Physics of the interactions between inertial particles and turbulence

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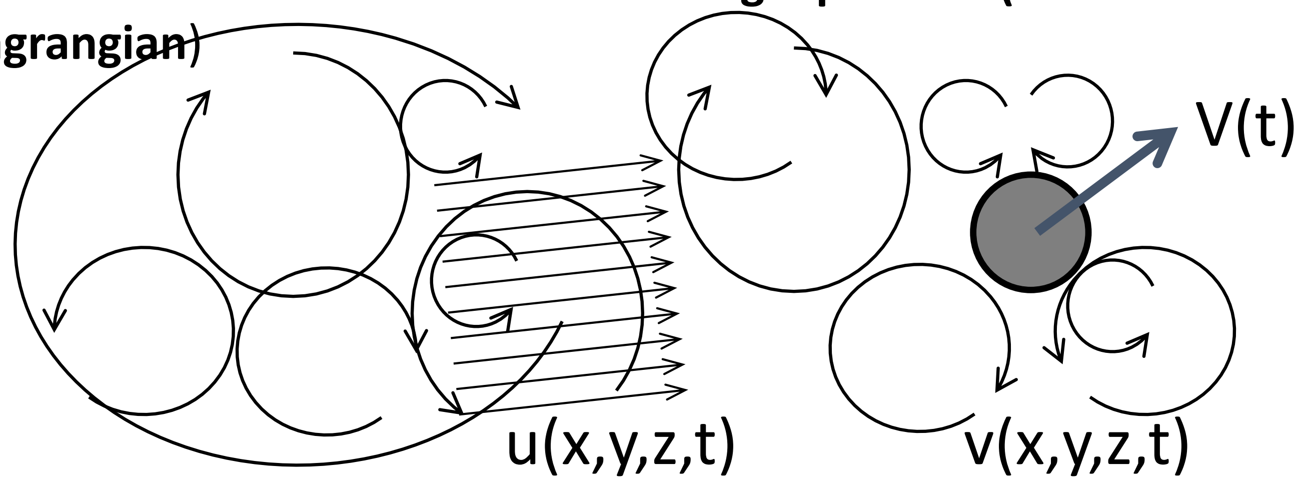
Particle dynamics in non-uniform flow: the modified Basset-Boussinesq-Oseen equation

Two Fluid Equations (Eulerian-Eulerian) VS

Particle Tracking Equations (Eulerian-Lagrangian)



u is the unpertubed fluid velocity
v is the actual fluid velocity influenced by the presence of the spherical particle
Y(t) is the position of the center of the sphere

"Equation of Motion of a small rigid spherical particle in a non-uniform flow". Maxey, M. and Riley, J., Physics of Fluids, 1983

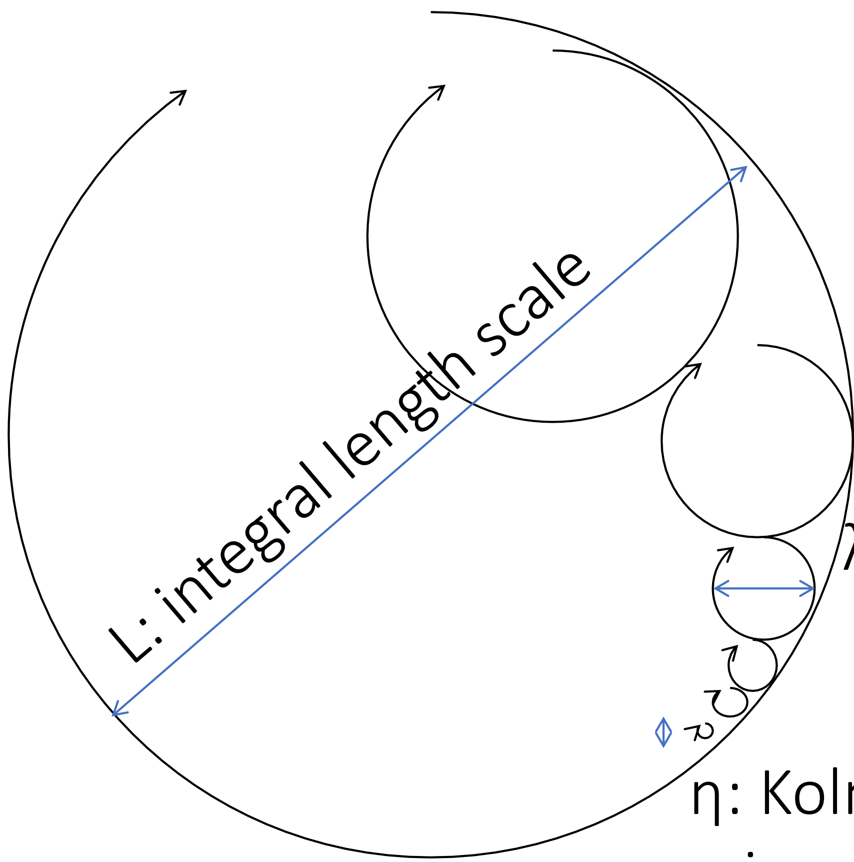
Particle dynamics in non-uniform flow: the modified Basset-Boussinesq-Oseen equation

When we group the added mass together with the left hand-side (for the particle acceleration) and the fluid stresses (for the fluid acceleration) and divide the MR eq by the mass of the sphere, $\rho_p \pi d^3/6$, plus the fluid added mass, $1/2 \rho_f \pi d^3/6$, we get:

$$\begin{aligned}\underbrace{\frac{d\vec{V}}{dt}}_{0+3} &= \underbrace{\frac{\left(\rho_p-\rho_f\right)}{\left(\rho_p+\frac{1}{2}\rho_f\right)}\vec{g}}_{1} + \underbrace{\frac{3}{2}\frac{\rho_f}{\left(\rho_p+\frac{1}{2}\rho_f\right)}\frac{D\vec{u}}{Dt}}_{2+3} \\ &+ \underbrace{\frac{18 f_d \rho_f \nu_f}{\left(\rho_p+\frac{1}{2}\rho_f\right)\cdot d^2}\left(\vec{u}-\vec{V}\right)}_{4} + \frac{C_L}{2}\frac{\rho_f}{\left(\rho_p+\frac{1}{2}\rho_f\right)}\left(\vec{u}-\vec{V}\right)\times\vec{\omega} \\ &+ \underbrace{\frac{9\rho_f \nu^{1/2}}{\pi^{1/2}\left(\rho_p+\frac{1}{2}\rho_f\right)d}\int_0^t \frac{1}{\sqrt{t-\tau}}\frac{d}{d\tau}\left(\vec{u}-\vec{V}\right)d\tau}_{5}\end{aligned}$$

A note about the scales of turbulence

Richardson’s 1921 poem:
“Big whirls have little whirls
That feed on their velocity
and little whirls have littler whirls
and so on to viscosity”



λ: Taylor’s microscale

η: Kolmogorov’s microscale

A note about the scales of turbulence

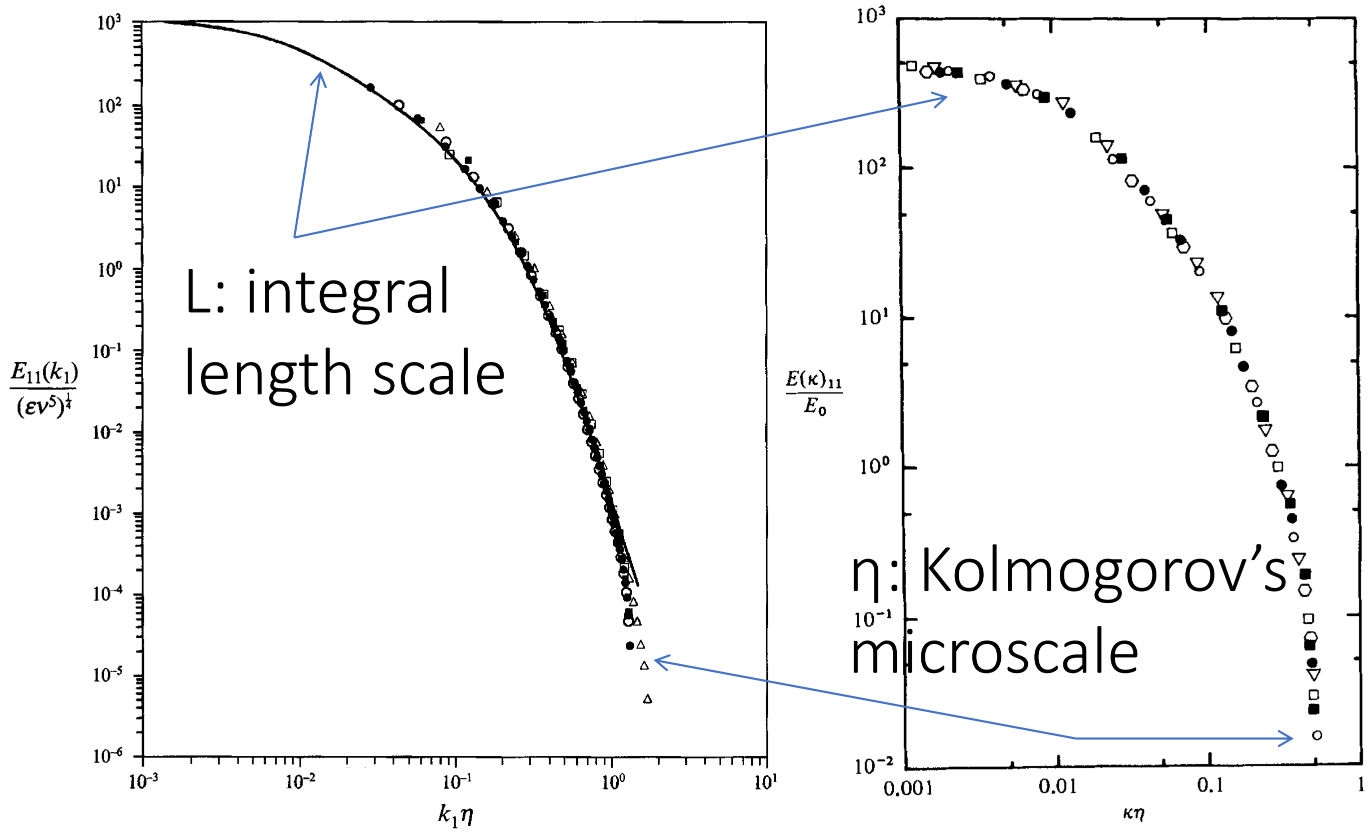


FIGURE 17. Longitudinal one-dimensional energy spectrum: ○, $x/M = 15$; □, 30; ⬡, 45; ▽, 60; ●, 75; ■, 90.

A note about the scales of turbulence

The ratio of L (integral length scale) to η (Kolmogorov's microscale) depends on the Reynolds number: $L/\eta \approx Re_{\lambda}^{3/4}$

The results from Maxey and Riley are valid when $D/\eta << 1$ and $Re_p = DV_t/\nu_f < 1$

Governing parameters

The dynamics of heavy particles in a homogeneous and isotropic turbulence depends on :

- **the droplet inertia**, characterized by the Stokes number:

$$St = \frac{\tau_p}{\tau_k}$$

$$\tau_p = \frac{(\rho_p / \rho_{air}) d_p^2}{18 \nu_{air}}$$

$$\tau_k = \left(\frac{\nu}{\varepsilon} \right)^{1/2}$$

- **the tern** **ty**, characterize ratio:

$$\frac{V_{Stokes}}{U_{Kolmogorov}}$$

$$V_{St} = \tau_p . g$$

$$U_k = (\nu \cdot \varepsilon)^{1/4}$$

For a 10 micron particle in 0.1 m^2/s^3 air turbulence

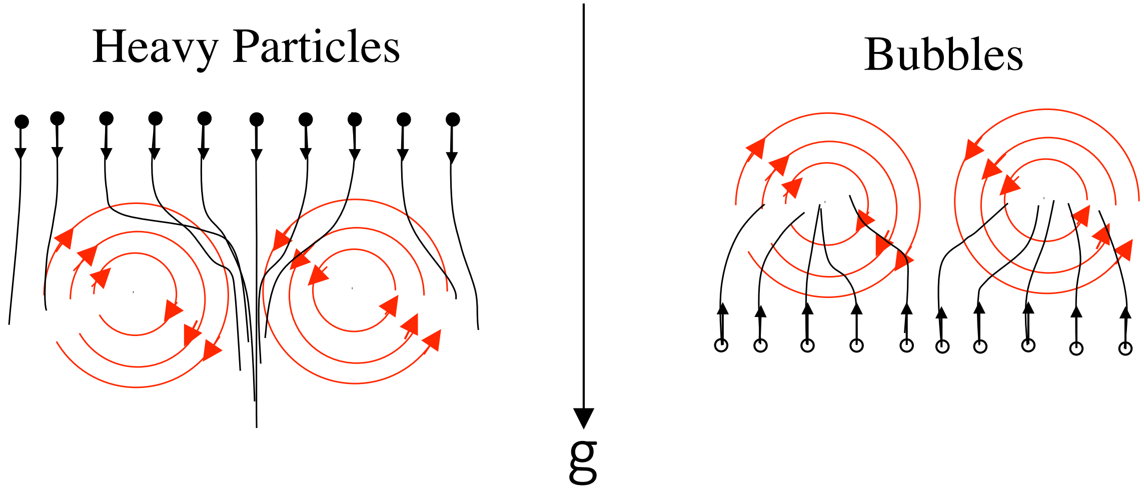
= 1.2 ms

= 12 ms

= 1.2 cm/s

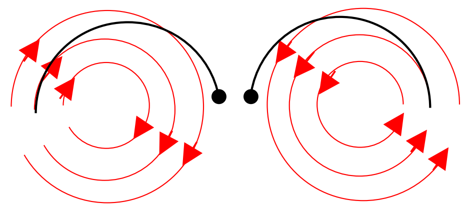
= 3.5 cm/s

Particle dynamics in non-uniform flow: a model for the interaction of particles with turbulence



$$\frac{d\vec{V}}{dt} = \frac{(\overrightarrow{u_f} - \overrightarrow{v_p}) + \overrightarrow{V_{St}}}{\tau_p} + \frac{3}{2}\frac{D\overrightarrow{u_f}}{Dt}$$

Turbulence-induced inertial dynamics: Preferential accumulation



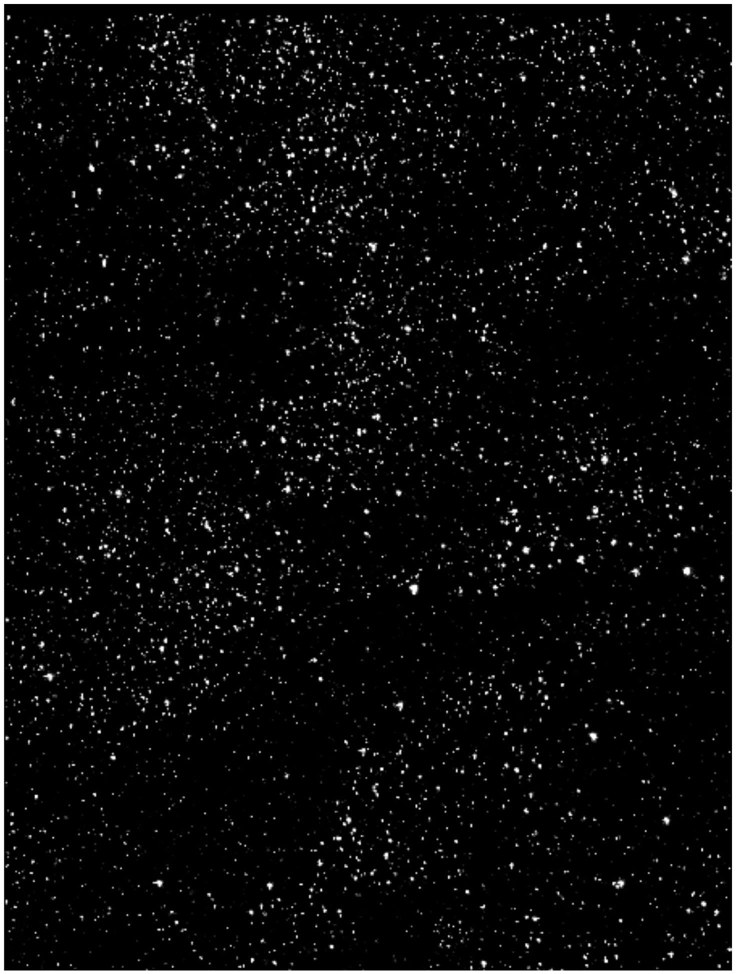
$$\frac{dV}{dt} = \left(u(Y,t) - V(t)\right) \big/ \tau_p$$

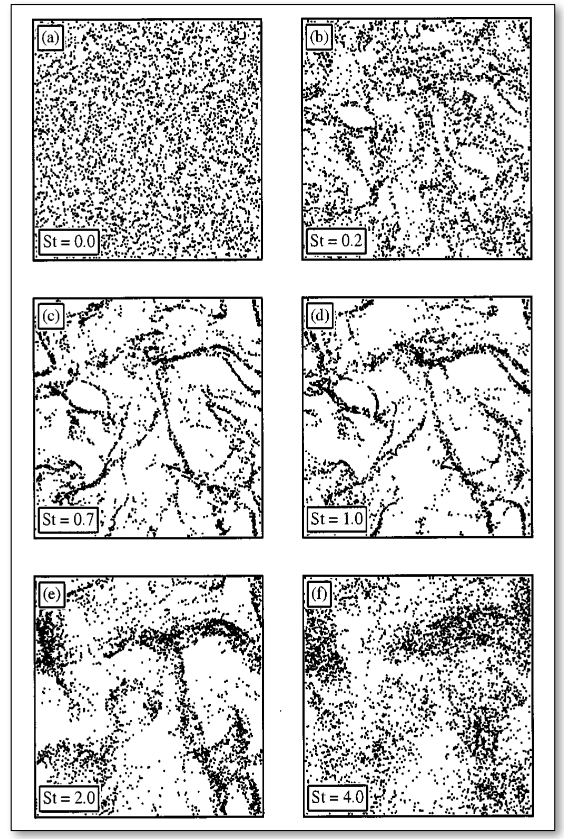
$$\frac{\rho_p}{\rho_f} > 1$$

The inertial bias mechanism

Heavy particles are centrifuged away from the vortex core, and thus tend to accumulate on the outer zone of the eddies, in regions of low vorticity and high strain.

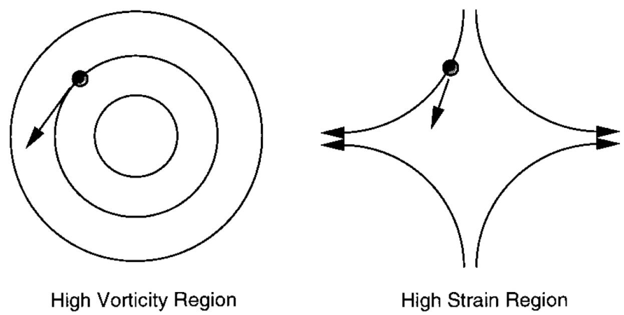
Preferential accumulation by the inertial bias mechanism

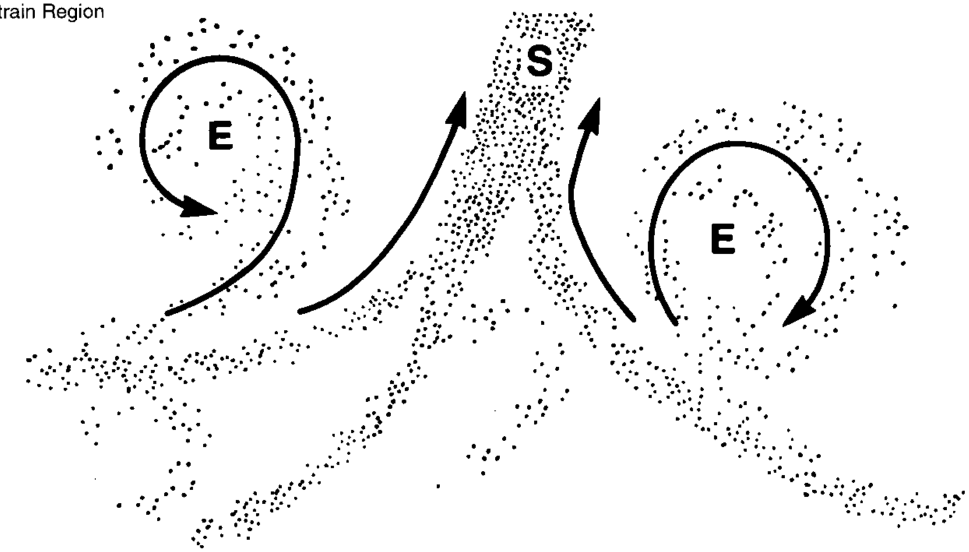




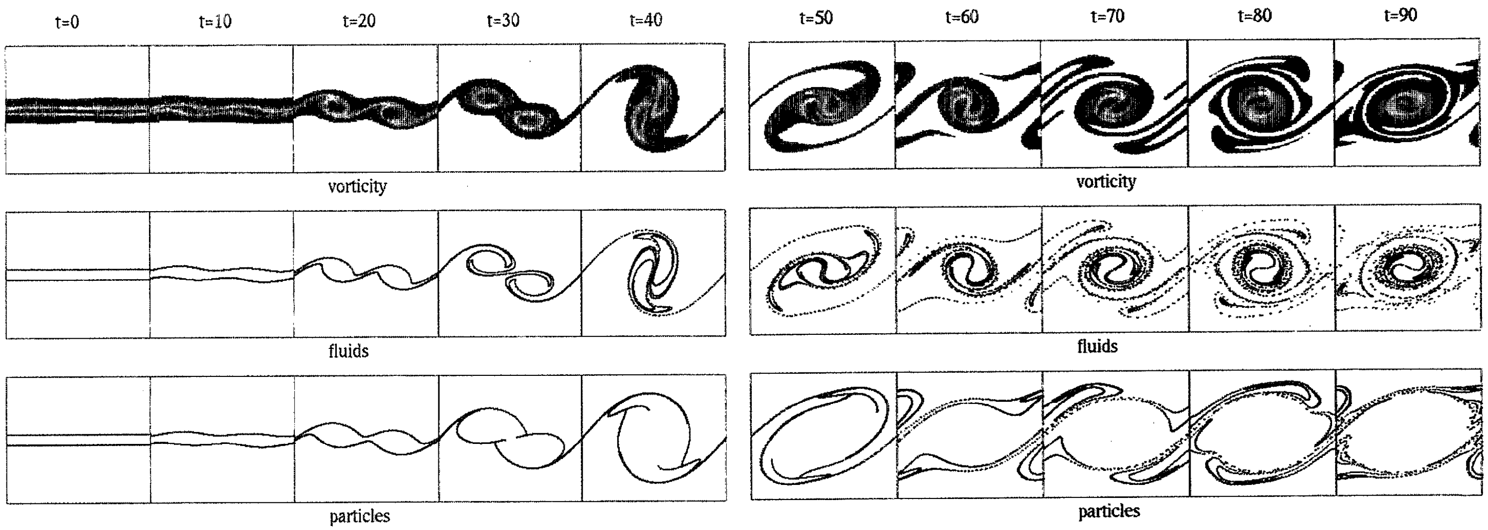
Walter C. Reade and Lance R. Collins. Effect of preferential concentration on turbulent collision rates. *Physics of Fluids*, 12(10):2530–2540, 2000.

Alternative view on the inertial bias mechanism: preferential accumulation in high strain regions



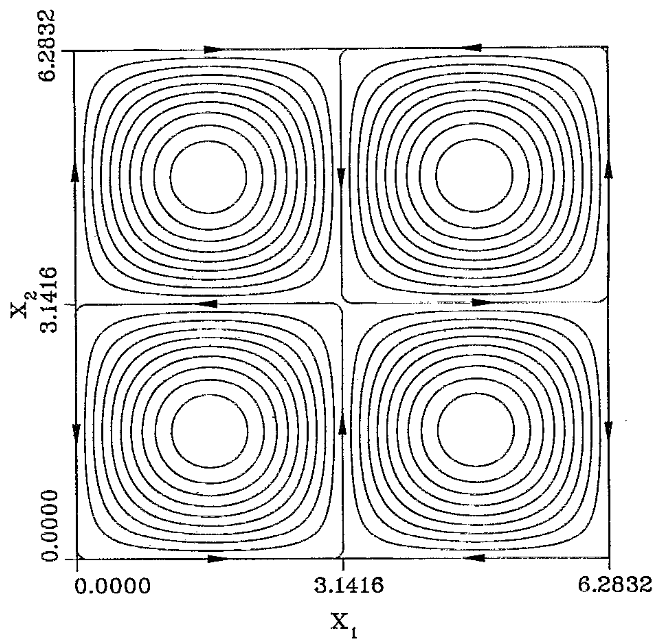


“Demixing” of heavy particles in a planar mixing layer



Wang, L.P. Maxey, M. Burton, T.D. and Stock, D.E. (1992) Chaotic Dynamics of Particle Dispersion in Fluids. Physics of Fluids A 4 (8)

Chaotic trajectories in a simple cellular flow



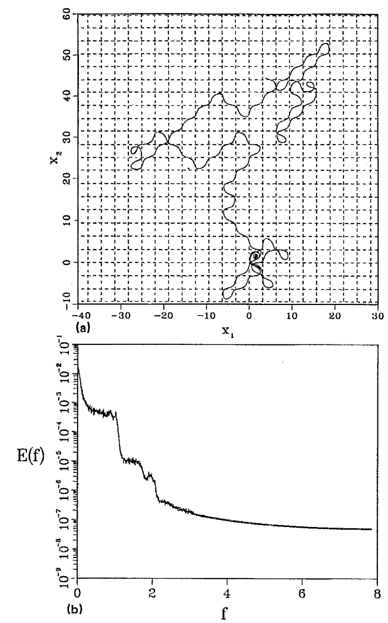


FIG. 5. (a) A typical trajectory of a particle in the chaos region with St=5 and R=0.3. Initial particle location is (1.6,1.6). Dashed lines mark the cell boundaries. (b) The power spectra for the particle velocities in the x_1 and x_2 directions for the same condition.

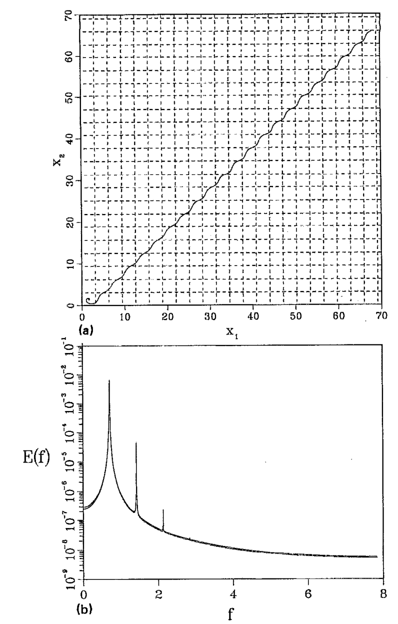


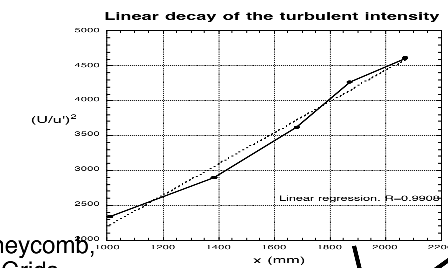
FIG. 6. (a) A typical periodic trajectory of a particle with St=5 and R=0.1. Initial particle location is (1.6,1.6). (b) The power spectra for the particle velocities in the x_1 and x_2 directions for the same condition.

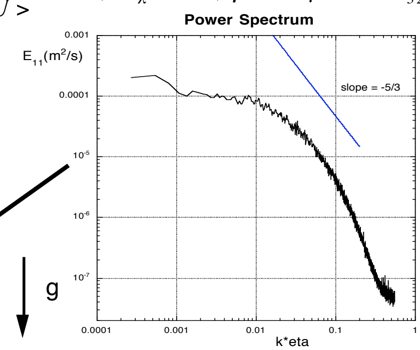
Wang, L.P. Maxey, M. Burton, T.D. and Stock, D.E. (1992) Chaotic Dynamics of Particle Dispersion in Fluids. Physics of Fluids A 4 (8)

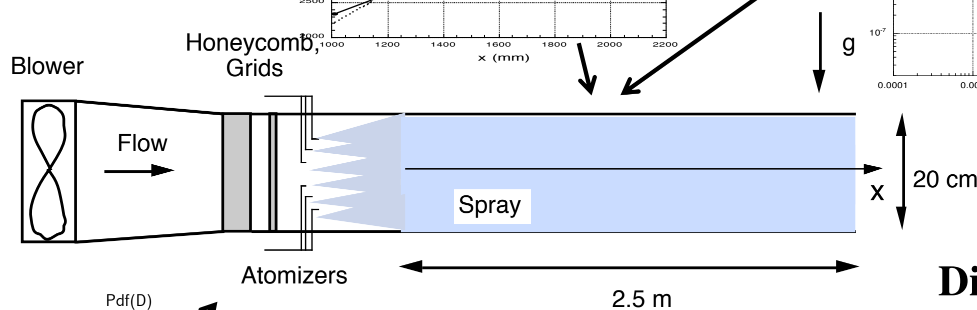
Experimental Setup.
Heavy particles in isotropic turbulence.

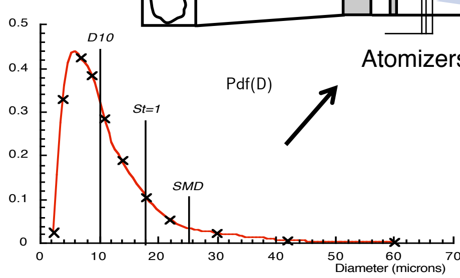
Continuous Phase Characteristics
Decaying homogeneous isotropic turbulence.

$$\frac{u'}{\langle U \rangle} = \frac{v'}{\langle U \rangle} \approx 2\% \; ;\mathrm{Re}_\lambda = 100 \, ; \eta = 200\,\mu m >> D_{32}$$









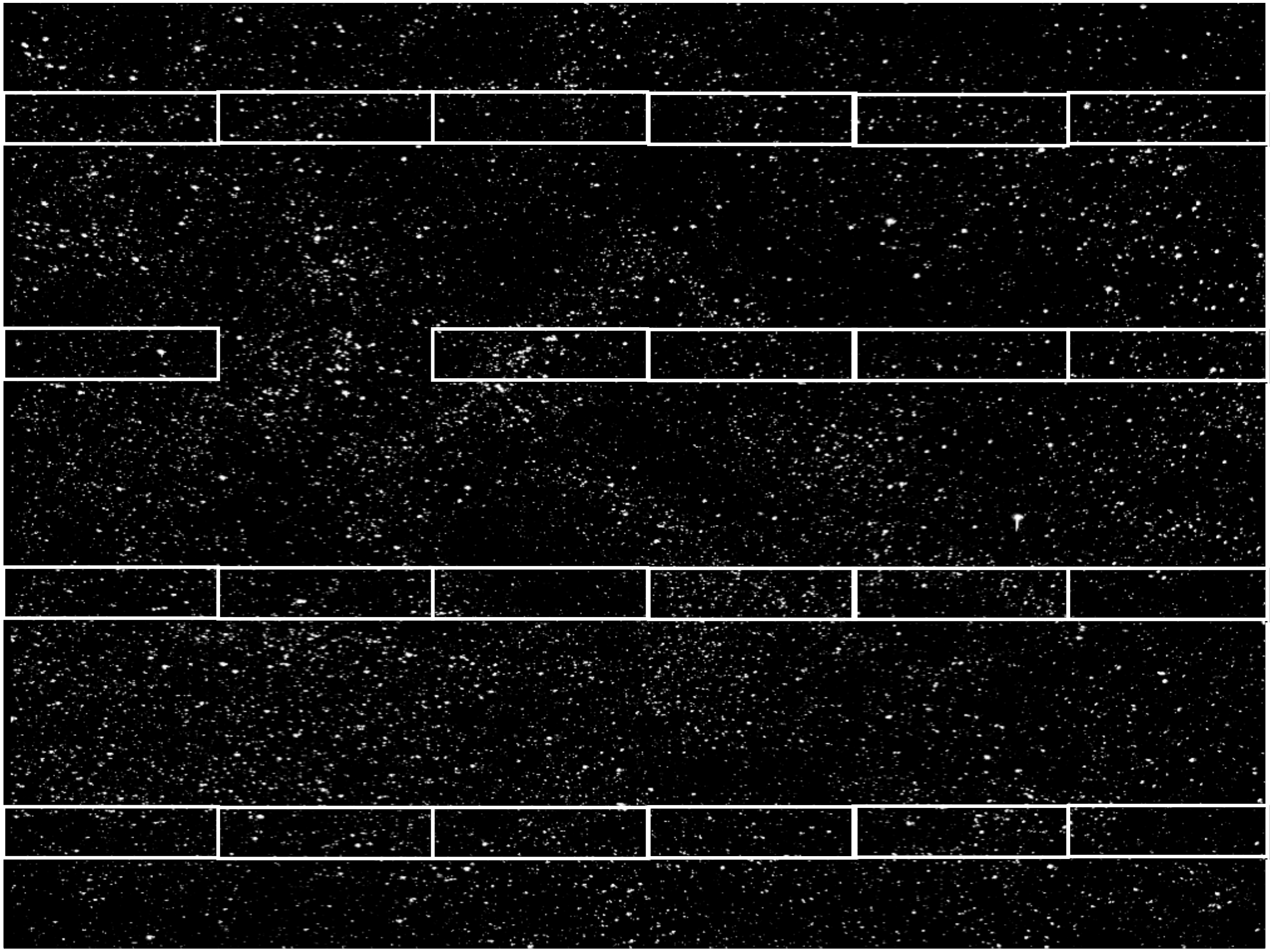
$D_{10} \approx 10\mu m$

$D_{32} \approx 25\mu m$

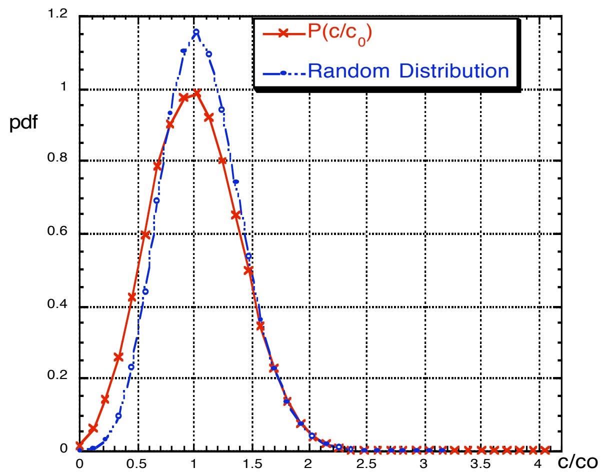
Disperse Phase Characteristics

Volume Fraction ranging from 8.10^{-6} to 8.10^{-5}

Aliseda, A., Hainaux, F., Cartellier, A., Lasheras, J.C. (2002)
Effect of preferential concentration on the settling velocity of heavy particles in homogeneous isotropic turbulence. J. Fluid Mech. 468: 77-105



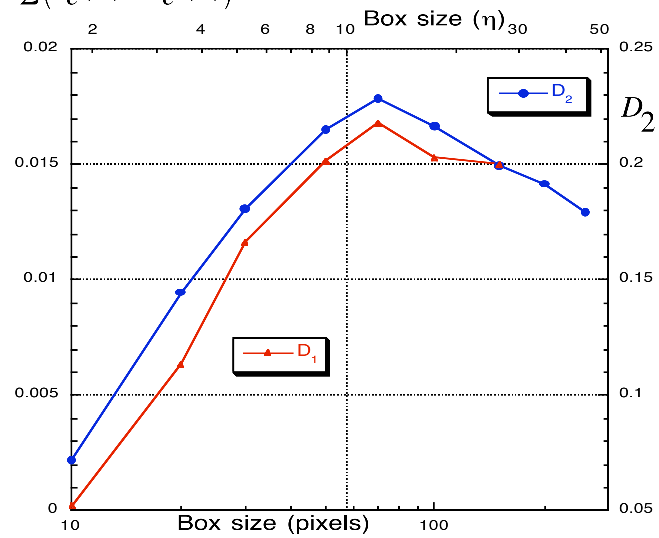
Quantifying the Deviation from Randomness



Compute the pdf of the concentration in boxes of different sizes

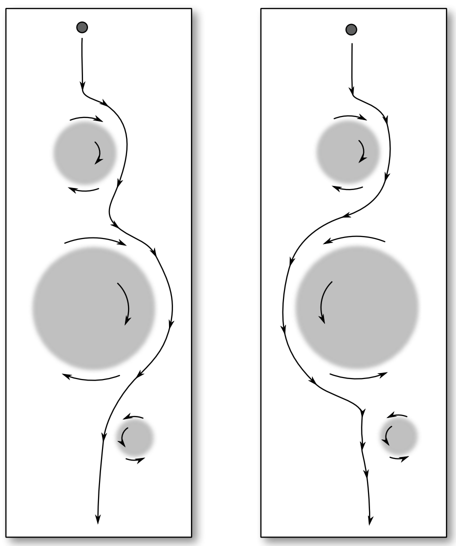
Comparing the difference with a random distribution a typical length scale of the clusters is exhibited.

$$D_1 = \sum \left(P_c(C) - P_c^r(C) \right)^2$$

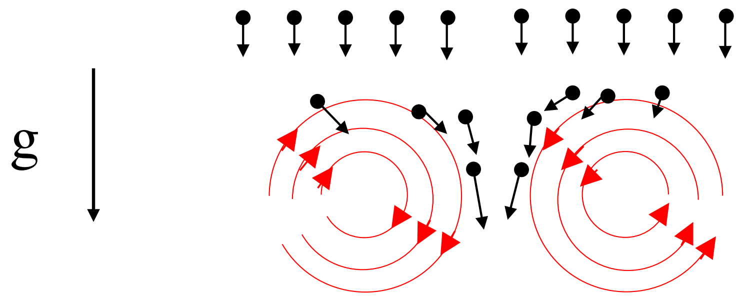


$$D_2 = \frac{\sigma - \sigma_R}{\mu}$$

Turbulence-induced inertial dynamics: Enhanced settling



$$\frac{dV}{dt} = \left(u(Y,t) - V(t) + V_{St}\right) \Big/ \tau_p$$



The preferential sweeping mechanism

In the presence of gravity, settling particles are preferentially swept to the downward side of eddies, and thus spent more time in regions where the vertical velocity of the fluid is negative.

Enhanced settling preferential sweeping mechanism

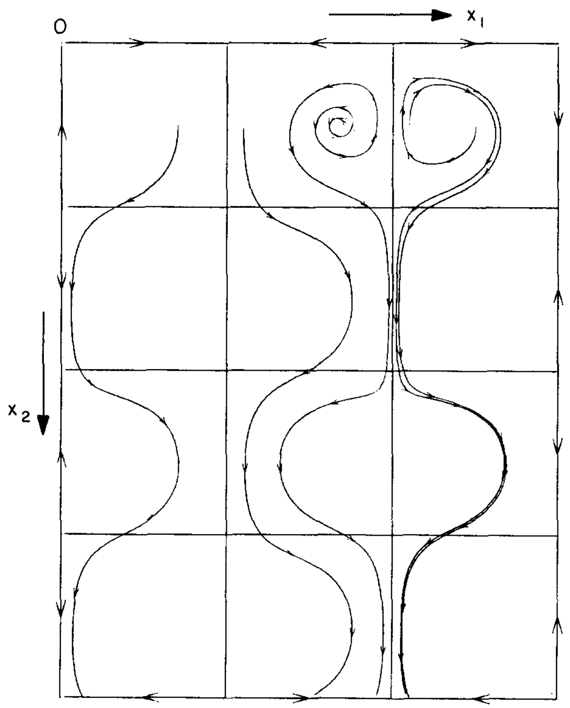


FIG. 5. Trajectories for aerosol ($R = 0$) particles settling under gra
the cellular flow field, Eq. (25), for $W = 0.5$ and $A = 5$. Particles star
rest, and initial positions $(Y_1(0)/\pi, Y_2(0)/\pi)$ are (0.7,0.5), (1.
(1.7,0.5), (2.1,0.5), and (2.5,0.5). Arrows on trajectories are drawr
tervals $\Delta t = 2.0$; arrows on the cell boundaries indicate the circulai
the adjoining cell.

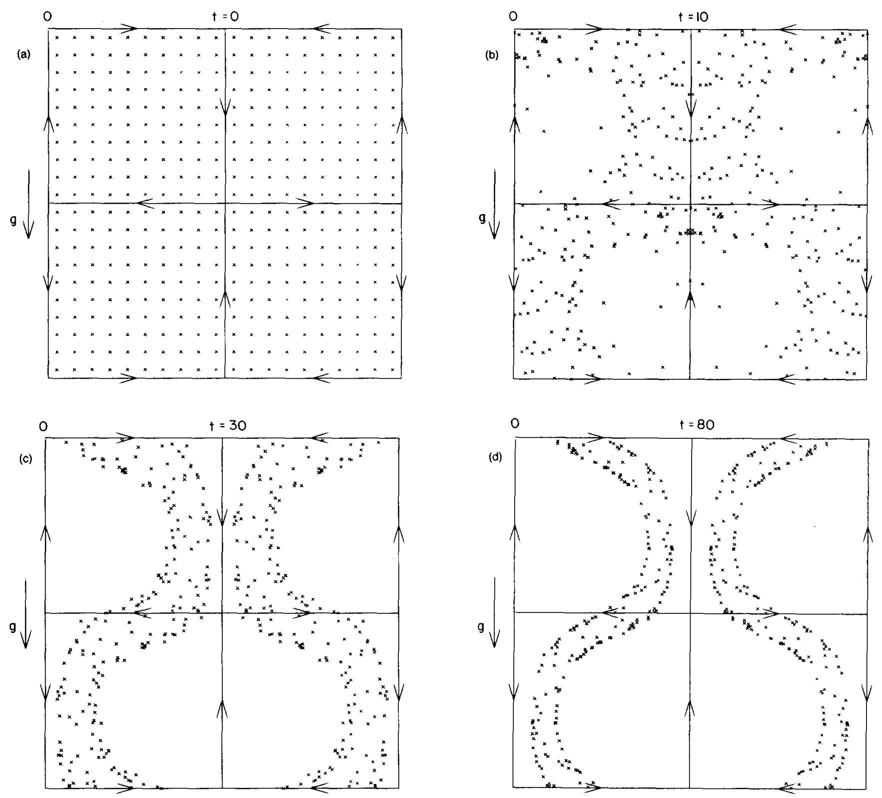
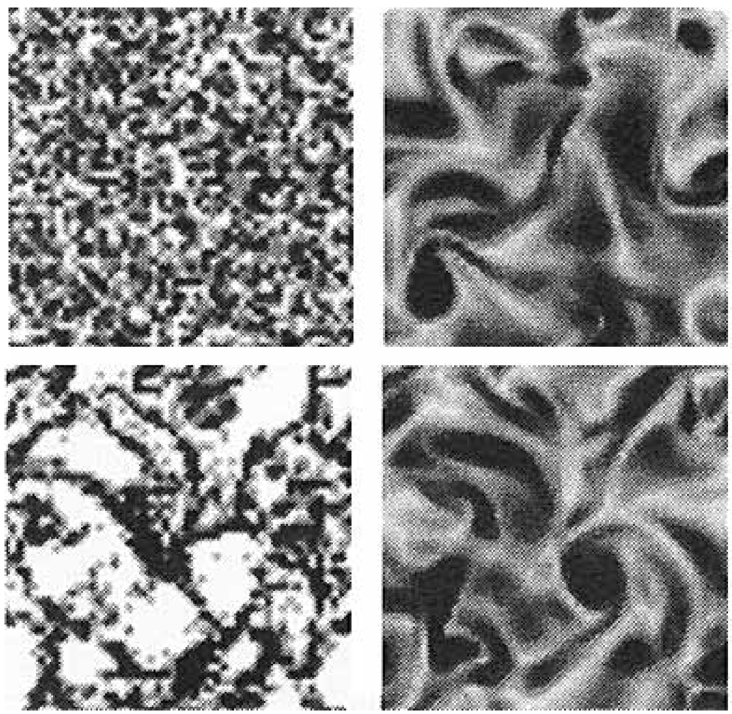
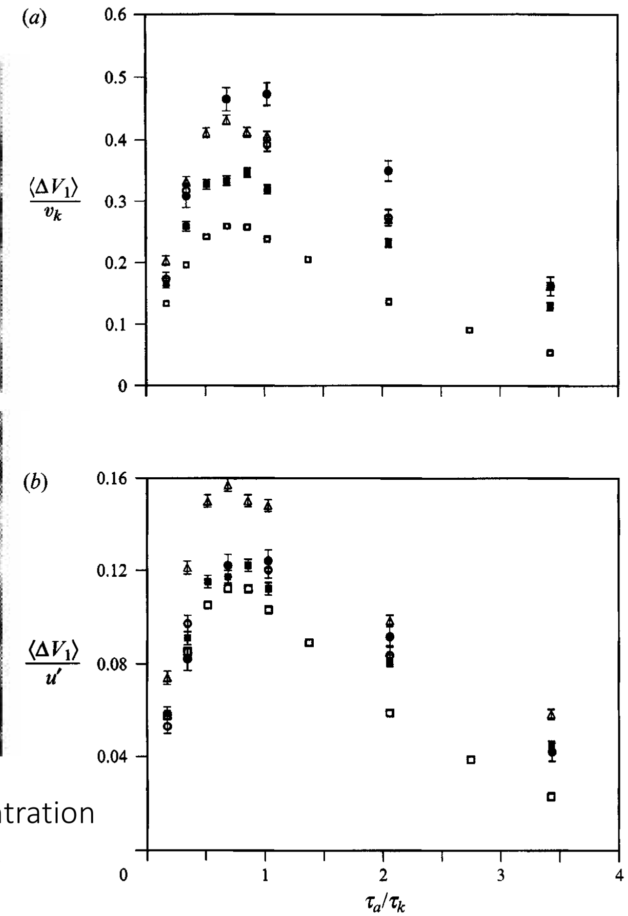


FIG. 6. Time sequence of particle position plots for aerosol ($R = 0$) particles settling under gravity for $W = 0.5$ and $A = 5$: (a) $t = 0$; (b) $t = 10$; (c) $t = 30$; (d) $t = 80$. Particle positions are marked x; arrows on the cell boundaries indicate the circulation in each cell; gravity acts in the positive x_2 direction.

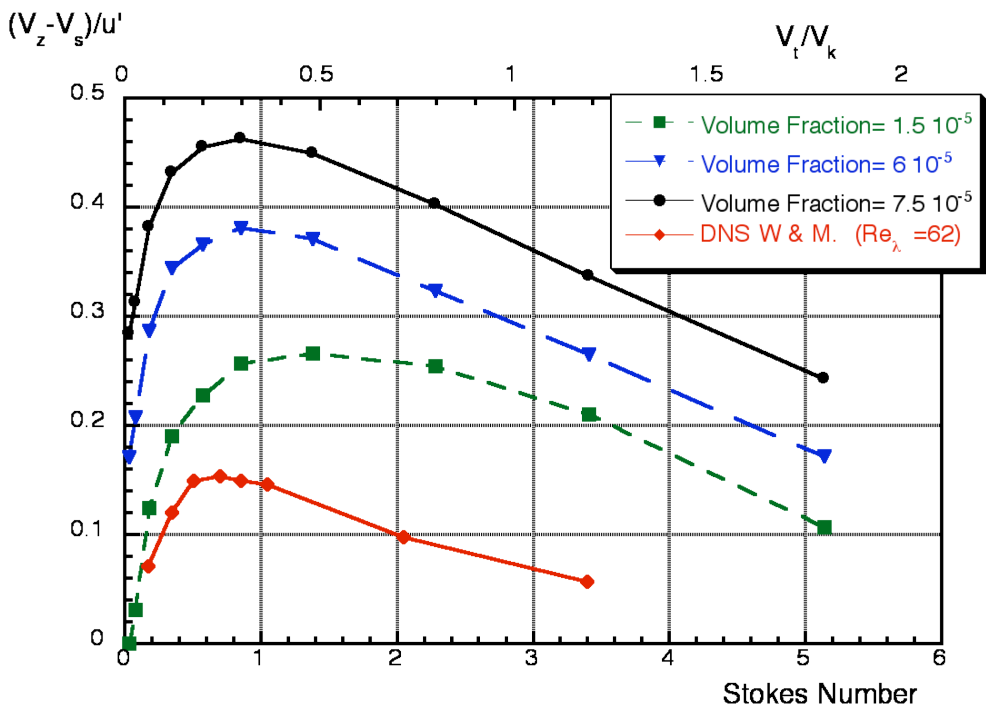
Enhanced settling preferential in Direct Numerical Simulation of Homogeneous Isotropic Turbulence





Wang, L.P. Maxey, M. (1993) Settling velocity and concentration distribution of heavy particles in homogeneous isotropic turbulence. J. Fluid Mech. 256: 27-68

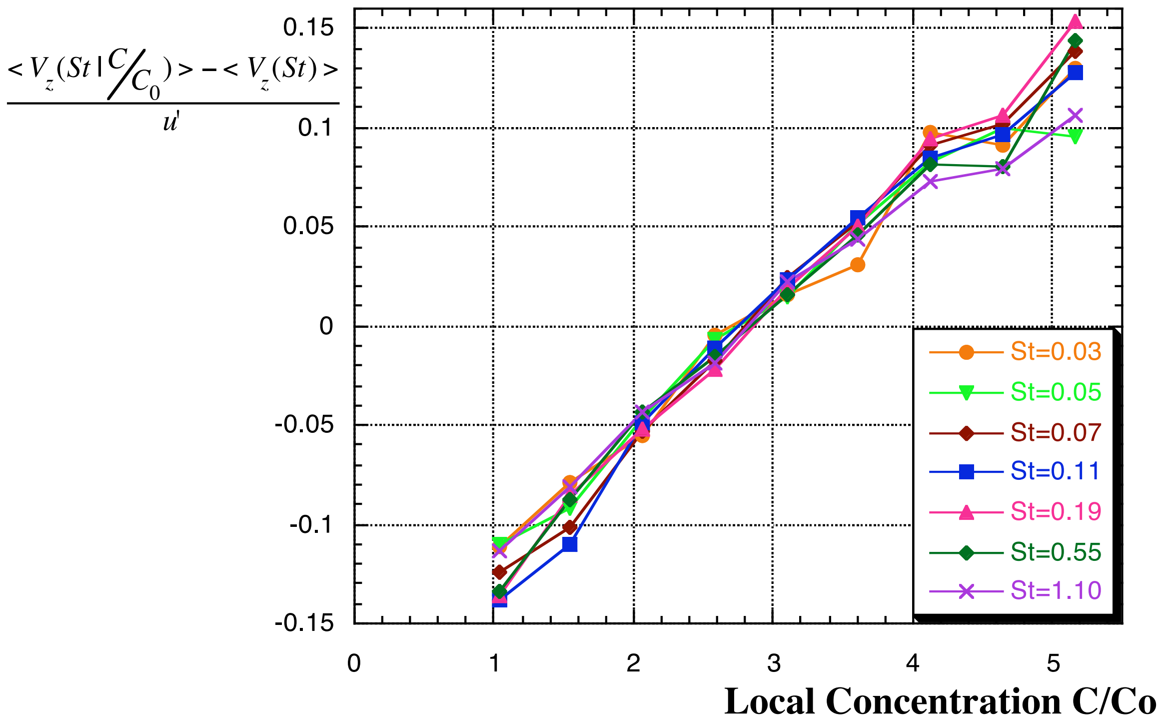
Enhanced Settling Results



Interaction of inertial droplets with turbulence leads to:

- **Increased settling velocity (preferential sweeping and collective behaviour/clustering)**
- Increased local concentration (preferential accumulation)

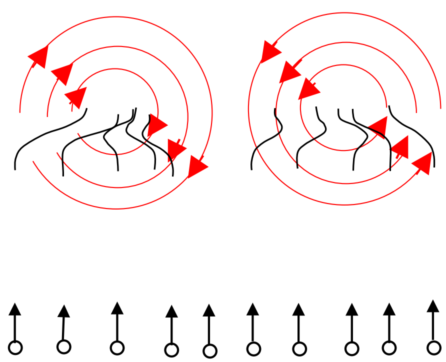
Settling velocity enhanced by clustering



• The settling velocity of a particle inside a cluster can be up to 30% of u' higher than outside a cluster (for a particle of St~1 that represents 200% of V_{St}).

• The settling velocity increases linearly with the local concentration, independent of the St number.

Bubble dynamics



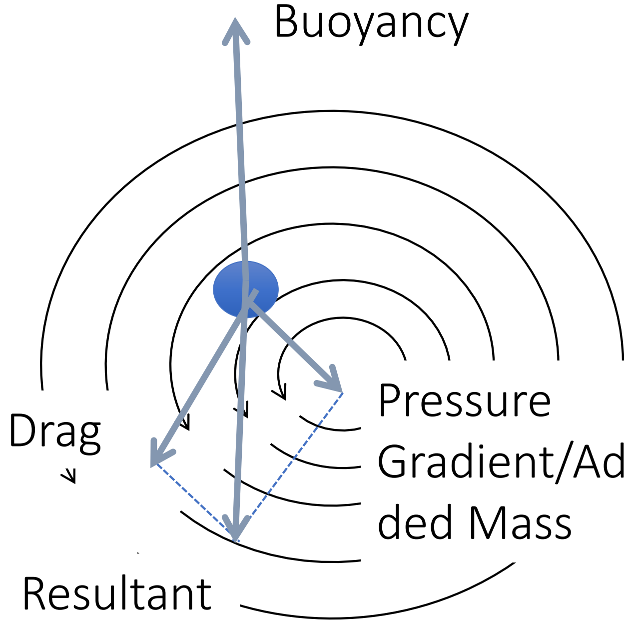
BBO equation for bubbles: $\frac{\rho_p}{\rho_f} << 1$

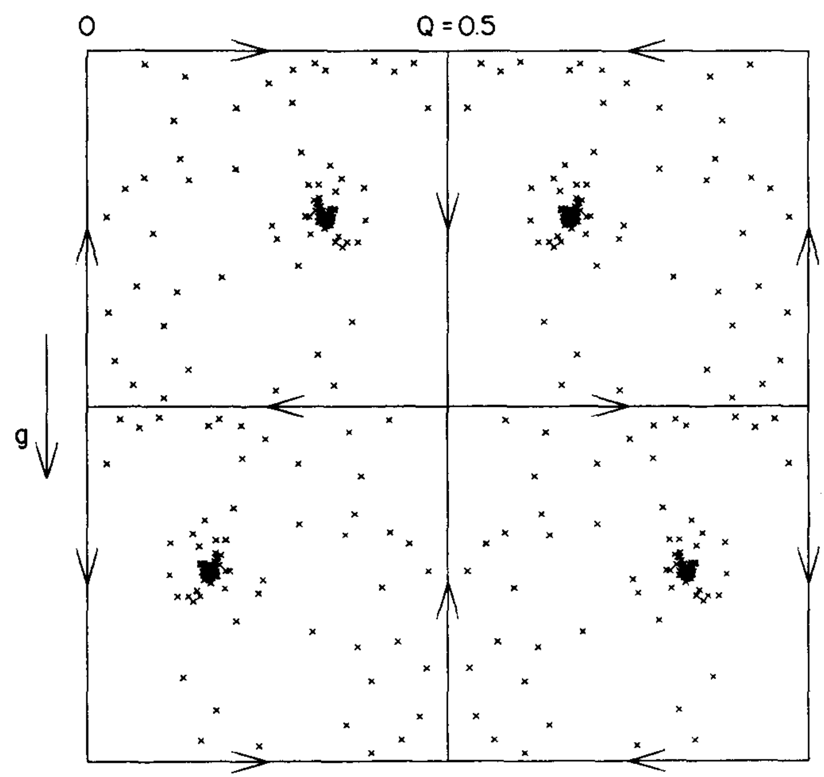
$$\frac{dV}{dt} = \left(u(Y,t) - V(t) + V_{St}\right) \Big/ \tau_B + 3\frac{Du}{Dt}$$

Preferential accumulation and sweeping also occur when bubbles interact with the vorticity field of a turbulent flow.

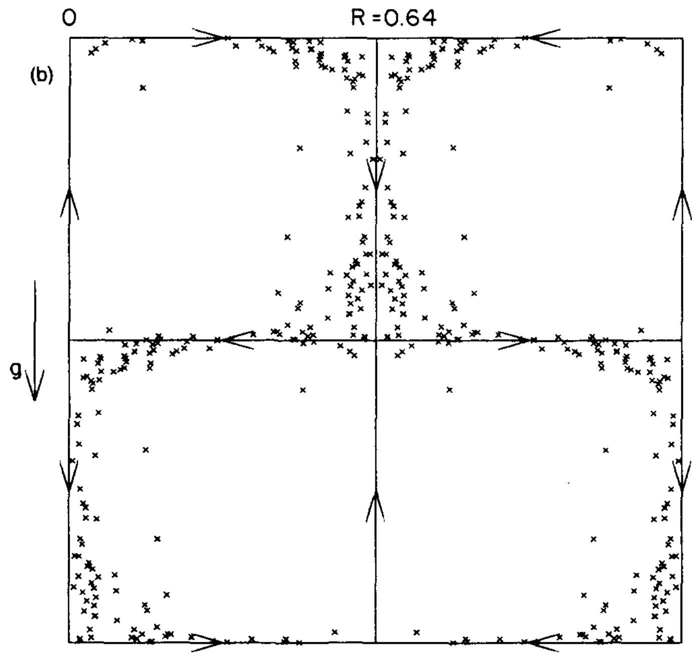
Because bubbles are less dense than the surrounding fluid, they are attracted to the vortex cores, where pressure is minimum.

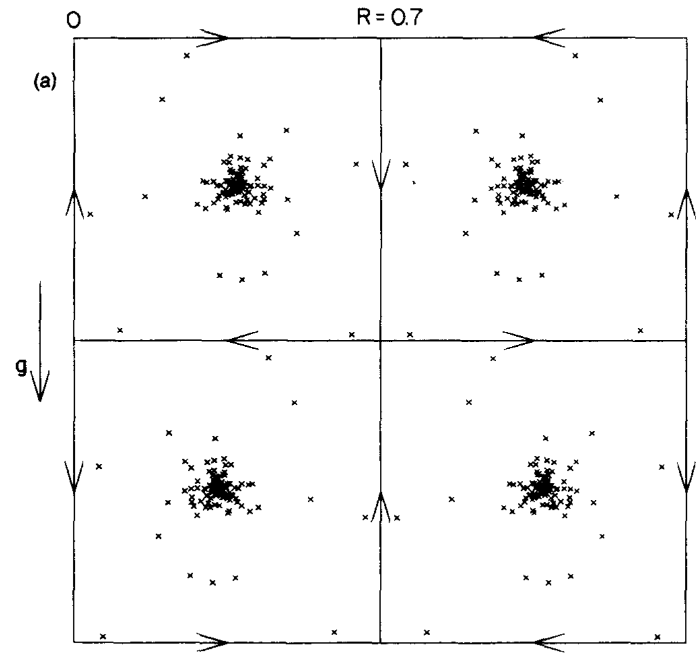
Preferential sweeping for bubbles: stable equilibrium point





Preferential sweeping:
comparison of bubble and particle behaviour



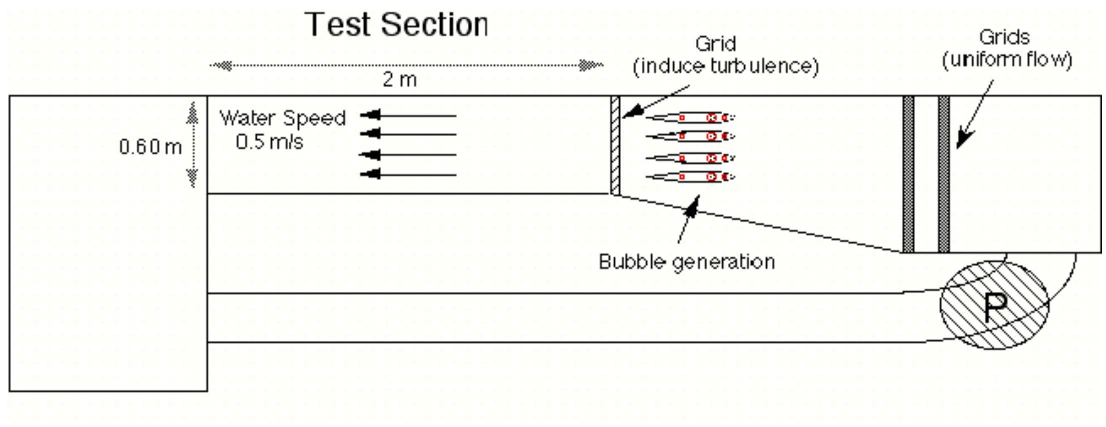


Heavy Particles are centrifuged away from vortex cores and concentrate in regions of high strain)

Bubbles accumulate near their equilibrium point (high vorticity)

[12] M. R. Maxey,, "The motion of small spherical particles in a cellular flow field," Phys. Fluids **30**, 1915 (1987).

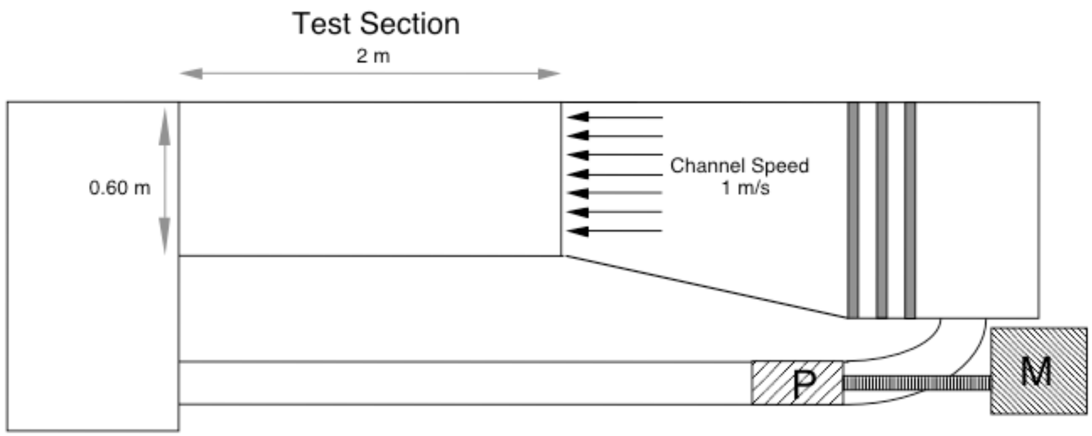
Experimental Setup



Re_{λ} = 200

Bubble Stokes number

St = 0.1 – 3



$Re_{1\,m}$ = 2- 6 10^5

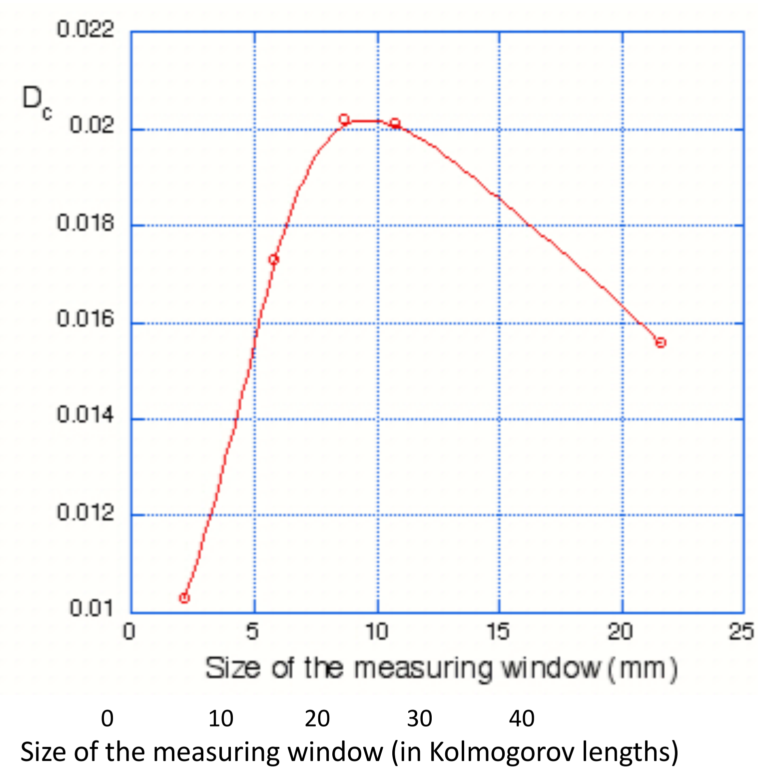
Re_{θ} = 0.7- 2 10^3

Initial Bubble Void fraction

Φ= 1 - 5 %

Preferential accumulation of bubbles

$$D_C = \sum_{c=1}^{N_b} (P(c) - P^{Poisson}(c))^2$$

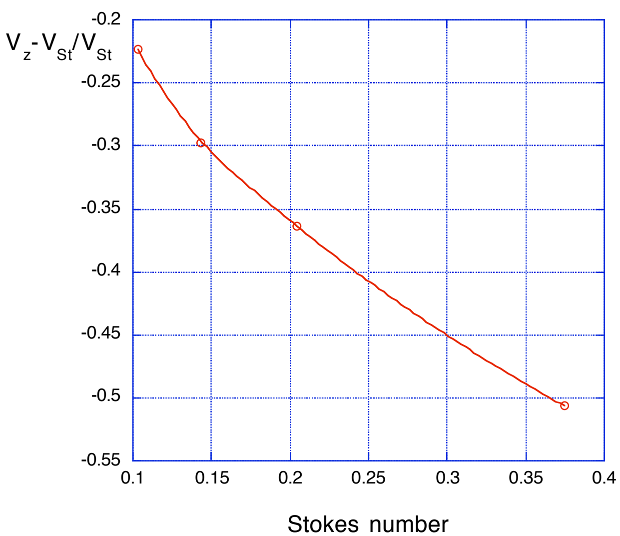


The instantaneous concentration of bubbles was found to be highly non-random.

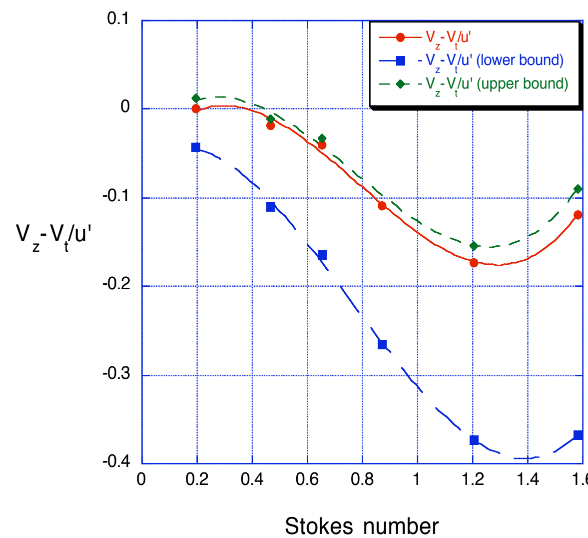
The deviation from randomness was maximum at a length scale equal to 20 times the Kolmogorov length scale of the turbulence.

Rise velocity of the bubbles

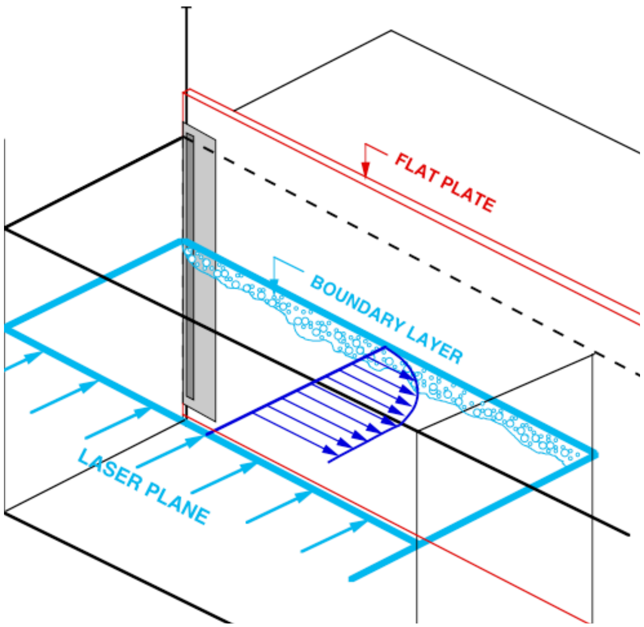
from PDF evolution



from PDPA measurements

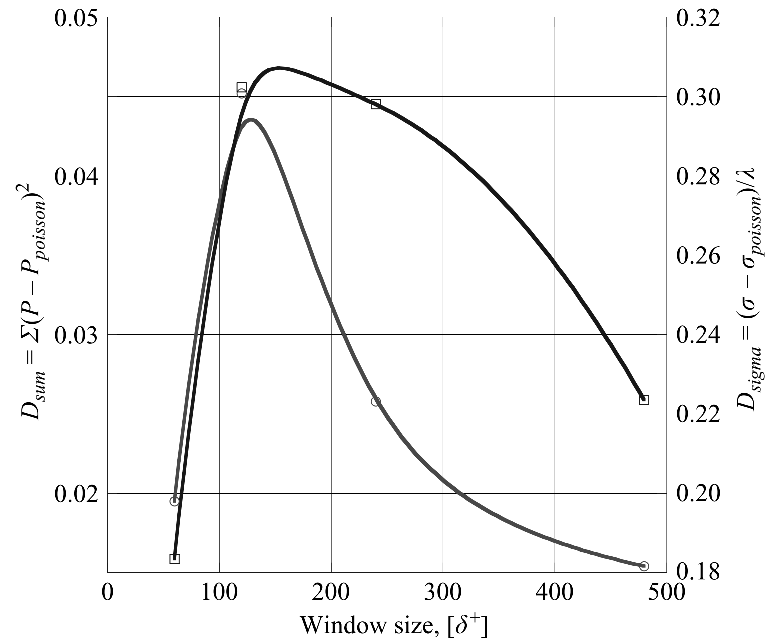


Flow Visualization.
Horizontal Cut.



- Horizontal cuts show large inhomogeneities in the bubble concentration.
- Significant vertical vorticity created by the mean horizontal shear is apparent.

Bubble clustering due to the turbulent structures

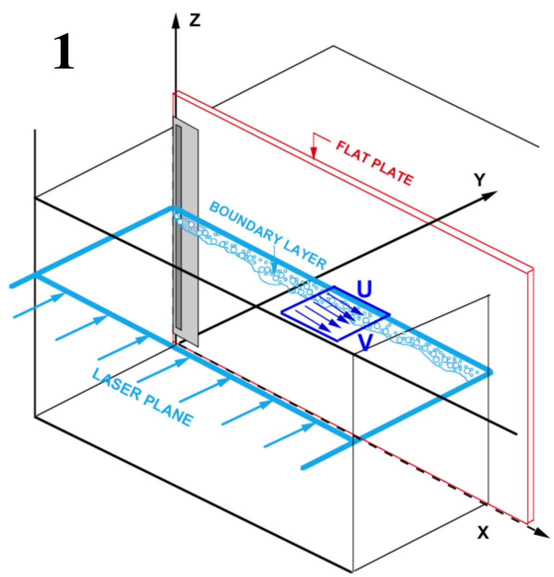


The preferred length scale for accumulation is of the order of 100 wall units ($\delta^+ = \nu / u_\tau$)

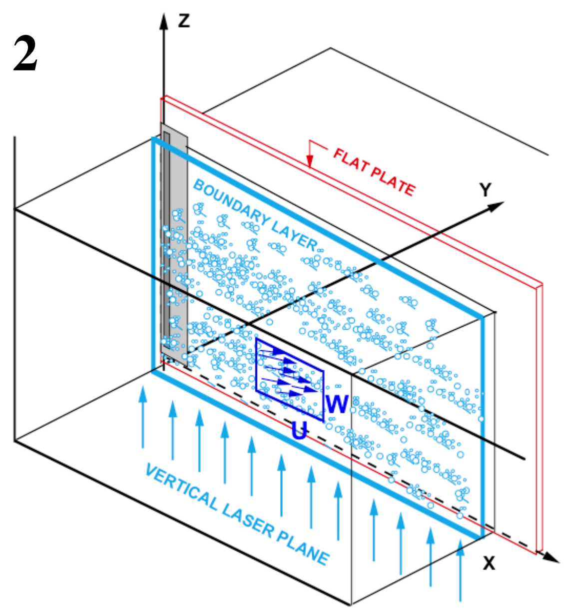
Velocity field.

PIV and Particle tracking measurements.

1



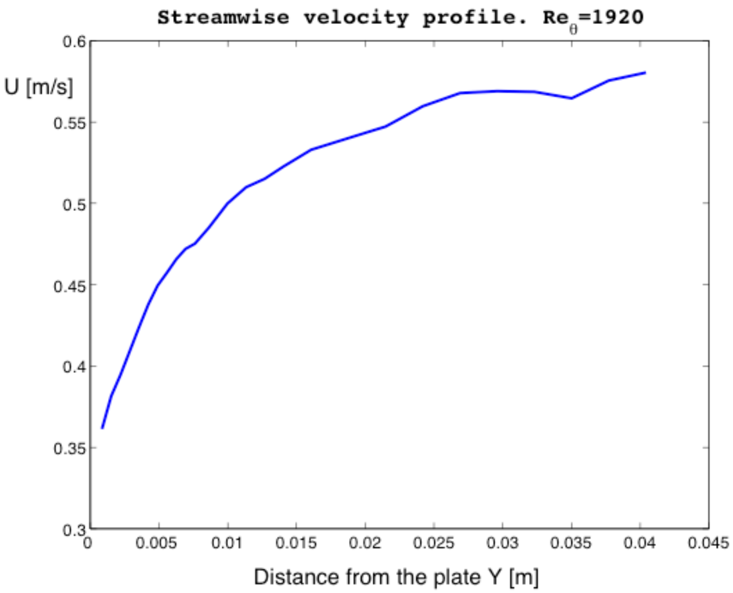
2



Two sets of measurements:

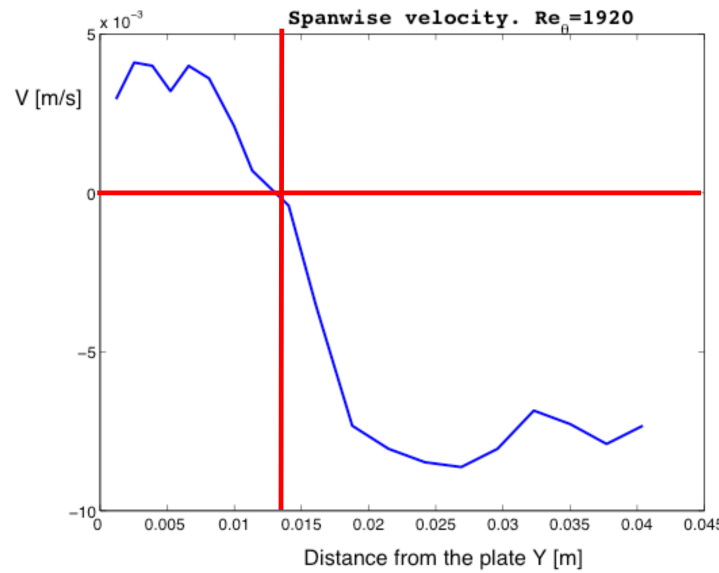
- 1. Streamwise Horizontal laser plane: U, V velocity components.
- 2. Streamwise Vertical laser plane: U, W velocity components.

Streamwise and Spanwise Velocity Profiles
Re = 1920



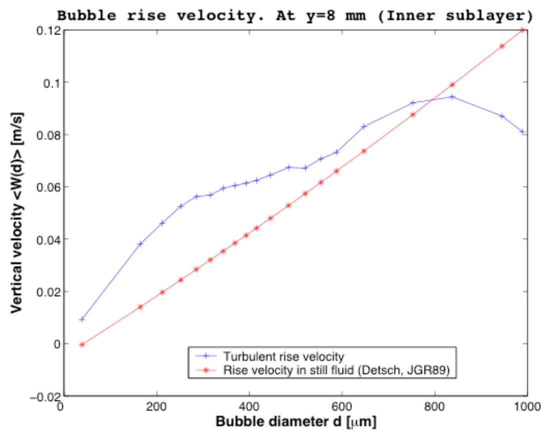
The streamwise velocity profile U is typical of a turbulent boundary layer.

The spanwise velocity profile V is that of a convergent flow.

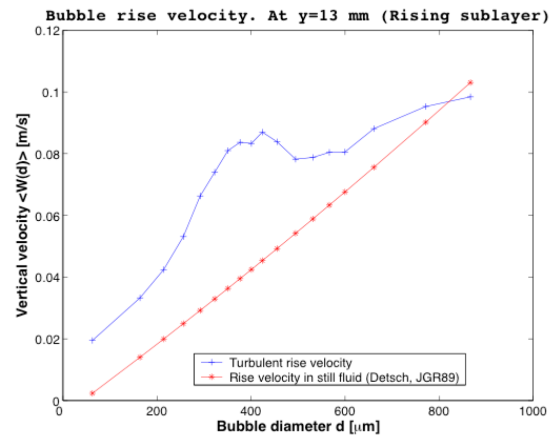


Bubble rise velocity vs diameter

Inner sublayer



Intermediate (rising) sublayer



Outer sublayer