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Classification of Turbulence Modification by Dispersed Spheres Using a Novel Dimensionless Number

Tomohiko Tanaka* and John K. Eaton+

Department of Mechanical Engineering, Stanford University, 488 Escondido Mall, Bldg. 500, Stanford, California 94305-3030, USA

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A novel dimensionless parameter, the particle moment number Pa, was derived using dimensional analysis of the particle-laden Navier-Stokes equations, in order to understand the underlying physics of turbulence modification by particles. A set of 80 previous experimental measurements where the turbulent kinetic energy was modified by particles was examined, and all results could clearly be divided into three groups in Re-Pa mappings. The turbulence attenuation region was observed between the augmentation regions with two critical particle momentum numbers.

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Previous studies of particle-laden flows have shown that a dilute dispersion of fine particles can either augment or attenuate the carrier-phase turbulent kinetic energy (TKE). Although broad patterns can be observed in the literature on turbulence modification, the factors controlling the level of turbulence modification are not well understood. It is possible to examine the effects of dispersed particles on both the production of turbulence and the dissipation rate using either experiments or simulations. However, knowing only these two rates does not allow one to know the equilibrium level of the turbulent kinetic energy in a statistically stationary flow. Many experiments have shown little change in the mean fluid phase velocity profile with the addition of particles. Therefore, there should be little change in the production of turbulence due to mean velocity gradients. Also, heavy particles falling through the flow are giving up gravitational potential energy and introducing random velocity fluctuations to the fluid phase. This suggests that the addition of particles would increase the turbulence level, a phenomenon observed in many flows. However, in other particle-laden flows, extra dissipation caused by local distortion of the turbulence around inertial particles leads to an overall reduction in the turbulent kinetic energy. To date, no method has emerged that is capable of accurately predicting turbulence modification by particles over a broad range of parameters. Gore and Crowe [1] successfully categorized the turbulence modification into augmentation and attenuation by proposing an intuitive parameter, d_p/l_e. This parameter is the ratio of the particle diameter, d_p, to a characteristic size of large eddies, l_e. However, the classification does not describe the effects of changing particle material density, nor does it predict the magnitude of the turbulence modification. The single parameter cannot capture the effects of other important parameters such as the mass loading or particle Reynolds number. Other single parameter classifications, including those based on Stokes number and particle Reynolds number, are even less effective than d_p/l_e. In order to describe the turbulence modification accurately, other physical approaches are necessary.

In this Letter, we first consider appropriate dimensional parameters to describe turbulence modification by nondimensionalizing the particle-laden Navier-Stokes equation and introduce a resultant nondimensional parameter. Next, we propose a mapping method to classify turbulence augmentation and attenuation with the resultant nondimensional parameter. We evaluate the utility of the parameter for both turbulence augmentation and turbulence attenuation by examining previous turbulence modification experiments for internal flows [2–11]. The experiments are summarized in Table I. The reader is referred to the original papers for details of the experiments. Generally, the turbulence level was determined using phase-sensitive optical methods (LDA or PIV) to measure the fluid phase velocity field with and without particles. These turbulence modifications are clearly categorized by using the nondimensional number as described later.

The governing equations for particle-laden flows are concisely described assuming a Newtonian fluid and spherical particles. For the particulate phase, the linear momentum and angular momentum equations are applied.

$$\frac{dx_i^p}{dt} = U_i^p, \qquad m^p \frac{dU_i^p}{dt} = F_i, \qquad I^p \frac{d\Omega_i^p}{dt} = \Theta_i, \quad (1)$$

where x_i^p, U_i^p, and Ω_i^p are, respectively, the ith component of the position, the velocity, and the angular momentum of

TABLE I. Summary form of the previous turbulence modification experiments for internal flows [2–11].

Ref.	Flow descriptions	Carrier	Particles
[2]	Horizontal channel	water	glass
[3]	Downward channel	air	glass or copper
[4,5]	Horizontal channel	air	glass
[6]	Upward pipe	air	glass
[7]	Horizontal pipe	air	polyacrylate.
[8]	Downward channel	water	glass or cellulose
[9]	Downward channel	water	ceramic
[10]	Horizontal pipe	air	plastic
[11]	Vertical pipe	air	polyethylene

the sphere. The operator, d/dt, denotes a derivative along the particle trajectory, m^p is the mass of the sphere and I^p is the moment of inertia. The force, F_i, is the integration of the pressure and the viscous stress over the sphere surface. Body forces are neglected. The total torque acting on a particle, Θ_i, is calculated by integrating the contributions from the shear stresses.

For the fluid phase, the incompressible Navier-Stokes equation is applied:

$$\frac{DU_i}{Dt} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x_i} + \nu\frac{\partial^2 U_i}{\partial x_j \partial x_j}, \tag{2}$$

where, U_i, p, ρ_f, and ν denote the fluid velocity in the ith direction, the pressure, the fluid density and the fluid kinematic viscosity. The operator, D/Dt, denotes a substantial derivative along the fluid path. The particle and fluid equations are coupled through the total force, F_i, the total torque, Θ_i and the boundary conditions at the particle surface: namely, the no penetration and no slip conditions. Fluid motion in particle-laden flows is ideally described by two basic equations: the continuity equation and the Navier-Stokes equation. However, the complicated issue is the boundary condition at the moving particle surface. Since it is fairly difficult to describe the momentum exchange for only a single particle as mentioned in the previous section, it is currently impossible to accurately determine interactions between a large number of particles and fluids in particle-laden turbulence.

The momentum equation for fluid flows containing dispersed particles can be constructed by considering very small differential volume of the Navier-Stokes equation.

$$\frac{DU_i}{Dt} = -\frac{1}{\rho_f}\frac{\partial p}{\partial x_i} + \nu\frac{\partial^2 U_i}{\partial x_j \partial x_j} - \frac{1}{\rho_f} f_i. \tag{3}$$

The total force per unit volume exerted by the fluid onto particles, f_i, is the integration of the surface force, $\mathcal{S}$ (per unit surface), over the particle surface inside the control volume.

$$f_i = \lim_{\Delta V_{\rm cell}\to 0}\frac{1}{\Delta V_{\rm cell}}\int \mathcal{S}_i(\vec{e}_i \cdot \vec{n}_s)ds. \tag{4}$$

The unit vectors, $\vec{e}_i$ and $\vec{n}_s$ are oriented in ith direction and normal to the surface, respectively. Note that the particle force term, f_i, behaves like the Dirac delta function at the particle surface when the control volume size shrinks to zero ($\Delta V_{\rm cell} \to 0^+$), while it becomes zero elsewhere. The force is not defined inside particles since Eq. (3) is constructed only for the continuous phase. By integrating over the surface of one particle, the total force can be described as

$$F_i = \int \mathcal{S}_i(\vec{e}_i \cdot \vec{n}_s)ds. \tag{5}$$

Next, we nondimensionalize the particle-laden Navier-Stokes equation proposed in Eq. (3) to find relevant nondimensional parameters. We follow the same scaling as the Reynolds' nondimensionalization [12] for the convection, pressure, and viscous stress terms, since the only difference from the Reynolds' nondimensionalization is the particle term. Thus,

$$U_i^* = \frac{U_i}{U_L}, \qquad x_i^* = \frac{x_i}{L}, \qquad p^* = \frac{p - p_0}{\rho_f U_L^2}, \qquad t^* = \frac{tU_L}{L}, \tag{6}$$

where U_L and L are the large scale velocity and length scales, and the superscript, $*$, denotes a nondimensional variable. We nondimensionalize the particle term in two different ways related to particle Reynolds number, Re_p, or Stokes number, St.

Although the particle force term, f_i, is treated as a delta function in our description of the carrier-phase momentum equation, it can be scaled based on its volumetric average value as used in the point force momentum coupling method [13,14]:

$$f_i \sim \frac{1}{\Delta V_{\rm cell}}\sum_{n=1}^{N^p} F_{i,n}, \tag{7}$$

where N^p is the instantaneous number of particles inside a differential volume of the Navier-Stokes equation. The Stokes drag, $F_i^{\rm Stokes}(\equiv \frac{m^p}{\tau_p}(U_i - U_i^p))$, can be chosen as the representative force scale, since it is dominant ($F_i \sim F_i^{\rm Stokes}$), where τ_p is the particle aerodynamic relaxation time. By introducing the concentration of particles, or mass of particles per unit volume, ρ_C, $(\equiv N^p m^p/\Delta V_{\rm cell})$, the particle force term can also be described as

$$f_i^{\rm Stokes} = \frac{\rho_C}{\tau_p}\langle U_i - U_i^p\rangle, \tag{8}$$

where the angle brackets, $\langle\rangle$, denote an ensemble-averaged quantity. By replacing the slip velocity, $\langle U_i - U_i^p\rangle$, with the particle Reynolds number, $\mathrm{Re}_p(\equiv d_p|\langle\vec{U} - \vec{U}^p\rangle|/\nu)$, and the relaxation time, τ_p, with its definition under the Stokes flow approximation ($\tau_p \approx \frac{d_p^2}{18\nu}\frac{\rho_p}{\rho_f}$), the Stokes force per unit volume can be scaled as

$$f_i = 18\rho_C\frac{\rho_f}{\rho_p}\frac{\mathrm{Re}_p\nu^2}{d_p^3} f_i^{*\mathrm{Re}}, \tag{9}$$

where the superscript, $*$Re denotes the nondimensional variable based on the particle Reynolds number and ρ_p is the particle density.

Finally, the normalized momentum equation becomes

$$\frac{DU_i^*}{Dt^*} = -\frac{\partial p^*}{\partial x_i^*} + \frac{1}{\mathrm{Re}_L}\frac{\partial^2 U_i^*}{\partial x_j^* \partial x_j^*} - 18\phi\frac{\rho_f}{\rho_p}\frac{L^3}{d_p^3}\frac{\mathrm{Re}_p}{\mathrm{Re}_L^2} f_i^{*\mathrm{Re}}, \tag{10}$$

where the mass of particles per unit volume, ρ_C, is expressed with the mass loading ratio, ϕ $(\equiv \rho_C/\rho_f)$.

The particle term in Eq. (10) includes the ratio of the large scale to the particle diameter raised to the third power. This indicates that variations in L/d_p have a large effect on the carrier-phase flow. The Reynolds number squared term, Re_L^2, in the denominator is also prominent in the particle force term, and this probably relates to the ratio of the particle diameter and the Kolmogorov scale since the large scale Reynolds number fixes the Kolmogorov scale.

Another possible nondimensionalization is based on the Stokes number, St. Though the nondimensionalization based on Re_p is reasonable, it is sometimes difficult to estimate the slip velocity. Re_p is not given for most experiments in the literature, while St is easier to obtain. The slip velocity may be scaled by the Kolmogorov velocity scale, v_K $(\equiv \nu/\eta)$ as $v_K \sim |\langle \vec{U} - \vec{U}^p \rangle|$, where η is the Kolmogorov length scale. The assumption is valid when $\mathrm{Re}_p \sim d_p/\eta$. The particle force term becomes

$$f_i = \frac{\rho_C}{\tau_p} v_K f_i^{*\mathrm{St}}, \tag{11}$$

where the superscript, $*\mathrm{St}$, denotes the nondimensional value based on the Kolmogorov velocity scale. Using the definitions of Kolmogorov time scale, $\tau_f (\equiv \eta^2/\nu)$, and velocity scale, v_K, the nondimensionalized particle-laden Navier-Stokes equation can be written:

$$\frac{DU_i^*}{Dt^*} = -\frac{\partial p^*}{\partial x_i^*} + \frac{1}{\mathrm{Re}_L}\frac{\partial^2 U_i^*}{\partial x_j^* \partial x_j^*} - \frac{\phi}{\mathrm{St}\,\mathrm{Re}_L^2}\left(\frac{L}{\eta}\right)^3 f_i^{*\mathrm{St}}, \tag{12}$$

where the Stokes number is defined as $\mathrm{St} \equiv \tau_p/\tau_f$.

Based on the dimensional analysis above, we define a nondimensional parameter called the particle momentum number, Pa, to simplify the carrier-phase Navier-Stokes equation in the presence of particles. The dimensionless Navier-Stokes equation is

$$\frac{DU_i^*}{Dt^*} = \frac{\partial p^*}{\partial x_i^*} + \frac{1}{\mathrm{Re}_L}\frac{\partial^2 U_i^*}{\partial x_j^* \partial x_j^*} - \frac{\phi}{\mathrm{Pa}_\dagger} f_i^{*\dagger}, \tag{13}$$

where the dagger, $\dagger$, is either Re or St.

$$\mathrm{Pa}_{\mathrm{Re}} = \frac{1}{18}\frac{\mathrm{Re}_L^2}{\mathrm{Re}_p}\frac{\rho_p}{\rho_f}\left(\frac{d_p}{L}\right)^3, \tag{14}$$

$$\mathrm{Pa}_{\mathrm{St}} = \mathrm{St}\mathrm{Re}_L^2\left(\frac{\eta}{L}\right)^3 = \frac{1}{54\sqrt{2}}\frac{\mathrm{Re}_L^2}{\mathrm{St}^{1/2}}\frac{\rho_p^{3/2}}{\rho_f^{3/2}}\left(\frac{d_p}{L}\right)^3. \tag{15}$$

For the same flow conditions, $\mathrm{Pa}_{\mathrm{St}}$ is proportional to St, since Re_L, η and L depend only on the unladen flow conditions. The right-hand side of Eq. (15) shows another form of $\mathrm{Pa}_{\mathrm{St}}$ that was derived by eliminating η.

In Eq. (13), there are three dimensionless parameters in the carrier-phase momentum equations: large scale Reynolds number, Re_L, mass loading ratio, ϕ, and particle momentum number, Pa. The mass loading ratio can be considered to be only related to the magnitude of the modification, since there are no experiments in which both attenuation and augmentation occur by simply changing the mass loading. Therefore, the critical parameters, which determine if turbulence is augmented or attenuated, are inferred to be Re_L and Pa.

A set of turbulence modification measurements [2–11] is examined. In the present study, the level of turbulence modification, χ, is defined using the fluctuating fluid velocities, ξ, as

$$\chi = (\xi_{\mathrm{st}}^{\mathrm{pl}}/\xi_{\mathrm{st}}^{\mathrm{ul}})^2, \tag{16}$$

where the subscript, st, denotes the streamwise direction and the superscripts, pl and ul, denote particle-laden and unladen cases, respectively. Thirty different combinations of Pa and Re out of the 80 measurements are plotted in Fig. 1 as a function of Re_L and (a): $\mathrm{Pa}_{\mathrm{St}}$ and (b): $\mathrm{Pa}_{\mathrm{Re}}$. The particle Reynolds number is estimated by assuming the slip velocity can be approximated by the terminal velocity, since Re_p was difficult to obtain for most of the previous experiments. We mapped only cases with modification levels larger than 5%. We rejected the small modification cases because in most cases the mass loading was small, and the measurements might be contaminated by measure-

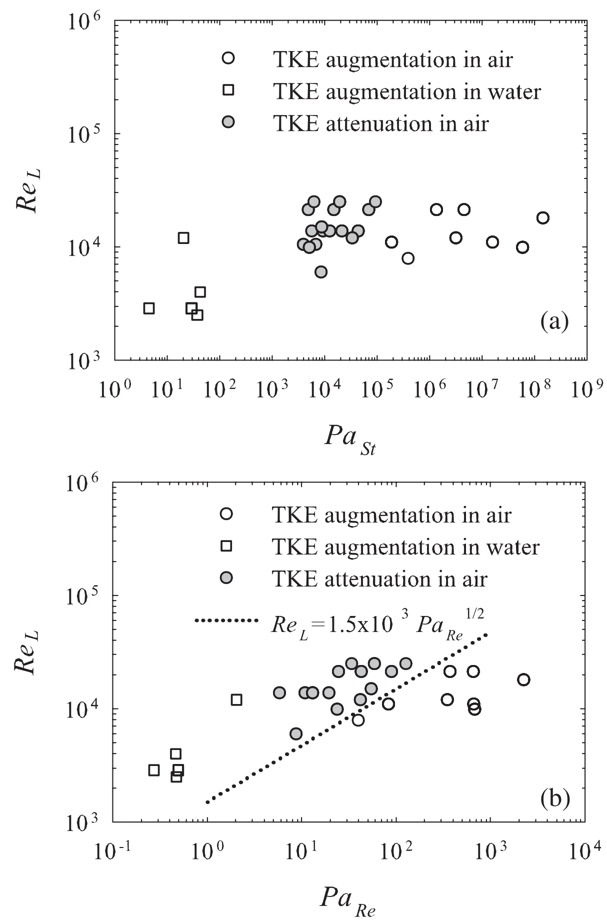

FIG. 1. Mapping of the previous turbulence modification experiments based on (a) $\mathrm{Pa}_{\mathrm{St}}$ and Re_L, and (b) $\mathrm{Pa}_{\mathrm{Re}}$ and Re_L. The circle and square symbols represent air and water turbulence, respectively. The open symbols represent TKE augmentation and filled symbols show TKE attenuation.

ment errors. The circle and square symbols represent air and water turbulence, respectively. The open symbols represent TKE augmentation and filled symbols show TKE attenuation.

The variables used in Fig. 1(a) separate the cases into augmentation and attenuation groups. Several important observations can be made about Fig. 1(a). First, the classification mainly depends on the particle momentum number, Pa. The Reynolds number appears to have little effect. However, it is important to note that the Reynolds number for these laboratory scale experiments all fell into the same range. Second, the classification of the turbulence modification is not monotonic, as we see the turbulence attenuation region is between the augmentation regions with two critical particle momentum numbers, $\mathrm{Pa}_{\mathrm{St}}^{C}$ as: $\mathrm{Pa}_{\mathrm{St}}^{\mathrm{CW}} \approx 10^3$ and $\mathrm{Pa}_{\mathrm{St}}^{\mathrm{CA}} \approx 10^5$. This indicates an interesting feature of particle-laden flows; particles with large Pa augment the TKE, while particles with smaller Pa attenuate TKE, and particles with much smaller Pa (much smaller St) again augment TKE. For even smaller Pa, particles eventually become tracers and cannot modify the turbulence. Though this consideration is counterintuitive, similar results are reported by Druzhinin and Elghobashi [15] and Druzhinin [14]. They obtained larger turbulence augmentation for smaller Stokes number. The small Stokes number might correspond to the large Pa region of the water augmentation. Though homogeneous and isotropic turbulence used in their studies cannot simply be compared to the current internal flow mappings, we expect a similar trend.

Figure 1(b) shows a similar trend to Fig. 1(a). Turbulence attenuation occurs in the range $3 < \mathrm{Pa}_{\mathrm{Re}}^{C} < 200$. Since there is apparently some dependence on Re_L, another modification regime boundary might be considered. The dotted line in Fig. 1(b) follows the relation $\mathrm{Re}_L^C = 1.5 \times 10^3 (\mathrm{Pa}_{\mathrm{Re}}^C)^{1/2}$. Thus, the classification can be described as $\mathrm{Re}_L < 1.5 \times 10^3 \mathrm{Pa}_{\mathrm{Re}}^{1/2}$ or $\mathrm{Pa}_{\mathrm{Re}} < 3$ for the augmentation cases, and $\mathrm{Re}_L > 1.5 \times 10^3 \mathrm{Pa}_{\mathrm{Re}}^{1/2}$ and $\mathrm{Pa}_{\mathrm{Re}} > 3$ for the attenuation cases.

For comparison, a mapping using Re_L and St, which is one of the most important parameters to describe turbulence modification, is shown in Fig. 2. The two parameters plotted in these figures are almost independent, and these figures do make it clear that St is not the parameter which controls turbulence modification.

In summary, the Navier-Stokes equation in the presence of particles was normalized to obtain a novel dimensionless parameter, the particle momentum number, Pa. Previous experiments of the turbulence modification by particles were successfully categorized into three groups based on Pa. The turbulence attenuation region is between the augmentation regions with two critical particle momentum numbers. It can be concluded that the particle momentum number, Pa, is an essential parameter to describe turbulence modification by particles. The maps classifying particle-laden flows can be used to help choose parameters for future experimental investigations.

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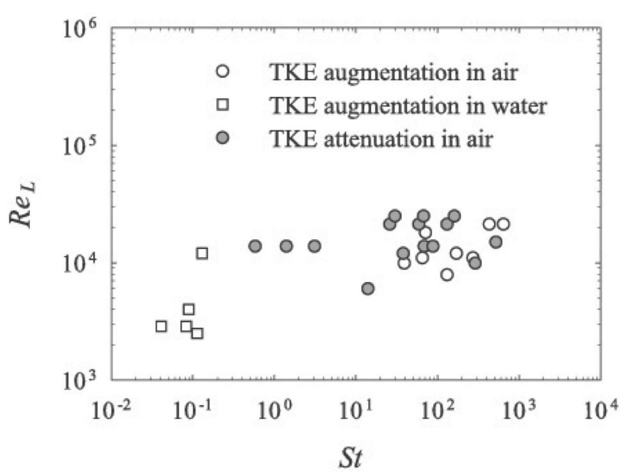

FIG. 2. Mapping of the previous turbulence modification experiments based on St and Re_L. The circle and square symbols represent air and water turbulence, respectively. The open symbols represent TKE augmentation and filled symbols show TKE attenuation.

*Present address: Central Research Laboratory, Hitachi Ltd.
tomohiko.tanaka.nx@hitachi.com
†eatonj@stanford.edu

[1] R. A. Gore and C. T. Crowe, Int. J. Multiphase Flow **15**, 2, 279 (1989).
[2] K. T. Kiger and C. J. Pan, J. Turbul. **3**, 1 (2002).
[3] J. D. Kulick, J. R. Fessler, and J. K. Eaton, J. Fluid Mech. **277**, 109 (1994).
[4] J. Kussin and M. Sommerfeld, Exp. Fluids **33**, 143 (2002).
[5] S. Lain, M. Sommerfeld, and J. Kussin, Int. J. Heat and Fluid Flow **23**, 647 (2002).
[6] S. L. Lee and F. Durst, Int. J. Multiphase Flow **8**, 125 (1982).
[7] C. Ljus, B. Johansson, and A. E. Almstedt, Int. J. Multiphase Flow **28–7**, 1075 (2002).
[8] Y. Sato and K. Hishida, Int. J. Heat and Fluid Flow **17**, 202 (1996).
[9] Y. Suzuki, M. Ikenoya, and N. Kasagi, Exp. Fluids **29**, S185 (2000).
[10] Y. Tsuji and Y. Morikawa, J. Fluid Mech. **120**, 385 (1982).
[11] Y. Tsuji, Y. Morikawa, and H. H. Shiomi, J. Fluid Mech. **139**, 417 (1984).
[12] F. M. White, *Viscous Fluid Flow* (McGraw Hill, New York, 1991).
[13] K. D. Squires and J. K. Eaton, Phys. Fluids A **2**, 1191 (1990).
[14] O. A. Druzhinin, Phys. Fluids **13–12**, 3738 (2001).
[15] O. A. Druzhinin and S. Elghobashi, Phys. Fluids **11**, 602 (1999).