2.1) It is assumed that the strains is uniform throughout, therefore the relations between true stress/strain and engineering stress/strain are applied such that: \( s = s(1+e) \) and \( e = \ln(1+e) \).

For \( s = 62.2 \) ksi and \( e = 0.0098 \) in/in respectively, \( s = 62.8 \) ksi and \( e = 0.0098 \).

For \( s = 90.8 \) ksi and \( e = 0.0898 \) in/in respectively, \( s = 98.95 \) ksi and \( e = 0.0860 \).

At the lower stress/strain values, presumably in the elastic region, the true stress/strain and engineering stress/strain are nearly identical, but as the stress/strain increases into the plastic region, the true stress become greater than the engineering stress and the true strain becomes less than the engineering strain.

2.2) For a 50 mm initial diameter and a 250 mm initial gage length. Structural steel at room temperature.

<table>
<thead>
<tr>
<th>Force (MN)</th>
<th>Deflection (mm)</th>
<th>Comment</th>
<th>Eng Stress (MPa)</th>
<th>Eng Strain (m/m)</th>
<th>Tru Stress (MPa)</th>
<th>Tru Strain (m/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>Start</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.33</td>
<td>0.2</td>
<td>Elastic</td>
<td>168</td>
<td>0.0008</td>
<td>168</td>
<td>0.0008</td>
</tr>
<tr>
<td>0.37</td>
<td>0.25</td>
<td>Drop</td>
<td>188</td>
<td>0.001</td>
<td>189</td>
<td>0.00099</td>
</tr>
<tr>
<td>0.33</td>
<td>1</td>
<td>Constant</td>
<td>168</td>
<td>0.004</td>
<td>169</td>
<td>0.0039</td>
</tr>
<tr>
<td>0.53</td>
<td>50</td>
<td>Ultimate</td>
<td>270</td>
<td>0.2</td>
<td>324</td>
<td>0.182</td>
</tr>
<tr>
<td>0.30</td>
<td>64</td>
<td>Fracture</td>
<td>df=30 mm</td>
<td>153</td>
<td>0.256</td>
<td>424</td>
</tr>
</tbody>
</table>

Elastic modulus, \( E = \frac{\sigma}{\epsilon} = \frac{\sigma_{\text{elastic}}}{\epsilon_{\text{elastic}}} = 210.084 \) GPa

Upper yield, \( S_{\text{upper}} = \frac{P_{\text{upper}}}{A_0} = 188.4 \) MPa

Lower yield, \( S_{\text{lower}} = \frac{P_{\text{lower}}}{A_0} = 168.1 \) MPa

0.2% yield, \( S_{\text{yp}} = \frac{P_{\text{lower}}}{A_0} = 168.1 \) MPa

Ultimate tensile strength, \( S_{\text{uts}} = \frac{P_{\text{max}}}{A_0} = 269.9 \) MPa

Engineering stress and strain at necking, \( S_{\text{uts}} = 269.9 \) MPa, \( \epsilon_u = 0.20 \) m/m
True stress and strain at necking, \( s_u = 323.9 \text{ MPa}, e_u = 0.18 \text{ m/m} \)

Strain hardening exponent, \( n = e_u = 0.18 \)

Percentage Reduction in Area, \( \% \text{ RA} = 100 \times \frac{d_0^2 - d_t^2}{d_0^2} = 64 \)

True fracture ductility, \( e_t = \ln \frac{L_f}{L_0} = 0.23 \text{ m/m} \)

or, \( e_t = \ln \frac{A_f}{A_0} = \ln \frac{d_0^2}{d_t^2} = 1.02 \text{ m/m} \)

True fracture strength, \( s_f = 424.4 \text{ MPa} \)

Strength coefficient, \( K = \frac{s_u}{e_u^0} = 441.8 \text{ MPa} \)

2.11) For the material, the values of \( E = 30,000 \text{ ksi}, n' = 0.202 \) and \( K' = 74.6 \text{ ksi} \) are given. The cyclic stress-strain curve is determined from 0 to 0.02 in/in strain using the relation, \( e = \frac{s}{E} + \left( \frac{s}{K'} \right)^{1/n'} \) by substituting the material properties into the equation for sequential values of stress, \( s \), until data sets of \( s \) and \( e \) have been determined from 0 to 0.02 in/in strain. Using a similar approach, the hysteresis curve for the stress and strain amplitude is determined from 0 to 0.04 in/in strain amplitude using the relation, \( ?e = \frac{?s}{E} + 2 \left( \frac{?s}{2K'} \right)^{1/n'} \) by substituting the material properties into the equation for sequential values of stress amplitude, \( ?s \). Until data sets of \( ?s \) and \( ?e \) have been determined from 0 to 0.04 in/in strain amplitude.

These results are plotted on the same scale as shown overleaf.
To develop the complete stabilized hysteresis stress-strain curve in terms of stress and strain (as opposed to stress and strain amplitudes), all points on the $s$ vs $\varepsilon$ curve are shifted by the half the maximum stress amplitude and half the maximum strain amplitude such that:

$$s = s_{\text{max}} - 0.5s_{\text{max}}$$

and

$$\varepsilon = e_{\text{max}} - 0.5e_{\text{max}}.$$ 

The top part of the hysteresis loop is formed this way by plotting the shifted $s - \varepsilon$ values. The bottom part of the hysteresis loop is formed by negating the $s - \varepsilon$ values determined from the top part of the loop as shown in the figure above.

- 3 -
Note that as the strain amplitude increases, the hysteresis loops become broader and more dominated by the nonlinear behavior. In particular, at a strain amplitude of 0.001 in/in, the hysteresis loop is primarily linear.

The broadest and most nonlinear is for a strain amplitude of 0.02. Finally, the cyclic stress-strain curve is plotted on top of the hysteresis loops and as expected it passes through the tips of the hysteresis loops.

2.16) a) Log-log plot of elastic, plastic, and total strain amplitudes as functions of the number of reversals is shown in the figure.

b) From the plot, it appears that \( 2N_t \) is about 100. From the relation,

\[
2N_t = \left( \frac{E\varepsilon_t}{s_t} \right)^{1/(b-c)},
\]

\( 2N_t \) is calculated as 100.

c) The total strain amplitude at \( 2N_t \) is determined from the Strain Life Relationship at \( 2N = 100 \), such that

\[
\frac{?}{2} = \frac{s}{E}(2N_t)^b + \varepsilon_t(2N_t)^c = 0.0127 \text{ in/in}.
\]

d) The strength coefficient can be estimated from the relation

\[
K' = \frac{s}{\varepsilon_t} = 440 \text{ ksi}
\]

using the strain hardening exponent from the relation, \( n = b/c = 0.17 \).

e) 100 reversals is rather low although it is also equal to the transition life. Because few cycles is usually associated with plastic strain dominated behavior, one way to increase the transition life and hence the total fatigue life is to increase the material’s ability to absorb plastic strains. As shown in Figures 2.19 and 2.20, decreasing the hardness, and hence increasing the ductility (i.e.,
increasing $e$ gives better fatigue resistance. By annealing this steel, a normalized condition could be achieved, hence increasing ductility.

f) At higher fatigue lives, elastic strains usually dominate. In this case, increasing the yield strength by heat treating to a quenched and tempered condition (see Figures 2.19 and 2.20) can increase fatigue resistance.

2.24) The data are manipulated to give the total, elastic and plastic strain components.

<table>
<thead>
<tr>
<th>$\frac{\Delta \varepsilon}{2}$</th>
<th>$\frac{\Delta \varepsilon}{2} = \frac{\Delta \sigma}{2E}$</th>
<th>$\frac{\Delta \varepsilon_p}{2} = \frac{\Delta \varepsilon - \Delta \varepsilon_e}{2}$</th>
<th>$2N_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00202</td>
<td>0.00131</td>
<td>0.00072</td>
<td>416714</td>
</tr>
<tr>
<td>0.0051</td>
<td>0.00186</td>
<td>0.00324</td>
<td>15894</td>
</tr>
<tr>
<td>0.0102</td>
<td>0.00214</td>
<td>0.00806</td>
<td>2671</td>
</tr>
<tr>
<td>0.0151</td>
<td>0.00222</td>
<td>0.01288</td>
<td>989</td>
</tr>
</tbody>
</table>

From power law fits of the elastic and plastic strains as functions of the reversals, the various coefficients can be determined from the relation,

$$\frac{\Delta \varepsilon}{2} = \frac{s_i^f}{E} (2N_f)^b + e_i^f (2N_f)^c$$

For example, for the elastic strain amplitude,

$$\frac{\Delta \varepsilon}{2} = \frac{s_i^f}{E} (2N_f)^b = 0.00429 (2N_f)^{-0.09038}$$

and for the plastic strain amplitude,

$$\frac{\Delta \varepsilon_p}{2} = e_i^f (2N_f)^c = 0.3491 (2N_f)^{-0.4796}$$

By inspection, $b = -0.09038$, $c = -0.4796$, $e_i^f = 0.3491$, and $s_i/E = 0.00429$ (where $s_i^f = 858$ MPa). From the relations,

$$K' = \frac{s_i^f}{e_i^f} = 1046 \text{ MPa} \text{ and } n' = b/c = 0.188.$$ 

a) $K' = \frac{s_i^f}{e_i^f} = 1046 \text{ MPa} \text{ and } n' = b/c = 0.188.$

b) $b = -0.09038$, $c = -0.4796$, $e_i^f = 0.3491$, $s_i^f = 858$ MPa

c) $2N_f = \left( \frac{Ee_i^f}{s_i^f} \right)^{1/(b-c)} = 81,006$ reversals

d) $0.0075 = \frac{\Delta \varepsilon}{2} = \frac{s_i^f}{E} (2N_f)^b + e_i^f (2N_f)^c$ such that $2N_f = 5660$ reversals.

2.34) The material has properties of $E = 10.6 \times 10^3$ ksi, $K' = 95$ ksi, $n' = 0.065$, $b = -0.124$, $c = -0.59$, $e_i^f = 0.22$, $s_i^f = 160$ ksi. The strains can be determined from the
stresses such that $e = \frac{s}{E} + \left(\frac{s}{K'}\right)^{1/n'}$ and $\frac{\Delta e}{2} = \frac{s}{E} + 2\left(\frac{s}{2K'}\right)^{1/n'}$. The relations used for non-zero mean stress include:

Morrow: $\frac{\Delta e}{2} = \frac{s_f - s_o}{E} (2N_f)^b + e_i (2N_f)^c$

Manson Halford: $\frac{\Delta e}{2} = \frac{s_f - s_o}{E} (2N_f)^b + e_i \left(\frac{s_f - s_o}{s_f}\right)^{c/b} (2N_f)^c$

Smith, Watson, and Topper: $s_{max}^2 = \frac{s_f - s_o}{E} (2N_f)^{2b} + s_f e_i (2N_f)^{(b+c)}$

Strain Life Results are plotted for A and B as shown using the properties given for the material and the values shown in the tables.

For Level A (compressive mean stress)

For Level B (tensile mean stress)
Results are tabulated as shown. Note that predictions appear to be greater than experiments by about one order of magnitude.

<table>
<thead>
<tr>
<th>Level</th>
<th>Max/Min Stress (ksi)</th>
<th>Range Mean Stress (ksi)</th>
<th>Test Results (2Nf)</th>
<th>Morrow (2Nf)</th>
<th>M-H (2Nf)</th>
<th>SWT (2Nf)</th>
</tr>
</thead>
</table>
| A     | $\sigma_{\text{max}} = 213$  
       | $\sigma_{\text{min}} = -30.1$  
       | $\Delta \sigma = 51.4$  
       | $\sigma_{\text{a}} = \sigma_{\text{0}} = -4.4$  
       | 5.4$\times 10^5$  
       | 5.5$\times 10^5$  
       | 7.2$\times 10^5$  
       | 3.5$\times 10^6$  
       | 3.5$\times 10^6$  
       | 5.7$\times 10^6$  
| B     | $\sigma_{\text{max}} = 615$  
       | $\sigma_{\text{min}} = 101$  
       | $\Delta \sigma = 51.1$  
       | $\sigma_{\text{a}} = \sigma_{\text{0}} = 35.6$  
       | 5.6$\times 10^4$  
       | 6.4$\times 10^4$  
       | 6.8$\times 10^4$  
       | 4.8$\times 10^5$  
       | 3.9$\times 10^5$  
       | 1.0$\times 10^5$  