Review of the subsurface strain path approach to fatigue life assessment of components

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ABSTRACT Fatigue lives obtained from complex testing and monitoring of different components often involve some degree of discrepancy in results due to geometrical variations, even when they are tested under controlled conditions and have a similar surface cyclic strain range at the critical location. Recently, several fatigue models have been developed to improve the correlation of specimen lives using a critical ‘process zone’ that surrounds the damaged material. A review of such an approach is presented. The approach is based on critical subsurface strains and consists of a fatigue damage summation procedure in the affected area. The fatigue life-prediction model is applied to two structural materials using three geometries subjected to biaxial cyclic stresses. These include notched bars, rhombic plates and car components. The subsurface strains are evaluated by using a detailed elastic–plastic finite element analysis, and by considering critical subsurface fatigue strain paths. It is shown that in the several cases investigated, the subsurface approach appears to improve life predictions.

Keywords critical plane; geometrical aspects; life prediction; process zone; subsurface strain.

NOMENCLATURE

\[ D_n, D'_n = \text{subsurface model damage related to the strain gradient at each strain increment and the modified damage parameter at increment } n \]
\[ i = \text{subsurface model strain location number in a particular subsurface strain path from the surface to a critical distance } (n = i - 1) \]
\[ n = \text{subsurface model increment} \]
\[ N_{ln}, N_{fD} = \text{subsurface model number of cycles and modified number of cycles to failure at increment } n \]
\[ N_i = \text{cycles to crack initiation} \]
\[ \sigma, \epsilon = \text{local stress and strain} \]
\[ \theta = \text{surface intersection angle of subsurface plane} \]
\[ \phi = -\nu, +1, 0, -1 = \text{principal strain ratio minimum/maximum = uniaxial, equibiaxial, plane strain and pure shear, respectively} \]
\[ \bar{\epsilon}_i, \bar{\epsilon}_n = \text{subsurface model strains at a typical distance } i \text{ and average strain at a particular increment } n \text{ under the surface} \]
\[ \Delta \epsilon_n = \text{general subsurface strain increment} \]
\[ \Delta \epsilon_{ct} = \text{critical distance subsurface strain increment} \]
\[ \varepsilon_{eq}, \varepsilon_e, \varepsilon_p = \text{equivalent multiaxial strain parameter, elastic strain and plastic strain, respectively, using the Coffin–Manson–Basquin relation} \]
\[ \gamma^* = \text{shear strain on a plane of intersection at } 45^\circ \text{ with the surface} \]
\[ \varepsilon^*_n = \text{normal strain on a plane of intersection at } 45^\circ \text{ with the surface} \]

INTRODUCTION

Multiaxial fatigue theories developed in the past have fairly successfully predicted the fatigue life of components subjected to complex loads. Low cycle fatigue (LCF) theories

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have often used strain-based parameters that correspond to material deformation and microstructural behaviour. A similar microstructural approach has also been adopted to predict the failure of components subjected to high cycle multiaxial fatigue where elastic conditions prevail during the majority of life.

An example of geometrical aspect in fatigue is shown in Fig. 1. Results from biaxial fatigue of thin-walled specimens (1 mm thickness) are compared to uniaxial fatigue of solid specimens (8 mm diameter) using the Lohr–Ellison strain parameter. Simulation has shown that the thin-walled geometry of the biaxial specimen approaches a plane-stress state with almost no strain or stress gradient across the specimen wall. The fatigue lives of the uniaxial specimens, when tested under similar surface strain conditions, are about three times greater than the lives of the hollow biaxial specimens (Fig. 1). This difference between lives is associated with a decrease inwards, or radial stress/strain gradient, from the surface to the midsection of the solid specimens.

Several subsurface fatigue models have been proposed to overcome the stress or strain gradient effects on fatigue life. In general, the models have been used either for high cycle fatigue (HCF) and sometimes to modify the endurance limits, or for LCF where the plasticity is considered by using strain-based parameters. The fatigue models are based either on a critical plane multiaxial fatigue criterion or an energy approach. The subsurface models can be separated into those using a critical depth and those that accumulate the fatigue damage up to a certain critical depth. Other types of models have introduced linear elastic fracture mechanics (LEFM) principles to evaluate limit life of a notched component by employing a critical distance within the so-called ‘process zone’, and using the line or the point methods for short cracks fatigue calculations.

Figure 2 illustrates the basic types of ‘process zone’ models to account for the geometrical differences in life prediction, using subsurface parameters. These models typically use one of the following (Fig. 2): (1) Reference point; (2) Reference path; (3) Reference plane and (4) Reference volume.

**SUMMARY OF THE SUBSURFACE STRAIN PATH APPROACH**

The subsurface strain path model belongs to case 2 in Fig. 2, and uses the following assumptions:

(a) A critical high-strain path up to a critical depth is numerically calculated.

(b) A subsurface multiaxial strain parameter along a critical path is divided into equal increments and, using the material strain-life relation, the life corresponding to the average strain from each increment is obtained.

(c) The contribution to the fatigue damage process from each increment of strain under the surface is assessed and assumed to decrease with the distance from the surface.

(d) A linear accumulation of the subsurface damage is carried out along a critical path.

(e) The equivalent average strain from each increment is calculated as (Fig. 3a)

\[ \bar{\varepsilon}_n = \frac{\varepsilon_i + \varepsilon_{i-1}}{2}, \]

where \( \bar{\varepsilon}_n \) is the average incremental strain and \( n \) is the increment number, with \( n = i - 1 \).

The incremental damage parameter is calculated using the incremental strain gradient divided by the critical

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**Fig. 1.** Fatigue lives of biaxial hollow specimens and uniaxial solid specimens.
distance subsurface strain increment, $\Delta \varepsilon_{Cr}$:

$$D_n = \frac{\varepsilon_i - \varepsilon_{i-1}}{\Delta \varepsilon_{Cr}}.$$  

(2)

The relative distance from the surface of each strain increment is introduced through a function that modifies the damage values with regard to the surface distance, for example,

$$D^*_n = D_n \left( 1 - \left( \sum_{i=1}^{n-1} D^*_n \right) \right),$$  

(3)

where $D^*_n$ is the modified incremental damage parameter.

The total life to failure, then, is summed as follows (Fig. 3b):

$$N_{fD^*} = \sum_{i=1}^{n-1} (N_{fn} D^*_n),$$  

(4)

where $N_{fD^*}$ are the modified cycles to failure for a particular surface strain range and $N_{fn}$ is the number of cycles to failure at a certain depth along the critical path, corresponding to the average incremental strain $\bar{\varepsilon}_n$ at that depth.

The corresponding material multiaxial fatigue life, using the average incremental strain $\bar{\varepsilon}_n$, is calculated by using a Coffin–Manson–Basquin relation between the elastoplastic and plastic equivalent strains and the life to failure (Fig. 3b):

$$\varepsilon_{eq} = \varepsilon_p + \varepsilon_e = AN_f^{-\alpha} + BN_f^{-\beta},$$  

(5)

where $A, B, \alpha$ and $\beta$ are material constants and the incremental strain $\bar{\varepsilon}_n$ is equated to $\varepsilon_{eq}$.

The choice of multiaxial parameter is determined separately by correlating biaxial fatigue experiments for each material.$^{2,6}$ For example, in the case of the Lohr–Ellison parameter,

$$\varepsilon_{eq} = \gamma^* + C\varepsilon^*_n = \text{constant},$$  

(6)

where $C$ is material parameter. It is assumed that $C = 0.2$ for medium carbon structural steel.$^1$

**APPLICATIONS**

The subsurface strain model was used to estimate the life to failure of three different geometries; a car component, axisymmetric notched bar and a rhombic plate. In each case, a detailed elastic–plastic finite element analysis has been conducted using the appropriate cyclic loads for each testing condition. The simulations provide curves similar to those given in Fig. 3a, Eq. (5). The material master curves of strain life were used to calculate the incremental
subsurface life in critical paths. For the first two geometries (car component and notched bar) the material was medium carbon structural steel, where biaxial data appeared to be well correlated using the Lohr–Ellison parameter. Therefore, this parameter biaxial master curve was used in subsequent subsurface strain analysis. For the rhombic plate made of aluminium alloy, biaxial data were not available. Instead, it was assumed that biaxiality is taken into account by using maximum shear strain range master curve.

Fatigue of axisymmetric notched bar (isotropic and anisotropic structural steel)

Two batches, isotropic and anisotropic, of the structural steel EN15R (BS150M36) were used in an extensive experimental programme reported elsewhere. The cyclic strains at the notch root of the axisymmetric notched bar specimens were estimated using the elastic–plastic finite element method. The finite element simulations also provided subsurface strains that were used to evaluate life through the subsurface strain model. A separate analysis was carried out for each batch of the material.

Fatigue strain-life master curves were evaluated from uniaxial smooth solid and biaxial hollow specimen test results (Fig. 1), using the Manson–Coffin–Basquin relation and the Lohr–Ellison equivalent strain parameter. The surface and subsurface notch specimen lives were estimated by using elastic–plastic finite element strains and employing Eqs (1)–(4).

In Fig. 4, the experimental lives of the notched specimens are compared with predictions obtained by using the strain-life master curve obtained from biaxial fatigue testing of thin-walled specimens and the cyclic strains from finite element analysis of the notched specimens. The surface strain analysis predictions are shown in Fig. 4a and the subsurface strain analysis prediction in Fig. 4b. It is shown in Fig. 4 that the conservative trend due to the difference in specimen geometry is reduced when the subsurface model is used, particularly for the isotropic material.

Fatigue of rhombic plate under anticlastic bending (aluminium alloy 2024 T3)

Experimental results of a rhombic plate subjected to anticlastic bending to obtain cyclic biaxiality of stress and strain are reported elsewhere, along with the details of the elastic–plastic finite element analysis to estimate the surface and subsurface cyclic strains. The subsurface strain method was applied to the anticlastic bending test results, and the experimental lives of the rhombic plates were compared to surface life predictions (Fig. 5). The life prediction procedure was carried out at several subsurface paths by using the maximum shear strain obtained from finite element simulations.

The subsurface strain lives of the specimens were calculated up to 1 mm thickness at two different planes, and were compared to surface predictions (Fig. 5) using the uniaxial strain-life master curve. The predicted subsurface model lives are somewhat non-conservative, but the
trend is consistent with the notched specimens subsurface analysis to reduce the surface life prediction scatter.

**Fatigue of a Metro car suspension arm (structural steel)**

The subsurface strain path analysis was also used to estimate service component life—the Metro car suspension arm. The car component was made of the isotropic batch of the EN15R material mentioned previously. Critical surface and subsurface elastic–plastic strains were estimated from separate plane-stress and plane-strain finite element analyses, and these strains were used to calculate several biaxial fatigue cyclic parameters. The component life was predicted by using a biaxial fatigue master curve obtained from hollow specimen tests, similar to the notched specimens life prediction.

Fig. 5. Experimental and predicted lives of rhombic plate specimens using uniaxial solid bar data and two different strain paths.

In Fig. 6, the predicted lives are compared to the experimental data using strains from the plane-stress and plane-strain finite element analyses. The fatigue tests of the car component were carried under two nominal cyclic load ranges of 14.8 kN or 8.8 kN. Although the experimental results have shown much scatter in lives between components for the same cyclic load, the life prediction from the analysis using a critical subsurface path was less conservative in comparison to the surface analysis. This was independent of the type of finite element analysis used, and was in agreement with the analyses of the laboratory specimens shown previously to reduce scatter.

**DISCUSSION**

The reviewed subsurface strain path model appears to improve correlation between predicted and experimental multiaxial fatigue lives of components in comparison to correlation based on the critical surface strain state. The model is independent of the choice of biaxial strain fatigue parameter or critical path, and is particularly useful in situations when fatigue master curves obtained using plane-stress (e.g. thin-walled specimens) or plane-strain specimens (e.g. solid bar specimens) are used to predict lives of components that are in a different stress state. For example, it is argued that in case of plane-strain specimens, the material constraints and the strain gradient under the surface delay the fatigue failure process. This is further demonstrated in Fig. 7 where the ratio between the experimental lives to the subsurface model predicted lives is shown for the different tested components in terms of the maximum local strain range obtained from a finite element analysis. When the subsurface model is used to predict lives of specimens approaching plane-strain from plane-stress data (e.g. thin-walled tubes data and the car components) results are good or conservative.
subsurface model is used to predict lives of components approaching plane-stress from plane-strain data (e.g. solid bars data and the anticlastic specimens), the predicted lives are non-conservative. However, in all cases investigated, the scatter in prediction was improved by using the subsurface model in comparison to surface strains, as is shown in Figs 4–6.

The elastic–plastic finite element analysis requires an input of the cyclic stress–strain material response; in this review this was taken from fatigue tests at half life. A proper hardening law is also required to simulate the unloading part of the fatigue cycle. In the case of the medium carbon steel, an isotropic hardening law was used while kinematic hardening was used in the simulation of the aluminium alloy. For efficiency in some cases it is possible to use geometrical simplification that may not affect the estimated strains. For example an axisymmetric model was used to model notched bars and both plane stress and plane strain models were used to estimate the strains in the car component. Regardless of the type of finite element analysis used, the estimated strain results were validated using strain gauge measurements.

To calculate the model parameters, a critical fatigue strain path under the surface is required, and this is geometry and loading dependent. Sometimes the choice is obvious, as for example the paths used with the axisymmetric notched specimens and for the Metro car suspension arm life predictions. However, it may not be straightforward in other cases. For example, finding the critical path in the case of the rhombic plate tests. In this case, several paths, aligned at increments of 15° from the surface, were investigated. This required a very detailed finite element simulation and careful consideration of element meshing prior to the analysis. It may be argued that, in general, the shortest life is calculated for the critical subsurface path, and this could be obtained by using a numerical optimisation procedure.

Current limitations of the subsurface model include the following. It does not contain a direct calibration with material fatigue micromechanics damage and/or constitutive behaviour, and the subsurface distance in which the model applies is not well defined. Implementation of a subsurface critical distance parameter related to the material microstructure and the geometrical constraint at the critical areas requires further investigation. However, the model includes the use of the stress–strain response in the simulation stage, the choice of a suitable multiaxial fatigue parameter and the material strain–life relationship (master curve).

The subsurface strain model is predominantly influenced by the subsurface strain gradient, which is a function of the loading, the geometry and elastic–plastic material response. All could affect the calculations, for example a constant subsurface elastic strain gradient may exist in bending of plates or an exponential strain variation from the surface to the mid-section under elastic–plastic tension of solid notched bars. However, these strain gradient conditions are considered during the estimation stage of the subsurface strains (using finite element analysis) and therefore are not model-dependent for the life prediction analysis.

CONCLUSIONS

1 A previously developed, process zone type fatigue model, based on damage accumulation in a subsurface critical strain-path domain, appears to reduce the scatter in component life predictions in comparison to predictions using the surface strains.

2 The improvement in fatigue life scatter is demonstrated in estimates of cycles to failure of notched specimens, rhombic plates and car components under high-strain multiaxial fatigue. The life predictions are based on multiaxial parameters life-strain master curves for two structural engineering materials, medium carbon steel and aluminium alloy.

3 In general, this review indicates that an approach to fatigue damage that considers critical subsurface material domain instead of surface critical location may be favourable in fatigue cases having a geometrical dissimilarity between service components and tested laboratory specimens.

REFERENCES


