1) (Note: All answers are subjective (i.e., no right or wrong))

a) Child’s bicycle used in rough child's play could have cracked at the joint from high stress or strain due to fracture from either i) brittle fracture quasi static loading or ii) low cycle cyclic fatigue cracking
b) Aluminum bat develops cracks due to high stress or strain low cycle fatigue.
c) Corrosion/environmental cracking exacerbated by low cycle high strain fatigue. Brittle fracture occurred from pre-existing cracks.
d) Corrosion /environmental cracking initiating in a notch due to low cycle high strain fatigue. Brittle fracture from the pre-existing cracks.
e) Time-dependent deformation from creep (temperatures greater than 0.5 of the homologous)

2) (Note: All answers are subjective (i.e., no right or wrong))

Sail boat rudder experiences non constant amplitude cyclic loading during course corrections and tacking. Static loads are mainly due to gravity and/or buoyancy effects. Working loads are imposed during course corrections and tacking. Vibratory loads occur when running at high speeds (before the wind) or while motoring. Accidental loads can arise from hardovers or running aground.

3) (Note: All answers are subjective (i.e., no right or wrong))

Difficult to assess this failure based on the verbal description. However, the bracket no doubt has geometric transitions with attendant stress raisers. Because more than one sailboat has developed such cracks, it would be reasonable to assume that unusual loading conditions or “bad” material are probably not the cause of anomalous failures. Instead, a highly strained notch root under common cyclic loading conditions are probably causing crack initiation in relatively few cycles. These cracks then grow until noticed by the user.

To address this problem, the notch effect could be reduced by changing geometry. The bulk stresses can be reduced by changing dimensions or adding addition reaction points. Choice of material plays a major role as does surface finish, processing, etc.

4) a) This is a plane stress condition where the complete stress state can be represented as:

\[
\begin{align*}
\sigma_x &= 60 \\
\tau_{xy} &= 30 \\
\tau_{xz} &= 0 \\
\tau_{xy} &= 30 \\
\sigma_y &= 0 \\
\tau_{yz} &= 0 \\
\tau_{xz} &= 0 \\
\tau_{yz} &= 0 \\
\sigma_z &= 0
\end{align*}
\]

\[MPa\]
b) Principal normal stresses are determined by solving the eigenvalues matrix or using the “usual” equations for principal stresses.

For example in the x-y plane \( C = \frac{\sigma_x + \sigma_y}{2} = 30 \text{ MPa} \) and \( R = \sqrt{(\sigma_x - C)^2 + \tau_{xy}^2} = 42.4 \) MPa. From these, two of the principal stresses can be determined. The three principal normal stresses are: \( \sigma_1 = C + R = 72.4, \sigma_3 = C - R = -12.4, \sigma_2 = 0 \text{ MPa} \).

c) Maximum shear stresses are the radii of the three Mohr’s circles:
\[
\tau_{12} = (\sigma_1 - \sigma_2)/2 = 36.2; \tau_{23} = (\sigma_2 - \sigma_3)/2 = 6.2; \tau_{13} = (\sigma_1 - \sigma_3)/2 = 42.4 \text{ MPa}
\]

d) AISI 1020 is ductile (\%elongation = 36 is greater than \%elongation = 5 (rule of thumb for ductile materials)). Use Tresca criterion for simplicity
\[
FS = \frac{\tau_{max}}{S_{yp}} = \frac{\sigma_o/2}{(\sigma_1 - \sigma_3)/2} = \frac{\sigma_o \approx S_{yp}}{\sigma_1 - \sigma_3}
\]
Mean \( S_{yp} \) for as-rolled AISI 1020 steel is 260 MPa. Using results of (b) and (c), FS is 3.1. Nominally, this is considered ‘safe’ since FS > 1.

e) Silicon carbide is brittle (\%el = 0.3 is less than \%el = 5 (rule of thumb for ductile materials)). Use Rankine criterion for simplicity
\[
FS = \frac{S_{uts}}{Max(\sigma_1, \sigma_2, \sigma_3)}
\]
Mean \( S_{uts} \) for a SiC (Carborundum SA) for effective volume of 1100 mm\(^3\) is 280 MPa. Using results of (b) FS is 3.8. Nominally, this is considered ‘safe’ since FS > 1.

f) Since this is a static situation, the effects of fatigue cannot be assessed from a cycling standpoint. However, some broad considerations for loading include the amplitude of the cyclic stress, the mean stress component, the spectrum of loading. Considering the geometry, there are notch effects, surface finish, size of component, type of loading, etc. From a materials standpoint, some considerations include the endurance limit, the various cyclic fatigue exponents and coefficients and the crack propagation parameters, etc.

5) \( R = \frac{\sigma_{min}}{\sigma_{max}} = \frac{1-A}{1+A} \) is the stress ratio and describes the relation of the minimum to the maximum stress. It can be used to describe the loading ratio as well. Examples of R ratios that describe unique cyclic loading conditions are \( R = -1 \) for completely reversed (sometimes completely reversed rotating bending), \( R = 0 \) for zero-tension cycling, \( R = \infty \) for zero-compression cycling.

\[
R = \frac{1-A}{1+A} = \frac{\frac{1}{\sigma_{max} + \sigma_{min}}}{\frac{1}{\sigma_{max} - \sigma_{min}}} = \frac{\sigma_{max} + \sigma_{min} - \sigma_{max} + \sigma_{min}}{\sigma_{max} + \sigma_{min} - \sigma_{min} + \sigma_{max}} = \frac{2\sigma_{min}}{\sigma_{max}} = \frac{2\sigma_{min}}{2\sigma_{max}} = \frac{\sigma_{min}}{\sigma_{max}}
\]
\[ A = \frac{\sigma_{u}}{\sigma_m} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{\sigma_{\text{max}} + \sigma_{\text{min}}} = \frac{1 - R}{1 + R} \]
is the amplitude ratio and describes the relation of the amplitude to the mean stress. It can be used to describe excursion of the cyclic stress about the mean stress. Examples of A ratios that describe unique cyclic loading conditions are \( A = \infty \) for completely reversed (sometimes completely reversed rotating bending) where \( \sigma_{\text{m}} = 0 \) and \( A = 0 \) for no cycling where \( \sigma_{\text{a}} = 0 \)

\[ A = \frac{1 - R}{1 + R} = \frac{1 - \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}}{1 + \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{\sigma_{\text{max}} + \sigma_{\text{min}}} \]

\[ = \frac{2\sigma_{\text{max}} - \sigma_{\text{min}}}{2\sigma_{\text{max}} + \frac{\sigma_{\text{min}}}{2}} = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{\sigma_{\text{m}}} \]

\[ \Delta \sigma = 2\sigma_{\text{a}} = \sigma_{\text{max}} (1 - R) \]
is the stress range and is simply twice the stress amplitude (where the amplitude is the cyclic stress excursion from the mean stress). The stress amplitude is used to describe the cyclic stress state in all three fatigue analysis methods (stress life, strain life and crack propagation).

\[ \Delta \sigma = \sigma_{\text{max}} (1 - R) = \sigma_{\text{max}} \left(1 - \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \right) = \sigma_{\text{max}} - \frac{\sigma_{\text{max}} \sigma_{\text{min}}}{\sigma_{\text{max}}} = \sigma_{\text{max}} - \sigma_{\text{m}} = 2\left(\frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2}\right) = 2\sigma_{\text{a}} \]

\[ \sigma_{\text{m}} = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} = \frac{\sigma_{\text{max}}}{2} (1 + R) \]
is the mean stress is simply “average” stress applied to the component. The mean stress is used to describe the non-cyclic aspect of the stress state and it can have a pronounced effect on the cyclic fatigue behaviour of materials (see for example, mean stress effects in stress-life, strain-life and crack propagation analysis methods).

\[ \sigma_{\text{m}} = \frac{\sigma_{\text{max}}}{2} (1 + R) = \frac{\sigma_{\text{max}}}{2} \left(1 + \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} \right) = \frac{\sigma_{\text{max}}}{2} + \frac{\sigma_{\text{max}} \sigma_{\text{min}}}{2\sigma_{\text{max}}} = \frac{\sigma_{\text{max}}}{2} + \frac{\sigma_{\text{min}}}{2} = \frac{\sigma_{\text{max}} + \sigma_{\text{min}}}{2} = \sigma_{\text{m}} \]