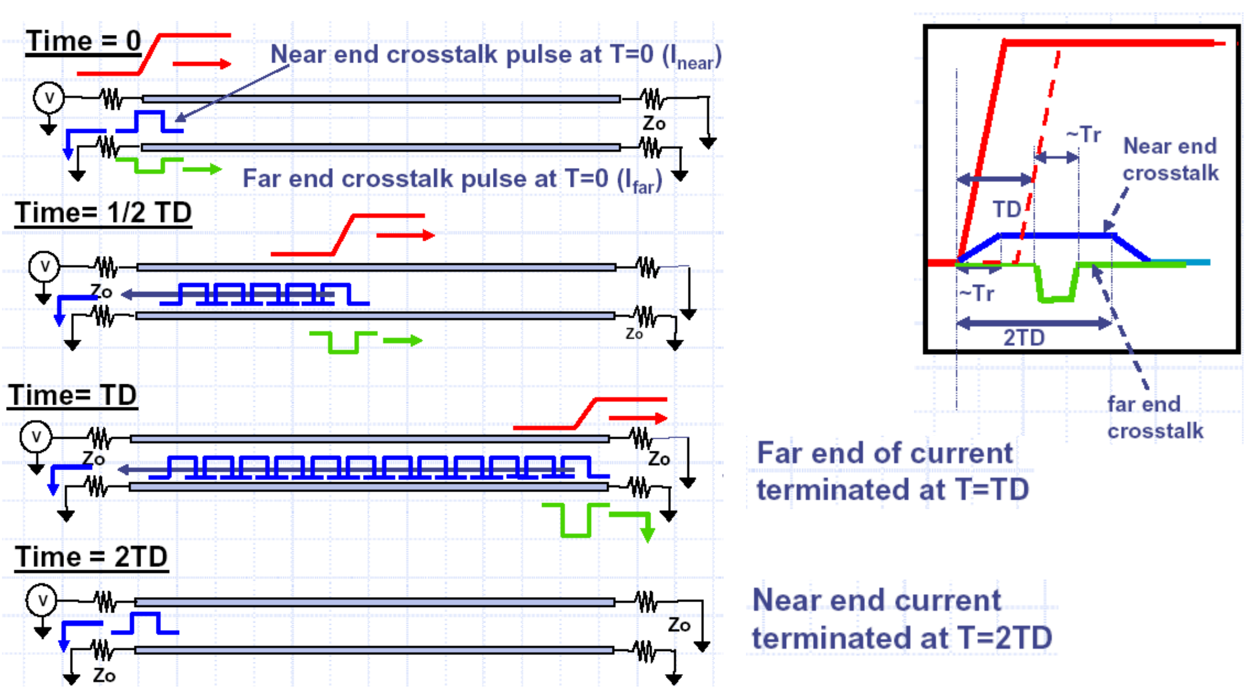
$$v_{NE} = \frac{1}{4}\frac{m}{L'} v_o$$

L' is the inductance per unit length

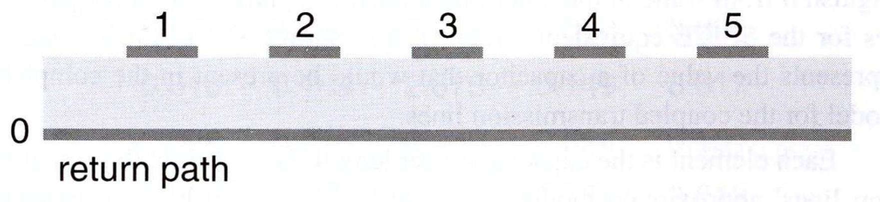
Total crosstalk = capacitive + inductive

$$v_{FE} = \frac{1}{2} d\left(Z_0 c_m - \frac{m}{Z_0}\right)\frac{dv_s}{dt}\ ; \quad v_{NE} = \frac{1}{4}\left(\frac{c_m}{C'} + \frac{m}{L'}\right) v_o$$

Graphical explanation

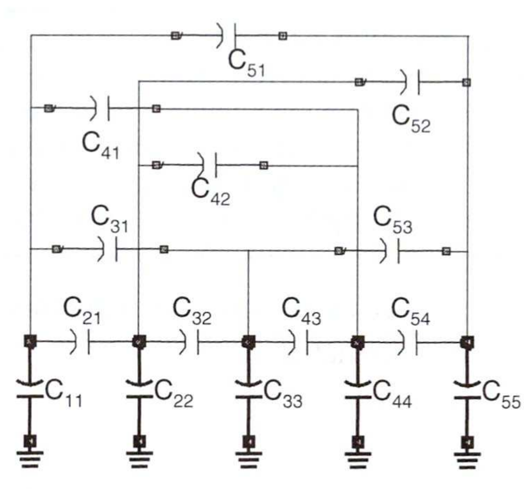


Equivalent circuit model for crosstalk analysis in multiple traces



Inductance matrix

$$L = \begin{bmatrix} L_{11} & L_{12} & \cdots & L_{1M} \\ L_{21} & L_{22} & \cdots & L_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ L_{M1} & L_{M2} & \cdots & L_{MM} \end{bmatrix}$$



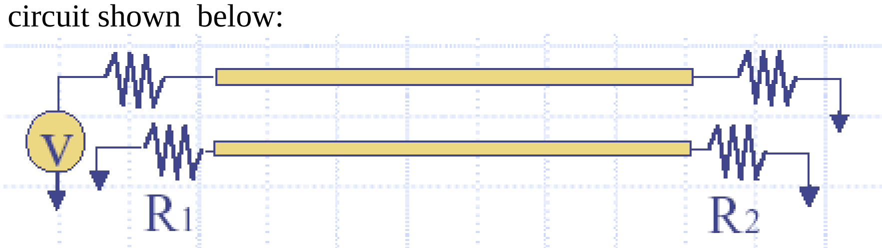
L_{MM} is the self-inductance of line M per unit length
L_{NM} is the mutual inductance between line M and line N per unit length

Capacitance matrix

$$C = \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1M} \\ C_{21} & C_{22} & \cdots & C_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ C_{M1} & C_{M2} & \cdots & C_{MM} \end{bmatrix}$$

C_{NN} is the self capacitance of line N per unit length where
C_{MN} is the mutual capacitance between lines M and N

Example: Calculate near and far end crosstalk-induced noise magnitudes and sketch the waveforms of circuit shown below:



Vinput = 1.0V, Trise = 100ps. Length of line is 2 inches. Assume the following capacitance and inductance matrix:

$$L = \begin{bmatrix} 9.869nH & 2.103nH \\ 2.103nH & 9.869nH \end{bmatrix} \text{(per inch)} \qquad C = \begin{bmatrix} 2.051pF & 0.239pF \\ 0.239pF & 2.051pF \end{bmatrix} \text{(per inch)}$$

- Far end cross talk => $v_{FE} = \frac{1}{2} d \left(Z_0 c_m - \frac{m}{Z_0} \right) \frac{dv_s}{dt}$

The characteristic impedance is $Z_O = \sqrt{\frac{L_{11}}{C_{11}}} = \sqrt{\frac{9.869nH}{2.051pF}} = 69.4\Omega$.

Because the input signal is assume to be triangle ramp $\frac{dv_s}{dt} = \frac{v_{in}}{T_{rise}} = \frac{1}{100}$ volt/psec

In this case, $c_m = C_{21}$, and $m = L_{21}$, therefore

$$v_{FE} = \frac{1}{2} d \left(Z_0 c_m - \frac{m}{Z_0} \right) \frac{dv_s}{dt} = \frac{1}{2} d \left(Z_0 C_{21} - \frac{L_{21}}{Z_0} \right) \frac{v_{in}}{T_{rise}}$$

$$= \frac{1}{2}(2 \text{ inches}) \left[(64.9\ \Omega)(0.239 \text{ pF/inch}) - \frac{2.103(\text{nH/inch})}{(64.9\ \Omega)} \right] \left(\frac{1}{100} \text{ volt/psec} \right) = -0.137 \text{ volts}$$

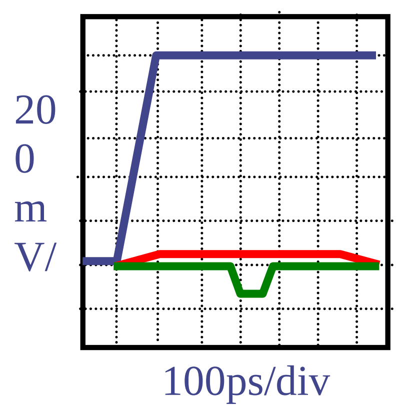
- Near end cross talk => $v_{NE} = \frac{1}{4} \left(\frac{c_m}{C'} + \frac{m}{L'} \right) v_o$

In this case, $c_m = C_{21}$, $m = L_{21}$, $C' = C_{11}$, and $L' = L_{11}$,

$$v_{NE} = \frac{1}{4} \left(\frac{c_m}{C'} + \frac{m}{L'} \right) v_o = \frac{1}{4} \left(\frac{C_{21}}{C_{11}} + \frac{L_{21}}{L_{11}} \right) v_o$$

$$= \frac{1}{4} \left(\frac{(2.051\text{pF/inch})}{(0.239 \text{ pF/inch})} + \frac{(2.103 \text{ nH/inch})}{(9.869 \text{ nH/inch})} \right) 1 \text{ volt}$$

$$= 0.082 \text{ volts}$$



Propagation delay of the 2 inch line is

$$\frac{d}{v_p} = \frac{d}{1/\sqrt{L'C'}} = d\sqrt{L_{11}C_{11}} = (2 \text{ inch})\sqrt{(9.869nH)(2.051pF)}$$

$$= 0.28454 \text{ nsec}$$

Useful references:

- Digital Signal Integrity: Modeling and Simulation with Interconnects and Packages by Brian Young
- Signal Integrity – Simplified by Eric Bogatin
- Fundamentals of Applied Electromagnetics by Fawwaz Ulaby