Department of Mechanical Engineering

ME534 Advanced Graduate Fluid Mechanics

Homework #2, due 05-04-18

Problem 1

We are interested in studying the flow induced by the decay of a point vortex (irrotational vortex) with circulation Γ. To think about this physical phenomena, imagine an infinitely thin rod rotating in an infinite expanse of fluid. At times $t < 0$, the velocity field in the flow is approximated by the irrotational field $V_\theta = \Gamma/2\pi r$, and the vorticity is approximately infinite at the rod's position (r=0). At time t=0, the rod suddenly stops rotating and vorticity starts diffusing away from the origin (where it was infinitely concentrated). The intensity of the motion is characterized by the circulation Γ.

Calculate:

- the spatial distribution of vorticity at times $t > 0$
- the resulting velocity in the flow field for time $t > 0$. In particular, it is interesting to analyze the regions $r << \sqrt{4\nu t}$ and $r >> \sqrt{4\nu t}$
- Extra Credit. Assume that the vorticity was not infinitely concentrated at the origin, but uniformly distributed along a core of radius R (so the initial vorticity is given by $\omega(r) = \Gamma/(2\pi R$ for $r \leq R$ and $\omega(r) = 0$ for $r > R$, for t¡0. Solve (possibly numerically) the spatial distributions of vorticity and velocity at times $t > 0$.