Bubble or Droplet Deformation

As particles move through a fluid, the surface stresses on them create a deformation that may impact the particles' motion.

For bubbles or droplets, the external stresses are counteracted by surface tension with tries to keep the particles spherical (for solid particles, elasticity would play the role of opposing deformation trying to return particles to their original shape).

Cohesive stresses due to surface tension scale as $\frac{\sigma}{R}$ while disruptive stresses from the fluid motion scale as ρV_∞^2 (for large Reynolds number, for low $\frac{\rho V_\infty R}{\mu}$ the stresses are $\frac{\mu V_\infty}{R}$. The ratio of this produces the Weber number $We = \frac{\rho V_\infty^2}{\sigma/R} = \frac{\rho V_\infty^2 R}{\sigma}$

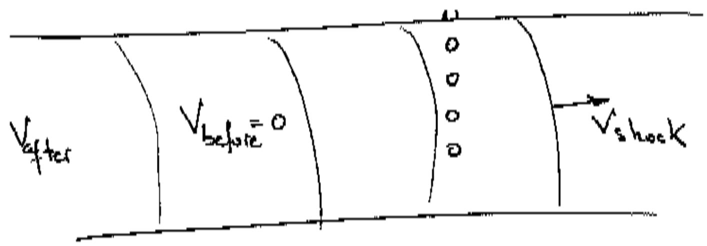
In low Reynolds number cases we get

$$\frac{\frac{\mu V_\infty}{R}}{\sigma/R} = \frac{\rho V_\infty^2 R}{\sigma} \cdot \frac{\mu}{\rho V_\infty R} = \frac{We}{Re}$$

Significant deformation will occur when $We > 1$ (for $Re \gg 1$) or when $\frac{We}{Re} > 1$ (for $Re \ll 1$).

To solve the previous problem, however, one needs to know the relative velocity between the fluid and the particle. In a shock wave tube, which has been used extensively to study droplet deformation, the fluid velocity is given by the shock strength and the droplet is initially stationary.

$V_\infty = V_{after}$



In flows where gravity is inducing the relative velocity this can not be determined a priori, it depends on the state of deformation of the particle. This can be represented by $F(Re, We, Fr) = 0$ where Fr is the Froude number

$$Fr = \frac{V_\infty}{\sqrt{gR\left|1 - \rho_p/\rho_f\right|}}$$

The Froude number is used in free surface flows when gravity waves are active and compete withe convective velocity to transmit information across the flow. The Froude number is analog to the Mach number in that acoustic (gravity) waves compete with the velocity to **propagate** across the fluid.

The problem of droplet/bubble deformation contain completely different physics and therefore the Froud number is not useful in understanding it.

Morton number: $M_o = \dfrac{g\left|1-\rho_P/\rho_f\right| \mu^4}{\rho_f\ \sigma^3} \left(= \dfrac{W_e^3}{F_r^2 R_e^3} \right)$

Thus, the problem of droplet/bubble deformation become

$F(R_e, W_e, M_o) = 0$ or $F(R_e, M_o, C_D) = 0$ since $V_{p\infty}$ is a function of C_D and R_e.

Alternatively, we can also use the Eötvös number

$E_o = \dfrac{g\left|1-\rho_P/\rho_f\right| R^2}{\sigma/\rho_f}$ This is used primarily in bubbl

Whereas the Bond number is used for droplets (but is define in exactly the same way).

For bubbles $\rho_P/\rho_f \ll 1$ and the Morton number is based on the properties of the surrounding fluid, gravity and the surface tension between the gas and the liquid. It is very small except for the highest viscosity liquids.

Bubbles deform when $Re > M_o^{-1/4}$ into ellipsoids first and for larger Re they eventually become spherical caps.

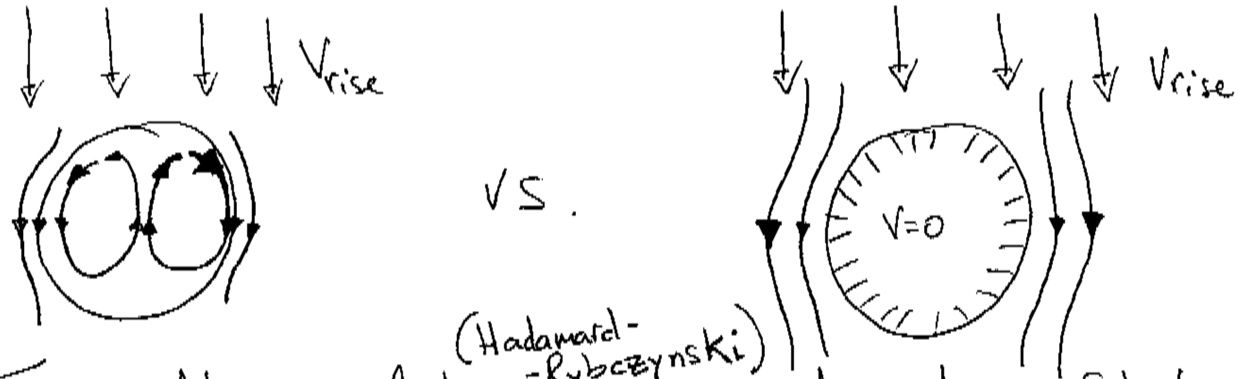
For $Re_b < 360$, the wake is laminar and is composed of a toroidal vortex. For larger Re, the wake transitions to turbulence. The rise velocity in still fluid asymptote to a value $V_\infty \simeq \sqrt{g R_b}$

MARANGONI EFFECTS

Surface tension gradients along the surface of a droplet and, specially, a bubble can create flow and deform the bubble/droplet. Temperature, electric field or contaminants (surfactants) concentration differences ca induce gradients of surface tension.

In bubbles, surfactants accumulate at the surfac where they decrease the value of surface tension. Additi lly they create a rigid layer that isolates the inner fluid (gas) from the surrounding liquid. This "crust changes the boundary condition at the free surfac from one of free stress to one of no-slip.

(technically continuity of stresses and velocities across the interface).



The resulting solution (Hadamard-Rybczynski) is similar to Stokes analytic solution but leads to a reduced drag coefficient $C_D = \frac{16}{Re}$ instead of $\frac{24}{Re}$.

In realistic conditions, we should expect water to be contaminated so that bubbles will actually collect surfactants quickly and saturate to the rigid interface (no slip) behaviour. Only in very select conditions will the free surface be pure enough to have stress free or at least intermediate behaviours.