Review of mathematical and experimental models for determination of service life of gears

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Abstract

For designing machines and devices the dimensioning with respect to service life is increasingly taken into account. This applies also for gearing which are still today one of very important components of almost all machines.

We have developed a stochastic model for determination of service life of gears. In our model we propose a new parameter by which we describe the fracture mechanics conditions in the tooth root where the defects, causing destruction, occur statistically most frequently as shown. We named that factor the tooth stress intensity factor $Z$. The value of the factor $Z$ is related to dislocation, propagation of the plastic zone, deformation and orientation of grain in case of short cracks and stress intensity factor $K$ in case of long cracks.

For determination of the service life for the area of short cracks we used Bilby, Cottrell and Swinden model which is based on the theory of continuously distributed dislocations and we complemented it with random generation of structure of material before cracks. For the long crack we have developed a stochastic model for determination of service life of gears.

For confirm mathematical models we developed different non-standard test pieces and on this pieces we used combination of mixed experimental methods. The aim of these combinations was to obtain as complete information about the individual influences as possible and to determine the interaction between different fracture mechanic magnitudes. In this way we confirmed the mathematical models as a whole and also determined some physical interpretations in models. With this we were able to ensure that the presented model is not purely a mathematical model.

Keywords: Short cracks; Stress intensity factor; Fracture mechanics

1. Introduction

Various standards such as American Gear Manufacturers Association (AGMA) standards or German DIN standards already indicate the possibility of dimensioning the so-called time gearing. For example, AGMA 2001 [1] for calculating the service life uses the Miner’s Rule. By means of Miner’s rule and coefficients of service life it is possible to determine the number of loading cycles:

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However, all these standard models are rough and give not enough accurate results since they do not take into account the actual conditions. Therefore we decided to develop models and procedures of calculations, that will give more reliable and, in particular, more accurate results. We limited ourselves to the tooth root, i.e., the defects occurring there, since the fracture causes instantaneous gear failure, which can have catastrophic consequences. Pressures at the contact between two gears during engagement cause the defect called pitting, which is a long and process with not so serious consequences.

For accurate determination of the service life we must take into account the loading, which are in most cases random loading of variable amplitude, the geometry of gears and properties of materials, from which the gears are made and which are known not to be constants. The more precise modelling of these input parameters the more precise and reliable the results.

In practice the following two basic problems arise when calculating the service life:

(a) During designing, the individual elements and the entire products are optimized particularly with respect to the service life. The basic requirement is that the service lives of the individual elements are approximately equalised. In this case the model of occurrence of defects and the model of crack propagation are important [2].
(b) A defect, i.e., a crack is detected during periodic inspection by non-destructive method. If the component concerned is not on stock or a considerable period of time would be necessary to manufacture it we are interested to know how long the damaged component will operate with full rated loading and/or what the loading for the desired service life is, i.e., for example the time necessary for the manufacture of a new component-gear.

We will limit ourselves to the first problem, i.e., prediction of the service live of the gearing during designing. Geometric particularities are taken into account by means of the shape factor and the tooth stress intensity factor \([3]\) and the real loading by means of mathematics modelling of actual gearing.

2. Tooth stress intensity factor

In our model we propose a new parameter by which we describe the fracture mechanics conditions in the tooth root where the defects, causing destruction, occur statistically most frequently as shown Fig. 1 [3].

We must emphasise that this factor does not apply in general, but only for gears and for calculation of conditions in the tooth root, i.e., for calculation of their service life. We named that factor the tooth stress intensity factor \(Z\). The value of the factor \(Z\) related to propagation of the plastic zone, deformation and orientation of grain in case of short cracks and stress intensity factors in case of long cracks.

There are three forms of the factors \(Z\) for three different area of crack length according to Fig. 2:

1. If \(a = 0\), i.e., if there is no crack, the factor \(Z\) is proportional to the stress concentration factor [4] occurring in the tooth root:
   \[Z \propto K_t \sigma\]

2. If \(0 < a < \sim 10\) grains (according to Taylor and Knott [5]), the factor \(Z\) depends on local plasticizing of grain, grain size, shear module, grain orientation, shape factor:
   \[Z = f(e, d, a, d - a, G, Y, \ldots)\]

3. If \(a > 10\) grains, such defect can be considered to be a long crack or a linear elastic fracture mechanics type crack and the following is obtained:
   \[Z \sim f(K_{IC}, K_{th}, K, Y(a/S), \sigma_f, \ldots)\]

Fig. 1. Critical cross section.
3. Multiaxial loading of variable amplitude

Since on the gear drives, which are most frequently used in crane and machine building, the loading on gearing are not known and are unstationary during operation, these problems cannot be approached deterministically. In the calculation in our model we take into account the actual loading so that the gear tooth is loaded at various points of engagement by currently corresponding force $F$. We can accurately take into account these loadings in the calculation only by mean of the mathematical model of a real gearing in the form of equation of motion, which in general represent a non-conservative non-linear system of differential equation of the second order. For a spur gear pair the replacement mechanical model is shown in Fig. 3.

For a gearing consisting of such gears we can write the system of equations of motion in the matrix form [6]:

$$ I\ddot{\Theta} + D\dot{\Theta} + K(\Theta + q) + F(t) = 0 $$

where:

$I = \text{diag}[I_1, I_2, \ldots, I_n]$ matrix of inertia moments
$D = \text{diag}[D_1, D_2, \ldots, D_n]$ damping matrix
$\Theta = (\Theta_1, \Theta_2, \ldots, \Theta_n)$ vector of angular turns
$K = \text{diag}[K_1, K_2, \ldots, K_n]$ rigidity matrix
$q$ vector of clearance between tooth flanks during engagement
$F(t)$ vector of external loadings

In general, depending on the number of gears we obtain the number of differential equation of second order, which, in our algorithm is solved by the conventional method of Runge Kutta. The results of calculation are angular turnings, which indicate the point of engagement on the tooth and loading occurring in this point.

4. Model of crack initiation

To be able to determine the properties of very short cracks, it is important in the analytical models describing the $S$–$N$ curves to consider the dominant properties of the fields of the crack tip as well as the crack growth rate, which is high in the beginning but then decreases [2]. This can be reached by considering the separate regimes in the Kitagawa–Takahashi diagram, which gives a significant advance in the understanding of short crack behaviour (Fig. 3). This diagram shows the effect of defect size on the fatigue limit stress; see, for example, Fig. 2. For large defects, the allowable stress for infinite life must be low, within linear elastic fracture mechanics regimes, and, therefore, the limiting condition is given by a straight line of slope minus one half of the threshold stress intensity factor $\Delta K_{th}$, such that

$$ \Delta K_{th} = Y(a/S)\Delta\sigma\sqrt{\pi a_{th}} $$

where $Y(a/S)$ is shape factor [3], $\Delta\sigma$ is stress ratio and $a_{th}$ is a threshold crack length. At the other end of the spectrum, for vanishing small defects, the allowable stress level must relate to the uncracked specimen fatigue limit. The Kitagawa–Takahashi diagram shows the observed behaviour between these two extremes.

It is known that the results are more accurate, if the number of data is great. By means of the generator of random numbers the number of input data can be numerically increased and on the basis of that new
4.1. Model for evaluation of lifetime for short fatigue crack initiation

The linear elastic fracture mechanics is inapplicable for describing properties of very short cracks, since elastic stress cannot accurately describe high deformation fields near the tip of small defects in high-strength materials. Therefore, we used the theory of continuously distributed dislocation of analysis of crystallographic slip deformations before the tip of short fatigue crack in combination with crystal limits and on the basis of it we formed an appropriate model [2,6]. The stress connected with dislocations can result in the slip of the inclusion or in generation of vacation. Dislocations can unite on any obstacle of the slip that causes their concentration in space. This results in nucleation of micro cracks and/or crack growth with uniting of remaining dislocations. The assumptions described above were used in our model for initiation and propagation of micro cracks.

The border between the area where the slip occurred and area where the slip did not occur is called the edge dislocation. The presence of the edge dislocation increases the free energy of crystal. In order to calculate, let us consider a cylindrical crystal of length \( l \) with spiral dislocation on which the Burger’s vector is located along its axis. The Burger’s vector \( b \) describes the size of shift and is the entire distance between...
atoms in case of crystal grating. The energy on the volume unit of this cylindrical crystal of radius \( r \) and thickness \( d \):

\[
\frac{dE}{dV} = \frac{1}{2} G r_0^2
\]

where

\[
\gamma_0 = \frac{b}{2\pi r}
\]

is the elastic stress in thin coil.

From Eq. (5) it is possible to calculate the deformation energy by integration:

\[
E = \int_{r_0}^{R} \frac{1}{2} G r_0^2 dV
\]

where \( R \) and \( r_0 \) are the upper and lower value of the variable \( r \). The \( R \) cannot exceed the dimension of crystal. The Eq. (7) show that the energy is proportional to the length of dislocation. Such a deformed dislocation has linear share stress \( T \) that is a vector and is calculated as follows:

\[
T = \frac{\delta E}{\delta l}
\]

And for equilibrium the integral equation is obtained [6]:

\[
\int_{D} \frac{f(x)}{x-x_0} \, dx = \frac{T(x_0)}{A}
\]

where \( A = G b / 2 \pi (1 - \nu) \) applies to edge dislocation and \( A = G b / 2 \pi \) for spiral dislocation. \( G \) is the shear modules, \( b \) is the Burger’s vector and \( T(x_0) \) is the resulting shear stress.

Muskhelishvili showed the solution of this integral equation aheadly in 1946 [6]. Thus the general notation of the distribution function in a slightly simplified compact form can be written:

\[
f(\xi) = \frac{\sigma_f}{\pi^2 A} \left[ \cosh^{-1} \left( \frac{1 - d \xi}{d - \xi} \right) - \cosh^{-1} \left( \frac{1 + d \xi}{d + \xi} \right) \right] + \frac{\sigma_f}{\pi^2 A} \frac{\xi}{(1 - \xi^2)^{1/2}} \left( 2 \sin^{-1} d + \pi \left( \frac{T}{\sigma_j} - 1 \right) \right)
\]

where \( \xi \) is dimensionless co-ordinate of crack tip.

For calculating the crack propagation the most important parameters is stress concentration before crack or, more precisely, before crack increased for the size of the plastic zone. Thus the additional stress for this point \(|\xi_0| > 1\) is

\[
\sigma(\xi_0) - T = A \int_{-1}^{1} \frac{f(\xi)}{\xi_0 - \xi} \, d\xi \quad \text{and} \quad |\xi_0| > 1
\]

By solving the integral Eq. (11) by using distribution function 10 and by taking into account that the value \( d \)—the location of the original crack tip, varies from 0 to 1 we obtain:

\[
\lim_{d \to 0} \sigma(\xi_0) = \frac{\xi_0}{(\xi_0 - 1)^{1/2}} T \quad \text{and} \quad \lim_{d \to 0} \sigma(\xi_0) = \frac{\xi_0}{(\xi_0 - 1)^{1/2}} (T - \sigma_j) + \sigma_f
\]

If we assume that \( \xi_0 = 1 \) than:

\[
\frac{\sigma(1)}{T} = \frac{1}{\sqrt{2(\xi_0 - 1)}} \left[ 1 - \left( \frac{2}{\pi} \right) \frac{\sigma_f}{T} \cos^{-1} d \right] + \frac{\sigma_f}{T}
\]
and the number of dislocation in the plastic zone is obtained by integration of the distribution function between \( n = 1 \) and \( n = d \). The tooth stress intensity factor \( Z \) as a function of plastic displacement is the product of number of dislocation and of the Burger’s vector. The results of this notation is \([2,6]\):

\[
Z = \frac{b}{\pi^2 A} \left( \frac{1}{T} \right) \left( \frac{TY(a/S)\sqrt{\pi}a}{\pi A} \right)^2 = \frac{b(1 - d^2)^{1/2}TY^2(a/S)a}{\pi A}
\]  

where the location of the original crack tip, which can be written in a simplified form, as follows:

\[
\frac{a}{c} = d = \cos \left( \frac{\pi T}{2\sigma_f} \right)
\]

By assuming that the crack growth rate is proportional to \( Z \) the present theory describes the model of calculation of service life for initiation and propagation of short cracks \([2,6]\).

### 5. Model of crack propagation

Now we will deal in detail with the model of crack propagation on the gear tooth as a basis of the algorithm for calculating the service life of gears and gearings.

Statistical analysis of experimental results shows that the material parameters in the equations of crack growth are random variables. As an example let as take the modified Paris–Erdogan equation:

\[
\frac{da}{dt} = C(\Delta Z)^m
\]  

or in stochastically form:

\[
\frac{da}{dt} = f(\Delta Z)A(t) = Cf(\Delta Z)[\mu + Y(t)]
\]

where \( \mu = E(A(t)) \) and \( A(t) \) is stochastic time process, and \( Y(t) \) is random process with mean \( \mu \). The two parameters \( C \) and \( m \) can be considered to be random function, have the following form:

\[
C = C(x) = C_0 + C(x) \\
m = M(x) = M_0 + M(x)
\]

where \( C_0 \) and \( M_0 \) are random variables describing random variation of mean value in different cases and \( C(x), M(x) \) are random fields describing the material micro-non-homogeneities independently of \( C_0 \) and \( M_0 \).

A good approximation is obtained if it is assumed that both functions, i.e., \( C(x) \) and \( M(x) \) are normal stationary random field with the zero mean and independent values (white noise) \([6]\).

#### 5.1. Stochastic modelling of crack growth

In our code we used a model, which randomises the equation of crack growth ration \([7]\):

\[
\frac{da}{dt} = q(a)X(t)
\]

where \( X(t) \) is a random process having two extreme cases. One of them is entirely an uncorrelated process with only two different times. It could correspond to the Gaussian white noise and the change of time, within which the specified crack size is obtained, would be minimal. The other extreme corresponds to the entirely correlated process. In this case the random process becomes a random variable \( X(t) = A \). We assumed that \( \log A \) was a normally distributed random variable. We used the stochastic model for prediction
of crack growth with variation of the crack size as a time function by modelling the random process \( X(t) \) as a random “train” of pulses:

\[
X(t) = \sum_{k=1}^{N(t)} Y W(t, \tau)
\]

(19)

where \( N(t) \)—homogeneous is Poisson’s process of counting which gives the entire number of pulses within the time interval \([-\infty, \tau]\), \( \tau \) is time of arrival of the \( k \)th pulse and \( Y \) is random amplitude of this pulse. For different \( k \) the amplitudes are independent, but they can be represented by the same statistical distribution so that they have equal probability function.

If we return to the crack growth ratio according to Eq. (18) and if we introduce the random variable \( A \) instead of process \( X(t) \), this equation obtains the following form:

\[
\frac{da}{dt} = q(a) A
\]

(20)

Let us integrate it by time

\[
\int_{a_0}^{a} \frac{da}{q(a)} = A \int_{t_0}^{t} dt
\]

(21)

where we wrote \( a \) for \( a(\tau) \). \( A \) is transformed as follows:

\[
A = \frac{1}{\tau - t_0} \int_{a_0}^{a} \frac{da}{q(a)}
\]

(22)

If the Eq. (22) is integrated the following is obtained:

\[
\int_{a_0}^{a} \frac{da}{q(a)} = \int_{t_0}^{t} X(t) \, dt
\]

(23)

It is evident that the left side of Eqs. (21) and (23) are identical. Therefore a new random process \( A(t) \) can be introduced:

\[
A(t) = \frac{1}{\tau - t_0} \int_{a}^{a(t)} \frac{da}{q(a)}
\]

(24)

From the comparison of Eqs. (24) and (22) we see that both of them are identical on the basis of which we can write:

\[
A(t) = \frac{1}{t - t_0} \int_{t_0}^{t} X(t) \, dt
\]

(25)

We assume that the random process \( A(t) \) follows the lognormal distribution with the probability function:

\[
f(A(t)) = \frac{\log e}{\sqrt{2\pi S_A} A(t)} \exp \left( -\frac{1}{2} \left[ \frac{\log A(t) - \log \mu_z}{S_z} \right]^2 \right)
\]

(26)

where \( S_z \) is the standard deviation for \( \log A(t) \); \( e \) is a basis of natural logarithm and \( \mu_z \) is the mean value \( A(t) \).

We also assume that the crack was initiated within the time \( t \) in the time interval \((0, \tau)\) where the size that crack was \( a(t) = a \). The size of the crack at the end of the time interval is designated \( a(\tau) \) and is obtained by using integration’s according to Eq. (23). It is evident that the crack size \( a(\tau) \) is a random
variable for fixed \( \tau \) immediately when the right-hand side of the equation is a time integral of the random process. Let us designate the conditional probability function of density \( a(\tau) \) with \( f(a|\tau) \) which gives the crack initiated in time \( t \). Thus, \( f(u|\tau) \) can be calculated from the probability density function for \( A(\tau) \) according to Eq. (26) with:

\[
f(u|\tau) = f(A) \left[ \frac{da}{du} \right]_{u(0)} = f(A); \quad u \geq a_0
\]  

(27)

where \( u \) is any crack in time \( \tau \), which is longer than the initial crack \( a_0 \). \( A(u) \) is calculated for the fixed \( s \) according to equation:

\[
A(u) = \frac{1}{\tau - t_0} \int_{a_0}^{u} \frac{da}{g(a)}
\]  

(28)

From Eqs. (26)–(28) we obtain the conditional probability function according to the equation

\[
f(u|\tau) = \log e \sqrt{2\pi S_2 q(u)} \int_{a_0}^{u} \frac{da}{q(a)} \exp \left( -\frac{1}{2} \left[ \log \left( \int_{a_0}^{u} \frac{da}{q(a)}(\tau - t_0) \right) \right]^{2} \right)
\]  

(29)

The probability function \( f(u) \) for crack size \( a(\tau) \) in the time \( \tau \) for \( u > a \) can be calculated from the conditional probability density function \( f_a(u|\tau) \), and the probability function \( f_t(t_0) \) from the time of crack initiation. Thus:

\[
f(u) = \int_{0}^{\tau} f(u|\tau)f_T(t)dt
\]  

(30)

By integrating the Eq. (30) we obtain the distribution of the crack size for a great number of cracks initiated at any possible moment between 0 and \( \tau \) with density corresponding to \( f_T(t) \).

6. Calculation of service life of gearing

On the basis of the model presented above it is possible to calculate the service life of the individual gear tooth. The service life of gear can be obtained from equation.

\[
L_i = f(u)N_i^{-\frac{1}{\varepsilon}}
\]  

(31)

The service life of gear drives is the sum of service lives of all components [8]:

\[
L_{GD} = \left( \sum_{i=1}^{n} L_i^{-\varepsilon} \right)^{-\frac{1}{\varepsilon}}
\]  

(32)

Weibull’s exponents are different for different elements and amount to \( \varepsilon = 2.5 \) for gears, \( \varepsilon = 1.5 \) for ball bearings and \( \varepsilon = 10/9 \) for roller bearing etc. If also the probability is taken into account the Eq. (32) has the following form:

\[
\log \left( \frac{1}{S_{GD}} \right) = \log \left( \frac{1}{0.95} \right) \left[ \left( \frac{L_{GD}}{L_1} \right)^{\varepsilon_1} + \left( \frac{L_{GD}}{L_2} \right)^{\varepsilon_2} + \cdots \right]
\]  

(33)

from which it is possible to calculate the relevant service life of the entire gearing \( L_{GD} \) for desired reliability different from 90% reliability.
7. Experimental determination of crack growth ratio

Since gears and gearings belong to the real complex structures by correctly selected and developed test pieces and carefully planned experiments we obtained results with which we confirmed and justified the use of the mathematical models [6,10] for calculating the tooth stress intensity factor $Z$ as well as the service life of gears.

7.1. Test pieces

We used two types of test pieces:

(a) Standard test piece for determination of fracture mechanics parameters of material AISI 4130 or 42CrMo4 according to DIN in the area of initiation and growth of short cracks according to ASTM-E606 as shown in Fig. 4. The tests were performed on the hydraulic testing machine INSTRON. Such a test piece was selected in order to be able to determine precisely the place of the beginning of occurrence of the crack at the point of the smallest cross section. The test piece was loaded by axial loading with the ratio $\Delta F = F_{\text{max}}/F_{\text{min}} = 0.1$, at the room temperature and with 50 Hz frequency. Thermal treatment of the test piece was the same as that of the gears. From the previous tests on three point test piece made of the same material and with the same heat treatment we obtained basic fracture mechanics properties of this material for long cracks [10].

(b) Non-standard test piece for determination of fracture parameters of gears. This type of test piece was gear test pieces developed during previous researches [10]. A test piece together with the testing device is shown in Fig. 5. We assumed that cracks would start in the critical cross section, i.e., the cross section with

![Fig. 4. Standard (ASTM-E606) test piece.](image)

![Fig. 5. Gear test piece.](image)
7.2. Measuring method used

As we wanted to observe particularly initiation and propagation of short cracks and to determine the conditions occurring in this case [6] (stresses, strains, moment of beginning of fracture etc.), we used mixed experimental methods: photo elastic method, method of measuring of stress/strain by means of strain gauges, replica technique for determination of prevailing initials and measuring of micro cracks by means of crack gauges. The a.m. methods were combined as follows:

- photo elastic examination with strain and crack gauges,
- strain and crack gauges and
- gauges with replica method.

The aim of these combinations was to obtain as complete information about the individual influences as possible and to determine the interaction between these magnitudes.

7.2.1. Replica on cylindrical test piece

During the tensile test of cylindrical test piece the observed test area had a cross section of $4 \times 10$ mm. The replica method was used for determining the length of short cracks with the replication interval of $2 \times 10^3$ cycles in case of very short cracks because we were particularly interested in these area, while for greater lengths the interval was increased to $2 \times 10^4$ cycles. The tests were stopped when the tensile loading was maximal so that the cracks remained open during replication of the surface area.

The test with surface area replication was effected with examination of loading of stress levels 892 and 696 N/mm$^2$, which corresponded to a service life of 22.385 and 80.150 cycles. We performed a few test for each loading ratio, which facilitated studying of a great number of cracks. The triangular form of the pulse at 50 Hz frequencies was used. The crack growth ratio in the first grain changes from its maximum to its minimum value, while the crack approaches or extends over the grain boundary. Since the crack length is measured only in one direction, i.e., along the surface area, the quantum step represents stopping of crack in the direction. In different direction e.g., into depth the crack may increase continuously, but we limited ourselves here only to the crack growth in measured direction. In this case most cracks are initiated and then propagate through grain where only a few cracks are initiated and propagate also along the crystal boundaries.

The replicas obtained at various stages of test were examined on an optical microscope Leitz with 150× to 250× magnification. In this way we determined and/or selected the cracks important for further observation. The length of crack was then measured on the basis of which we calculated the crack growth ratio $da/dN$. The length of the crack during destruction was then determined by observing the fracture surface area. For each experimental value $da/dN$ obtained from analysis of replicas we calculated the relevant value $\Delta Z$. By means of the least square method from equation:

$$\frac{da}{dN} = z_1(\Delta Z)^{z_2}$$

For each pair of data thus obtain with the same stress level we determined the corresponding value of coefficient of the material for short cracks:

$z_1 = 120, 571, 742 \quad \text{and} \quad z_2 = 3069$
The value of the tooth stress intensity factor \( Z \) is related to propagation of the plastic zone, deformation and orientation of grain in case of short cracks and stress intensity factors in case of long cracks. When we want to approximate the tooth stress intensity factor for general use for crack initiation, propagation of micro and macro crack, the stress intensity factor for gear tooth becomes a more complex form according to Eq. (14) and Fig. 1, where the shape factor [3] is

\[
Y(a/S) = \left( \frac{\cos \varphi - \frac{c}{L} \sin \varphi}{\frac{S}{6L}} \right) Y_m(a/S) - \frac{S}{6L} \sin \varphi Y_t(a/S)
\]

(35)

where \( Y_m(a/S) \) is the bending part and \( Y_t(a/S) \) is the known shape factor for the compact tension specimen.

For practice is more convenient polynomial form according to

\[
Y(a/S) = 2.2486135 - 3.7173537(a/S) + 33.95(a/S)^2 - 137.536(a/S)^3 + 210.91(a/S)^4
\]

(36)

7.2.2. Replicas on gears

By the replica method we followed up the propagation of short fatigue cracks in the observed direction, i.e., along the tooth width. The test was performed so that the loadings, i.e., \( F_{\text{max}} = 50 \) kN, \( F_{\text{min}} = 5 \) kN and the ration \( \Delta F = F_{\text{min}}/F_{\text{max}} = 0.1 \) was checked. The test was performed at the frequency 50 Hz. Replicas were made in intervals \( 3 \times 10^3 \) during the first three measurements, and then the step was increased to \( 6 \times 10^3 \) cycles. After \( 210 \times 10^3 \) the test was stopped and final impressions were made. All impressions were made in critical cross section at the point of maximum tensile stress. In the end we made ground section from the test piece on which we observed the process of occurrence of cracks and their size.

7.2.3. Measurements with crack gauges

For measuring the crack lengths greater than approximately 0.2 mm we used crack gauges made by Measurement Group Vishay Type TK-09-CPB02-005. The measuring gauges were glued to gear test pieces at approximately 0.11–0.2 mm from the edge as shown in Fig. 5. By means of the gauge and \( x-y \) plotter we recorded the crack growth from the initial value to the size of 2.6 mm in ten steps. By means of these measurements we obtained the \( a-N \) curve. On the basis of this diagram and measurements of short crack lengths we made the diagram \( da/dN-a \) and diagram \( da/dN-AZ \).

7.3. Measuring of strain and stress

7.3.1. Static measurements

We loaded and relieved the gear test piece with fixed strain gauges from 0–114 kN and measured deformations in the critical cross section of the gear tooth. On the basis of deformations we calculated the stress and residual stress, i.e., plastification in the tooth root. The results are average value of deformations along the entire width of the strain gauge. These data were used for evaluation of results of numerical analyses.

7.3.2. Measurements and fatigue

During the next step we started the fatigue test on the gear test piece at the room temperature and at 50 Hz frequency with the following loading: maximum force \( F_{\text{max}} = 50.1–50.2 \) kN, distance between supports \( L = 294 \) mm and ratio \( R = F_{\text{max}}/F_{\text{min}} = 0.1 \). During fatigue we measured by means of strain gauges the change of strain as function of number of cycles in the area of micro cracks. The basic aim of the measurements was to obtain a correlation between the micro crack length and stresses and/or how to conclude from the stress what happen in the material, when the crack occurs.

The number of cycles varied from 0 to 280.000 where the test was stopped. In the beginning the measurements of stresses were performed after \( 10^3 \) cycles afterwards the measurement step was increased to
2 × 10⁴ cycles. We found out that by such method of measurements of the stress/strain, thought the smallest strain gauge was selected, it was possible only to evaluate the local stress conditions since the influence of average values in the strain gauge is too great so that it is not possible to get precise insight into local micro conditions. On the basis of these researches it is possible to conclude when the changes of the deformation occur during fatigue.

Since by the measuring method with strain gauges we did not achieve desired results, we tried to measure stresses also by the photo elastic method. We glued photo elastic sheet to both teeth in contact. The data on this lining are: deformation optical constant \( F = 4.10 \times 10^{-6} \) m/rad, module of elasticity \( E = 2900 \) MPa, Poisson’s number \( n = 0.36 \) and thickness of lining \( h = 2 \) mm.

Because of the possibility of examination and because of comparison of different methods the following measuring components were glued in addition to the photo elastic plastic sheet on the gear test piece front side:

- one strain gauge on the tooth flank in the critical cross section,
- another strain gauge on the rear side in the critical cross section.

The basic aim of test was as follows:

1. To obtain a precise outline of stress fields at different points of the gear tooth, calibration of the individual methods and their comparison in the static area in case of different stresses.
2. To find out how the fatigue influences the distribution of the stress field and how it is possible from the change of stress distribution to assume the occurrence and growth of micro cracks in cases when they cannot be detected by non-destructive methods. Measurements of stresses by strain gauges and by the photo elastic method were made in intervals \( 3 \times 10^{3} \) for the first three measurements, and then the step was increased to \( 6 \times 10^{3} \) cycles.

After \( 210 \times 10^{3} \) we repeated static measurements of stresses by the photo elastic method. We photographed the isochromates and isoclines and we measured stresses by strain gauges. Thus we obtain a precise comparison of stresses between the starting and final condition during this test. The basic problem of this method is that from the conditions on the surface we assume the processes in the interior. This deficiency on the gears is not excessive as long as the narrow gear are treated whose ratio is \( B/S < 2 \), i.e., where the plane stress fields can be used.

During the test, micro and macro cracks occurred in the interior of the gear. Cracks were detected by the replica method as well as by the subsequent fractographic analysis according to Fig. 6. Because of these cracks changes occurred in the field of isochromates.
8. Numerical example

In order to verify correctness of functioning of the presented model, we have compared the numerical results with experimental results [9,10].

For simplicity and cleanness we have selected a simple model, i.e., single-step gearing with external cylindrical gears. For the required output moment $M_t = 4900$ Nm, gear ratio $i = 5$, centre distance $a = 315$ mm, number of teethes $z_{pinion} = z_{gear} = 21/103$. We select the material AISI 4130 and nitration in bath, the tooth width $B = 80$ mm and coefficient of profile displacement $x_{pinion} = 0.8$ and $x_{gear} = 0.258$ which does not change the required centre distance.

The results are shown in Fig. 7. Figure shows the results of comparison between the experimentally obtained data and calculated values by applying the gear stress intensity factor and proposed mathematical model. On the basis of the experiment and by the applying standard methods we determine the diagram $D_{K-d}$ and we compared it with a similar calculated diagram i.e., $D_{Z-d}$. On the basis of this we determined and calculated the service life of gears.

9. Conclusion

It can be seen above that for accurate calculations of service life of gearing the use of deterministic methods according to different standards is not sufficient. Therefore, for calculations we have developed the gear stress intensity factor $Z$ that takes into account not only the stress of homogeneous material in critical cross section but also the stress concentration due to defects occurring there. On the basis of the factor $Z$ we have developed the model of growth of cracks by means of which it is possible to calculate accurately the actual service life. Fig. 7 show that with our models we have reached the deviation of 5–12% from experimental results, which is a very good accuracy in such calculations. The figure shows also that by means of the presented model we have reached good accordance with experiment particularly in the area, which is most interesting for technical practice, i.e., in the area of permissible sub critical cracks. We believe that such results are very favourable, since we know that the deviations between conventional methods and real conditions are 50% and more.
For the presented case the critical crack length is approximately $a_c \cong 3.5$ mm so the permissible crack is $a_p = 1.75$ mm. In the area where we are approaching the critical lengths and/or fracture impact strength of the material the deviation is slightly greater. However, it is maybe most important that our model gives reliable results since in the calculations the fracture does not yet occur in this area although in practice we never permit the crack to grow up to the critical length.

References