**Department of Mechanical Engineering**

**ME534 Advanced Graduate Fluid Mechanics II**

**Homework #1, assigned Friday, 04/06/18, due 04/20/18**

# Problem 1

We have used the boundary layer equations to model free shear flows in which the shear is not imposed by a solid wall, but by fluid streams with a velocity difference. One such example is the plane jet, in which fluid enters a semi-infinite reservoir (filled with the same fluid) through a narrow slot, forming a jet and inducing a two dimensional flow. A sketch of this configuration is shown in figure **??**.

The velocity distribution can be found from the boundary layer approximation on the Navier-Stokes equation for incompressible, two dimensional, steady flow. Assuming that there is no pressure gradient in the x direction. The boundary layer equations have a similarity solution, in terms of the stream function, of the form:

$$\psi(x,y) = 6\alpha\nu x^{1/3} f(\eta)\ ; where\ \ \eta = \alpha\frac{y}{x^{2/3}} \tag{1}$$

The resulting equation for the similarity function $f(\eta)$ is:

$$\frac{d^3 f}{d\eta^3} + 2\frac{d(f \cdot df/d\eta)}{d\eta} = 0 \tag{2}$$

And the solution is given by $f(\eta) = \sqrt{c_2} tanh(\sqrt{c_2}\ \eta)$, and the resulting stream function is $\Psi = 6\alpha\nu x^{1/3} tanh(\alpha y/x^{2/3})$.

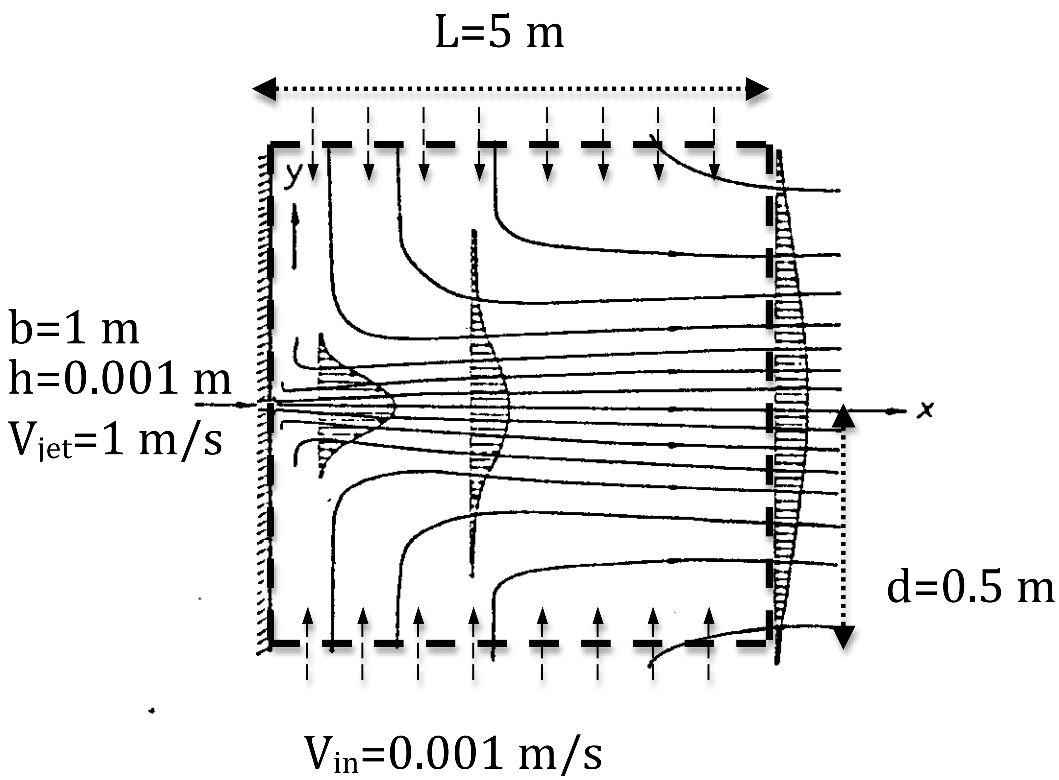
A control volume analysis can help in the interpretation of the results.

- Conservation of mass shows that the mass flux in the slot is negligible. Where does the mass that is leaving the control volume through the right boundary coming from? What is this mechanism, typical of jets and other free shear flows, called?

- Conservation of momentum show that the flux of x-momentum leaving the domain through the right boundary comes exclusively from the slot (and this equality sets the value of the undetermined constant $\alpha$). Since momentum is conserved, and therefore the integral of $v_x^2$ is constant between the left boundary (slot) and the right boundary, what can you say qualitatively about the fluxes of mechanical energy?

- Use the same control volume approach to sketch (not calculate rigorously) the balance of mechanical and internal energy. What are the sources of mechanical energy in this problem? What mechanism makes the amount of mechanical energy decrease and where does that energy go to? How do you establish conservation of energy (balance between mechanical and internal energy changes)?. Use the following dimensions to get a feel for the magnitude of the variables: Fluid Density: $\rho = 10^3 kg/m^3$, Slot height: $h = 10^{-3}\ m$, Slot width (into the paper): $b = 1\ m$, distance downstream to compute the fluxes with self-similar solution: $L = 5\ m$, uniform velocity at the slot: $v_{jet} = 1\ m/s$. Distance up and down for the horizontal control volume top and bottom surfaces: $d = 0.5\ m$, mean vertical velocity at these two horizontal surfaces: $v_{in} = 10^{-3}\ m/s$, viscosity: $\mu = 10^{-6}\ m^2/s$.

- If you have found the deficit in the mechanical energy, you can now estimate the average shear stress in the control volume and the average increase in temperature in the fluid leaving the control volume (heat capacity $C = 4,184\ J/kg\ K$).


L=5 m
y
b=1 m
h=0.001 m
Vjet=1 m/s
x
d=0.5 m
Vin=0.001 m/s