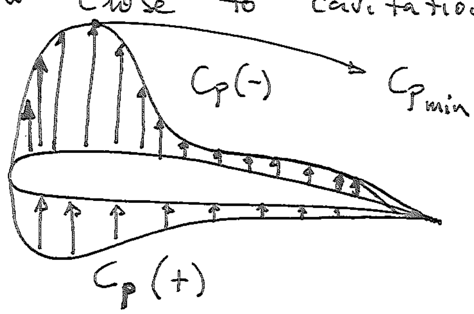
# CAVITATION

Cavitation occurs when the pressure in a liquid falls below a threshold and a vapour babble forms in the liquid. This can occar because gases come out of dissolution (saturation pressure is a function of pressure and temperature in the liquid) or because the pressure falls below the vapour pressure at that temperature and liquid vapour forms a babble.

Cavitation affects performance of turbomachinery and other hydrodynamic elements, generates noise and vibrations and damages solid surfaces in the surrounding area (pitting).

Cavitation number $\dfrac{P_\infty - P_v(T_\infty)}{\frac{1}{2}\rho_{liq} V_\infty^2}$ gives a measure of how close to cavitation the flow may be.




Ideally cavitation would occar at $\sigma^{cav} = \sigma_i^{cav}$ critical value for inception at the point where $C_p = C_{p_{min}}$

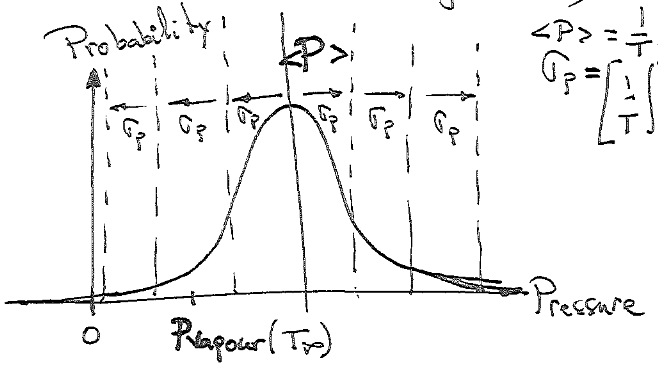
If cavitation were to occar where the local pressure reaches vapour pressure then $\sigma_i^{cav} = -C_{p_{min}}$

There are however a number of effects that delay cavitation from its theoretical inception.

- Ability of liquids to sustain tension: bubbles do not grow when $P \leq P_{vapour}(T_\infty)$. This tension strength depends on the presence of cavitation nuclei: microbubbles or microscopic particles that serve as starting points for vapour to come out of the liquid state. Dissolved gases also act in the same way.

- Residence time requirement for bubbles to grow: vapour or dissolved gases take time to come out of solution.

- Turbulent fluctuations: pressure fluctuates around the average, which is the quantity normally measured. But local fluctuations below the average can expose the liquid to cavitation conditions for long enough times to actually produce bubble growth, even when average conditions would not




$\langle P \rangle = \frac{1}{T}\int_0^T P(t)\,dt$ → average

$\sigma_p = \left[\frac{1}{T}\int_0^T (P-\langle P \rangle)^2\,dt\right]^{1/2}$ → root mean square

# STABILITY OF A VAPOUR / GAS BUBBLE

$P_\infty$

$(P_v + P_g)$

$$P_v + P_{g_{eq}} - P_\infty - \frac{2\sigma}{R_{eq}} = 0$$

This equilibrium may or may not be stable

The linearized Rayleigh - Plesset equation is

$$R_b \frac{d^2 R_b}{dt^2} + \frac{3}{2}\left(\frac{dR_b}{dt}\right)^2 + \frac{4\nu_{liq}}{R_b}\frac{dR_b}{dt} = \frac{\epsilon}{\rho_{liq}}\left(\frac{2\sigma}{R_{eq}} - 3n\gamma P_{g_e}\right)$$

If the perturbation is slow, so the partial pressure of gas inside the bubble is constant (gas comes in or out of solution to maintain the equilibrium partial pressure with the liquid as the bubble expands or contracts) then $n = 0$.

If the perturbation is very fast so that there is no mass transfer across the interface and the mass of gas inside bubble is constant, then $n = 1$.

If $\frac{2\sigma}{R_{eq}} > 3n\gamma P_{g_e}$ then the response $\frac{dR_b}{dt}$ and $\frac{d^2R_b}{dt^2}$ has the same sign as the perturbation $\epsilon \Rightarrow$ Unstable

If $\frac{2\sigma}{R_{eq}} < 3n\gamma P_{g_e}$ then the response $\frac{dR_b}{dt}, \frac{d^2R_b}{dt^2}$ is in the opposite direction as the perturbation $\epsilon \Rightarrow$ Stable

If $n=0$ $\Rightarrow$ the bubble is always unstable.

This is consistent with physical intuition: given sufficient time for gas transfer, any perturbation to a bubble is unstable. If $R$ decreases, $\frac{2\sigma}{R}$ the Poisson pressure inside the bubble increases, gas leaves to dissolution in the liquid, $R$ decreases further. If $R$ increases, $\frac{2\sigma}{R}$ decrease gas comes out of solution and into the bubble, etc.

For fast perturbations: $\underbrace{P_{ge} = \dfrac{m_g T_B R_g}{\frac{4}{3}\pi R_{eq}^3} > \dfrac{2\sigma}{3\gamma R_{eq}}}_{\text{Condition for stability}}$

so there is a critical radius $R_{crit} = \sqrt{\dfrac{9\gamma m_g T_b R_g}{8\pi\sigma}}$

$R_{eq} > R_{crit}$ can be stable, but $R_{eq} < R_{crit}$ are unstable.

All cavitation nuclei larger than $R_c$ will grow explosively and cavitate. The lower the pressure level the smaller the critical size and more nuclei will be activated.

$$P_{\infty c} = P_v - \frac{4\sigma}{3}\sqrt{\frac{8\pi\sigma}{9\gamma m_g T_b R_g}} \quad ; \quad R_c \simeq \frac{4\sigma}{3(P_v - P_\infty)^*}$$

The rate of growth can be approximated from solution to the Rayleigh Plesset equation

$$\frac{dR}{dt} \rightarrow \sqrt{\frac{2}{3}\,\frac{P_v - P_\infty^*}{\rho_{liq}}} \quad \text{Which becomes} \quad \frac{dR}{dt} \sim U_\infty\sqrt{-\sigma_{cav} - C_{p_{min}}}$$

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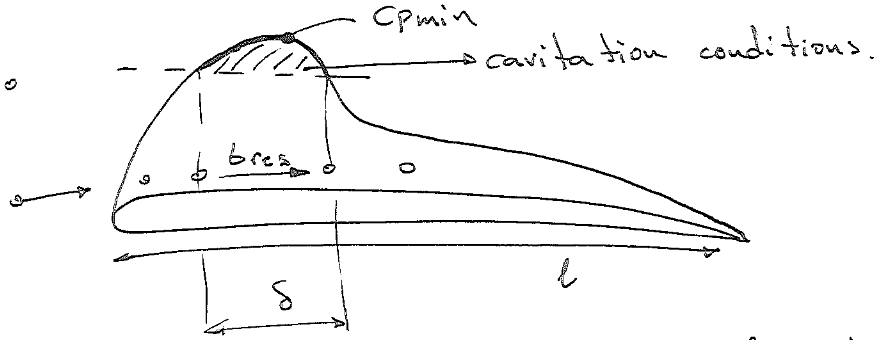
To estimate the size to which the bubble will grow under cavitation conditions, we need to know the time spent under those conditions. Assuming the pressure coefficient is represented by an inverted parabola, the time would be:




$\delta \sim \dfrac{l}{\sqrt{-\sigma - C_{pmin}}}$ and therefore time spent in it is $t_{res} \sim \dfrac{l/\sqrt{-\sigma-C_{pmin}}}{U_\infty}$

$$R_m \sim \cancel{U_\infty}\sqrt{-\sigma - C_{pmin}} \cdot \frac{l \cdot \sqrt{-\sigma - C_{pmin}}}{\cancel{U_\infty}}$$

$$\boxed{R_m \sim l\left(-\sigma - C_{pmin}\right)}$$

# CAVITATION BUBBLE COLLAPSE

If the maximum radius before the start of the collapse is $100\,R_0$, the pressure at that point will be of the order of $10^{-6}$ bar (assuming an initial pressure $P_{g_0} = 1$ bar). Therefore the pressure at collapse will be of the order of

$$P_{\min\,Radius} = P_{g_0}\left[(\gamma-1)\frac{P_\infty^{*} - P_\infty(0) - P_{g_{max}} + \frac{3\sigma}{R_0}}{P_{g_{max}}}\right]^{\frac{\gamma}{\gamma-1}}$$

$(10^6\ \text{bar})$

and the Temperature will be: $T_{\min\,Radius} = T_0 \left[\qquad\qquad\right]^{1} \simeq 4\cdot 10^{4}\,T_0$