SHORT FATIGUE CRACK BEHAVIOUR AND ANALYTICAL MODELS: A REVIEW

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Abstract—The use of engineering materials in critical applications necessitates the accurate prediction of component lifetime for inspection and renewal purposes. In a cyclic loading situation, it is very important to be able to predict the growth rates of cracks from initiation to final fracture. Most of the investigations have been focused on the long fatigue cracks from the notched specimens, and linear elastic fracture mechanics is used to analyse the crack growth data. The behaviour of the short fatigue cracks less than a millimeter in length cannot be analysed by linear elastic methods because of large-scale plasticity effects. A number of modifications to LEFM and new models have been introduced to correlate the behaviour of long and short fatigue cracks. This paper addresses the critical view of the modifications and some of the models proposed for the analysis of short fatigue cracks.

Keywords—analytical models, fatigue, short cracks.

1. INTRODUCTION

The problem of the short fatigue cracks received attention for the first time when Pearson [1] observed that linear elastic fracture mechanics (LEFM) failed to correlate the crack growth rate of very small cracks (0.006–0.5 mm) with that of long crack, as shown in Fig. 1. Since then interest has heightened in the study of short/small fatigue crack behaviour [2–15].

It is now well recognized that small pre-existing defects are an inherent feature of engineering components and structures. They may be formed as a result of material forming and fabrication techniques, or may be an adventitious results of careless transportation or handling. The size of such defects may range from the order of microns, for metallurgical inhomogeneities such as nonmetallic inclusions, to several millimeters for the case of welding defects such as slag inclusions or heat-affected zone cracks. Over the majority of this range of crack sizes the major portion of the fatigue life is spent whilst the crack is smaller than the non-destructive inspection (NDI) limit, as illustrated in Fig. 2. It is therefore convenient to define the upper bound of the small crack problem to correspond to the general NDI limit of around 0.5–1.0 mm. The importance of the short crack regime in fatigue is apparent in Fig. 2, and it is useful to apply the principles of linear elastic fracture mechanics and in particular the defect tolerant design approach. However, there are certain practical and fundamental difficulties in existing fatigue design codes when including such small cracks.

1.1. Effect of closure and microstructure

Several authors have proposed that changes in crack closure are the cause of observed short crack behaviour [19, 20, 22, 24]. The use of an effective stress intensity range (ΔKe) is one method of accounting for effect of closure on crack growth. Crack closure, microstructure and plasticity on short crack growth may either act independently or in conjunction.

It is well understood that closure reduces the stress intensity range at the tip of a growing crack through premature crack face contact. There are several mechanism that can cause this premature crack face contact, and they can be classified as (i) plasticity-induced closure, (ii) roughness-induced closure, (iii) oxide-induced closure, and (iv) transformation-induced closure. The actual closure level of a particular crack may be a combination of more than one of these mechanism.

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Plasticity-induced closure is considered to arise from the elastic constraint in the wake of the crack tip of material elements permanently stretched within prior plastic zones, leading to an interference between mating crack surfaces. Based on experimental compliance measurements in 2024-T3 aluminium alloy, Elber [25] proposed an empirical relationship between the ratio of the closure stress intensity to maximum stress intensity and load ratio. Lindley and Richards [26]
work indicated that the plasticity-induced closure plays a far less significant role in influencing fatigue crack growth.

Roughness-induced crack closure arises due to the roughness of the fracture surface near threshold stress intensity. Roughness-induced crack closure is promoted by small plastic zone sizes at the crack tip which induce a single shear mechanisms, small crack tip opening displacements, coarse grained materials and microstructure, crack deflection and low load ratios where the minimum crack tip opening displacements may be substantially smaller than the size of the surface asperities. Purushothaman and Tien [27] first modelled the roughness-induced closure. However, the model does not incorporate the role of mode II displacements, which influence the roughness-induced closure.

Oxide-induced crack closure concept is based on the phenomenon that at low load ratios, near threshold growth rates are significantly reduced in moist environments, compared to dry environment. Moist environment lead to the formation of oxide layers within the crack, which are thickened at low load ratios by fretting oxidation. The formation of corrosion deposits and the process of oxide-induced crack closure are promoted by (i) small crack tip opening displacements (CTOD), (ii) highly oxidizing environment, (iii) low load ratios, (iv) rough fracture surfaces, and (v) low strength materials where the plasticity-induced closure mechanism is dominant. A number of independent studies by Paris et al. [28], Tu and Seth [29], Skelton and Haigh [30], Kitagawa et al. [31], Stewart [32] and Suresh and co-workers [17, 33-35] suggested the possibilities of crack closure due to corrosion debris affecting near threshold growth rates and $\Delta K_{th}$ values.

Transformation-induced crack closure can result in the materials which undergo stress- or strain-induced phase transformations under cyclic loading. Such transformation-induced closure will be promoted by metallurgical phase changes which show a large increase in volume and by conditions which enhance the transformation, i.e lower temperatures, higher strain rates, etc. This closure effect arises from the constraint of surrounding elastic material on the transformed regions. Hornbogen [36] and Mayer et al. [37] reported that the pronounced crack growth retardation in the metastable alloy is due to the transformation of part of the austenitic phase into martensite in the stress field of the crack tip, accompanied by a volume increase and consequently, residual compressive stresses.

The understanding of the many sources of closure outlined above has enabled a much more consistent explanation to be obtained for the effects of many mechanical, microstructural and environmental factors which have been documented to influence near-threshold growth rates and the values of the fatigue threshold. The direct experimental closure measurements have confirmed the role of crack closure at low load ratios with long cracks. At high load ratios or with small cracks, the closure effect may well be severely diminished, non-existent or cannot be so effective as in case of long cracks.

James and Morris [22] suggested that in mechanically and physically small cracks, large-scale near-crack-tip plasticity or less extent of crack closure may be a main reason why small cracks propagate appreciably faster than long cracks subjected to the same nominal value of the stress intensity factor range. Jono and Sugeta [38] founded that fatigue crack longer than 0.2 mm were shown to have a residual tensile deformed region in the wake of the crack tip and exhibited retardation after load reduction due to the difference in closure levels. However, for crack smaller than 0.1 mm, closure is insufficient to control the crack growth behaviour and the high level loading may give rise to an acceleration of crack growth.

In spite of different findings of different researchers, crack closure has been successfully used to explain the anomalous behaviour of short fatigue cracks. The work of Dowson et al. [39] on the early stage of short crack growth suggests that after cracks outgrow the grain boundary, the amount of crack closure increases significantly. It appears that roughness-induced closure plays an important role in this case, due to crack deflection after crossing the grain boundary. It has been noticed that a combination of plasticity- and roughness-induced closures dominates the early stage of fatigue crack growth. According to Morris [40] stage I interaction with grain boundaries is found to induce closure stress close to the maximum applied stress, explaining the effective crack retardation of stage I cracks by grain boundaries observed for Al-7075. In these cases the closure load was found to slowly fall as cracking continues by a stage II mode into a second grain. For stage II cracking it is possible to estimate closure load from the cracking path.
as observed from the surface. Tanaka et al. [41] suggested that the anomalous behaviour of short cracks is due to reduction in the ratio $K_{op}/K_{max}$ as crack size decreases. Recently Gall et al. [42] presented an elastic–plastic finite element analysis of mode I crack growth and plasticity induced crack closure. The model shows that plasticity induced crack closure will occur when the crack tip plastic zone is smaller than a grain size. Closure levels are found to vary as the crystallographic orientation of the model with respect to the crack line is changed. For some orientations the closure levels are found to be negligible, while in others the levels approach 0.35 of the maximum applied load. Closure levels also vary as the angle between the slip planes is varied.

The variation in growth rates due to different crystallographic orientation is reported by Tanaka and co-workers [43, 44] during the study of microstructurally small fatigue cracks. The orientation of the individual grains in the direction of crack growth may increase, decrease or arrest the crack growth. Since the growth behaviour of short cracks can influence the fatigue predictions, it is very important to consider the microstructure to model short crack growth behaviour.

Crack retardation at microstructural interfaces is one explanation for the initially decreasing growth rates shown schematically for short cracks in Fig. 3. This decrease occurs despite the fact that $\Delta K$, for constant amplitude loading, is increasing steadily as the crack length increases. This observation has been attributed to two factors [47]. The first is the difficulty in propagating slip across an interface, which may give rise to an incubation period, depending on the type of the interface, e.g. high-angle grain boundary or interface with a second phase. The second factor is a possible increase in closure stress as the crack tip crosses an interface, caused by increased non-continuum crack tip plasticity due to slip extending to the next interface. In a closure dominated situation the decrease in growth rates can be the result of an increase in closure stress i.e. a reduce in $\Delta K_{eff}$. As the closure level approaches that of a long crack the increase in closure stress reduces gradually and the growth rate increases.

The effect of heat treatment is clearly demonstrated in two studies made on 0.4% C steel [2, 48]. James and Smith [48] studied a quenched and tempered 0.4% C steel with ferrite grain size of 10 µm. They observed no microstructural dependence of growth for small cracks. Higher growth rates were found for cracks whose surface length was less than 100 µm. The short crack effect was explained by change in closure. However, in a study of a normalized 0.4% C steel with prior austenite grain size of 140 µm, tested at 350–400 MPa shear stress, Rios

![Fig. 3. Schematics illustration of types of short crack behaviour (dashed lines) which may be observed over the first millimeter or so of growth[16].](image)
Fig. 4. Development of a surface crack. (a) Optical micrograph of a replica taken before the test commenced, showing the ferrite phase on the prior austenite grain boundary. (b) SEM micrograph of a replica taken after 20,735,865 cycles. Two cracks, X and Y (arrows), have formed in the ferrite, and were unable to propagate any further. $\Delta \sigma = 350.51$ MPa. (c) SEM micrograph, 46,763 cycles after raising the stress level to $\Delta \sigma = 414.55$ MPa. The cracks have now joined-up and branching has already occurred at one end (X). (d) Optical micrograph, 649,907 cycles after raising the stress level. The crack has now branched at both ends[2].
Fig. 5. Micrographs of crack initiation sites: (a) crack initiated along a persistent slip band; (b) crack initiated along a twin boundary (acoustic microscope image); (c) crack initiated in a grain with no visible persistent slip bands [49].
Fig. 6. Sequence of crack development (plastic replica). Pearlite banding parallel to specimen axis (a) before testing, (b) after 158,000 cycles, (c) after 235,000 cycles, (d) after 600,000 cycles[50].
Fig. 7. Crack arrest at pearlite region (plastic replica), parallel to specimen axis [50].

Fig. 8. Sequence of crack development in C–Mn steel (plastic replica) under push–pull loading [74].
$N_f = 27,000$. 
et al. [2] observed microstructurally dependent growth with cracks arresting at ferrite–pearlite interfaces. The cracks were initiated in the grain boundary ferrite and grew rapidly, at an almost constant rate, until they reached the end of ferrite plates. The growth rate was dropped dramatically and the crack arrested at the end of ferrite plate. Figure 4 shows the development of some of these cracks. Although crack closure is an important factor it has not been measured for 0.4% C steel. Short crack behaviour was observed for cracks with surface lengths in the range 200–300 μm, which corresponded to the lengths of ferrite phase in which the cracks were initiated. In the study of nickel base super alloy, Wasplay, Yate et al. [49] observed three types of crack initiation: (i) cracks developed directly from persistent slip bands; (ii) initiated along a twin grain boundary; and (iii) initiated in grains which showed no evidence of slip bands, as shown in Fig. 5. Although there were three types of cracks initiation, all cracks were initiated at 45° to the principal axis in a relatively large grain. The surface crack growth rates slowed down and reached a minimum at the grain boundary. The grain boundary blocking effect was observed up to a few grains. In 0.4% C steel [50] it was found that cracks were initiated and propagated along the slip band; these are shown in Fig. 6. The pearlite region was acting as barriers to crack propagation, see Fig. 7. In a ferrite– Bainitic microstructure, Hussain et al. [74] observed the initiation of the crack in ferrite region situated at prior austenite grain boundaries. In this microstructure cementite plates were found to be strong barriers to crack growth. The crack initiation and propagation behaviour is shown in Fig. 8. de Lange [51] observed deceleration in fatigue cracks when they become as large as four to five grain diameters. In a high strength steel [52] and an aluminium alloy [21], retardation in growth was observed at crack lengths equal to the grain size. Figure 9 shows short and long fatigue crack growth behaviour in different materials. Forrest and Tate [56] observed that cracks developed along slip bands and propagated in shear mode, but slowed down at grain boundaries. The degree of retardation depends on the grain orientation. It is well documented that microstructural size affects the mechanical properties of the material, especially the monotonic properties. The Hall–Petch [58, 59] equation is the obvious example of the dependence of tensile strength on grain size. Similar approaches were used to relate the fatigue strength to the grain size [45, 55–57, 60].

1.2. Limitations

The fundamental difficulties arise primarily from two factors: (a) application of LEFM to high stress situations which usually characterize the growth of short cracks; and (b) the experimental observation of growth of short cracks at values of the applied stress intensity range well below the conventional “long crack” threshold values (ΔKth). Such growth may occur at rates several orders of magnitude greater than might be expected, from an extrapolation of the Paris law regime to include low stress intensity values, and can cause a component failures at fraction of predicted life time. A basic condition of the use of LEFM is that applied stress should be well below the yield stress and that the extent of crack tip plasticity should be small compared with crack size and component dimensions. Associated with this is a possible noncontinuum aspect to crack tip plasticity when the crack size is small compared with some microstructural parameter. In this situation several studies have indicated that the size of the crack tip plastic zone may then be determined primarily by the microstructure rather than the applied stress intensity factor [16].

The short fatigue crack problem in fact is one created by fracture mechanics through a breakdown in the analysis of short cracks. Fracture mechanics analysis of fatigue crack growth assumes microstructural independence. Kitagawa and Takahashi [3] and Taylor [11] found that cracks smaller than a critical length cannot be accurately defined by the LEFM and these are termed as “short” cracks. Different researchers [9, 12, 16, 46] defined the short cracks in different ways. In accordance with ASTM specification, if the plastic zone size (rp) > a/50, then the crack (a) is short; conversely if rp is smaller than this ratio then the crack growth kinetics are predictable by LEFM.

According to James and Knott [16], when the growth of a fatigue crack from small initial defect is analysed by LEFM, different types of behaviour may be observed over the first millimeter or so of growth, as shown in Fig. 3. In all cases, there is an initially high growth rate at ΔK values well below the conventional long crack threshold. The growth rate may then decrease to minimum as ΔK increases. Such anomalous growth behaviour has been observed in many
Crock stopped growing under the applied loading. 

\[ R = 0.1 \text{ Hz} \]

\[ \Delta K = \text{MPa} \sqrt{\text{m}} \]

Fig. 9. Example of short and long crack behaviour in different materials: (a) aluminium bronze\([53]\), (b) 7075-T651 aluminium alloy\([53]\), (c) C–Mn steel\([54]\).

Materials including steels\([18–20, 49, 50, 67]\), aluminium alloys\([21]\), titanium alloys\([22, 23]\), nickel-base super alloys\([24, 45]\) and cast bronze\([10]\). The short crack regime can be defined as the crack length which shows high growth rates as compared to the long cracks. The higher growth rates may be observed when:

(a) Cracks are comparable in length to the scale of local plasticity. This is particularly the case when the crack is growing in a region of bulk plasticity.

(b) Cracks are smaller in length to some microstructural parameter. This is essentially a breakdown in the homogeneous continuum requirement for the application of fracture mechanics\([10]\).

(c) The crack is long in terms of microstructure and local plasticity, higher growth rates have been observed for physically small cracks (0.5–1.0 mm). This has been proposed to be due to a breakdown in similitude concept of LEFM.
Kitagawa and Takahashi [3] reported that at crack lengths greater than 0.5 mm, the threshold stress intensity factor for fatigue crack growth is constant. The intensity factor deviates from values predicted by LEFM if the crack length is smaller than 0.5 mm. Due to the limitations, discussed above, the LEFM cannot be used to analyse the short fatigue crack growth behaviour. Several mathematical models have been developed along with the modification of LEFM, to predict the anomalous behaviour of short cracks. This paper addresses the critical view of the models and their application to analyse short fatigue crack behaviour. In some cases a comparison is made between predicted and experimental fatigue lifetime.

2. ANALYTICAL MODELS

Several characteristics of short cracks distinguish them from long cracks and contribute to differences in growth rate. One of these is that the plastic zone size at the crack tip of short crack is large with respect to the length of the crack, which is the one violation of the conditions prescribed for linear elastic fracture mechanics. The second violation, the overall applied load levels, may be high with respect to the yield strength of the material, is the small scale yielding appropriate to LEFM analysis. The third violation is that the level of crack closure is in a state of transition as the crack changes from a short to long crack. Crack closure reduces the effective range of the stress intensity factor, $\Delta K_{\text{eff}}$, and there will be a greater driving force for growth of a short crack as compared to long crack at a given $\Delta K$ level[62].

Contrary to long cracks, short crack growth strongly depends on microstructure. Investigators have attempted to modify LEFM to account for the behaviour of the short cracks. Mechanisms based on crack closure stress and crack deflection, elastic–plastic approaches such as the $J$-integral, or simple semi-empirical approaches have been introduced. The methods are sufficiently successful when the crack length is only a few times that of the relevant microstructural parameters. A number of crack growth models have been proposed to describe the short fatigue crack propagation, taking into consideration the microstructural effect on the growth behaviour.

2.1. Model based on crack tip strain

Chan and Lankford[61] modified the LEFM equation to consider the variation on the grain orientation and effects of the grain boundaries. The model was based on the assumptions that near threshold stress intensity for a long crack, the crack-tip opening displacement (CTOD) is larger for small/short crack than for a nominally equivalent long through-crack, i.e the plastic strain range associated with a small crack is higher than that of a long crack. Considering the influence of the crystallographic orientations of the neighbouring grains and the distance of the crack-tip from the nearest grain boundary, the plastic strain range at the crack tip was defined as:

$$\Delta \varepsilon_p = C \Delta K^{n_1} \left[1 - K(\Phi) \left(\frac{D - 2X}{D}\right)^{m_1}\right].$$

The crystallographic function $K(\Phi)$ in terms of resolved shear stress in the grain with small crack (grain A) and its neighbouring grain (B) was defined as

$$K(\Phi) = 1 - \frac{\tau_B}{\tau_A}.$$  

In this approach, the local plastic strain range at the crack-tip was used as a measure of fatigue damage. Crack advance by the failure of a crack-tip element of size $\Delta X'$ occurs when the accumulated local plastic strain exceeds a critical value $\varepsilon_p^*$. The number of cycles $\Delta N$ required for failure of crack-tip element was given by

$$\Delta N = \frac{\varepsilon_p^*}{\Delta \varepsilon_p}$$

and the crack growth rate was defined as

$$\frac{da}{dN} = C_1 \Delta K^{n_1} \left[1 - \left(1 - \frac{\tau_B}{\tau_A}\right) \left(\frac{D - 2X}{D}\right)^{m_1}\right]$$
From a physics and mechanics point of view, Chan and Lankford's model is a gross approximation to reality. The model predicts little or no deceleration in growth rate in the case of similar orientation of grains. A comparison between long crack (analysed by LEFM) and short crack growth behaviour [analysed by eq. (4)] is shown in Fig. 10.

2.2. Model based on modification of LEFM

McEvily et al.[62] proposed a short crack analysis considering large-scale plasticity effects, crack closure and the fatigue crack growth threshold. The proposed short fatigue crack analysis method is based on modification of elastic analysis in the presence of large-scale plasticity. They used Irwin's expression[63] to analyse the experimental data as

\[ K = \lim_{\rho \to 0} K_T \sigma \sqrt{\frac{\pi \rho}{4}} \]  

which relates the stress intensity factor \( K \), to the stress concentration factor \( K_T \). In this expression \( \sigma \) is the applied stress and \( \rho \) is the radius of the stress raiser. Irwin has shown that it is possible to develop a variety of stress intensity factors appropriate to large cracks from a knowledge of the corresponding stress concentration factor when the radius \( \rho \) is allowed to approach zero. In a case of a central notch \( 2a \) wide sheet specimen loaded in tension, \( K_T \) was defined as \( K_T = (1 + 2 \sqrt{a/\rho}) \) and therefore eq. (6) can be written as

\[ K = \lim_{\rho \to 0} \left( 1 + 2 \sqrt{\frac{a}{\rho}} \right) \sigma \sqrt{\frac{\pi \rho}{4}} \]  

![Diagram showing crack growth and arrest](image)

Fig. 10. The crack-tip strain model is used to simulate the anomalously rapid growth of small crack. The simulated behaviour also includes crack growth perturbation including arrest at a grain boundary[61].
which leads to the following expression:

$$K = \sigma \sqrt{\pi a}.$$  \hfill (8)

In their model, instead of allowing $\rho$ to approach zero, they allow it to approach a finite value $\rho_c$. This modification was justified as follows:

1. the fatigue crack tip is not of zero radius even at the minimum load;
2. the large plastic zone to length ratio of a short crack requires modification of the LEFM;
3. the stress range required to propagate a crack must remain finite rather than go to infinity when the crack length is extremely small.

The “$K$” was defined as

$$K = \left(\frac{\pi \rho_c}{4} + Y \sqrt{\pi a}\right)\sigma.$$

(9)

In the propagation of short crack, the applied stress levels are high with respect to the yield strength of the material. As a result the plastic zone will be large than predicted by LEFM, and therefore the effective crack length, $a_{\text{eff}}$, i.e. $[a + (1)/(2)\rho_p]$ was used to calculate stress intensity factor as

$$K = \sqrt{\frac{\pi \rho_c}{4} + Y \frac{\pi}{2} a \left(\sec \left(\frac{\pi \sigma_{\text{max}}}{2\sigma}\right) + 1\right)}\sigma.$$

(10)

The short fatigue crack growth rate was defined as:

$$\frac{da}{dN} = A(\Delta K_{\text{eff}} - \Delta K_{\text{effth}})^2$$

(11)

where $A$ is material constant, $\Delta K_{\text{eff}} = K_{\text{max}} - K_{\text{op}}$ and $K_{\text{op}}$ (crack tip opening $K = (1 - e^{-\xi})K_{\text{opmax}}$, $0 \leq K_{\text{op}} \leq K_{\text{opmax}}$ provides a means for taking into account the develop-

![Graph](image-url)

Fig. 11. A comparison of calculated and experimental crack growth rates in 1045 steel: (a) plain specimen, (b) specimen containing 0.1 mm hole[62].
ment of closure during the transition period. $K_{op}$ is the crack opening level transition region, $l$ is the length of the short crack, $\kappa$ is the parameter of unit mm$^{-1}$ and $K_{opmax}$ is the crack opening level for a long crack and is a function of stress ratio $R$. Then eq. (11) becomes

$$\frac{da}{dN} = A[\Delta K - (1 - e^{-\kappa l})K_{opmax} - \Delta K_{effth}]^2. \quad (12)$$

By using eq. (10), the above equation becomes

$$\frac{da}{dN} = A\left[\frac{\pi \mu \sigma}{4} + \frac{\pi}{2}a \left(\sec \frac{\pi \sigma}{2 \sigma_y} + 1\right) \Delta \sigma - (1 - e^{-\kappa l})K_{opmax} - \Delta K_{effth}\right]^2. \quad (13)$$

This equation is applicable to both long and short fatigue cracks. The graphical presentation of eq. (13) and the experimental data is shown in Fig. 11. The model predicts crack growth behaviour in a continuum manner, but does not deal with the usual acceleration and deceleration behaviour of short fatigue cracks. Since LEFM is unsuitable for application in short cracks, the $\Delta K$ approach becomes questionable in the analysis of short crack growth. Although McEvily's approach is interesting in its own right, it has not been adopted by other researchers.

2.3. Model based on strength of a slip

del os Rios et al. [2] proposed that crack growth rate is proportional to the strength of the slip band, assuming that from a smooth specimen crack will initiate from slip band. The shear stress ($\tau$) at slip band of length ($L$) and strength ($n_d b$) was assumed as

$$\tau = \frac{\mu n_d b}{L} \quad (14)$$

or

$$\tau = \alpha \tau_{app} - \tau_0 \quad (15)$$

where $\alpha$ is orientation factor, $\tau_{app}$ the applied shear stress, $n_d$ the number of dislocation, $b$ the Burger vector and $\tau_0$ internal friction stress. The growth rate was defined as

$$\frac{da}{dN} = f_1 \frac{\tau L}{\mu} \quad (16)$$

where $\mu$ is the shear modulus, "a" the crack length, $f_1$ the fraction of dislocation on the slip band which takes part in the process of crack extension, and $N$ the number of cycles. After a number of cycle, when the crack length is equal to "a", the length of slip band was shortened to $L - a$, and the crack growth rate was defined as

$$\frac{da}{dN} = f_1 \frac{\tau (L - a)}{\mu}. \quad (17)$$

A comparison of experimental crack growth and predicted growth rate by eq. (17) is shown in Fig. 12.

2.4. Model based on energy release rate

Later, del os Rios et al. [50] developed another model based on load and bulk energy considerations, assuming that the local energy, i.e. the energy at the slip band, should be equal to the crack extension energy for crack propagation. A second consideration of the model was the nature and strength of the barriers that should be overcome to propagate the crack into the next grain. The microprocesses of crack growth need to be related to the mechanics of crack extension. The rate of energy released in a cycle by the system, in order to extend the crack (da), is the energy released rate ($G$) and was defined as

$$G = \frac{dU}{dR} \quad (18)$$

where $U$ is the energy of the slip band and $dU/dR$ is the rate of change of slip band energy with
slip band length. During one cycle the slip band associated with the crack was simulated to a linear distribution of close packed dislocation and was defined as

$$U = \frac{1}{2} \int_{r_0}^{R} \frac{\mu (n_d b)^2}{r} dr$$

(19)

where $\mu$ is the shear modulus, $n_d$ the number dislocation in the slip band, $b$ the Burgers vector, $r_0$ the core radius of the first dislocation, $R$ the slip band length, and $r$ the variable length in front of the crack and along the slip band. Then eq. (19) can be written as

$$U = \frac{\mu (n_d b)^2}{2} [\ln R - \ln r_0].$$

(20)

As the crack length $"a"$ increases, $R$ decreases, and therefore the change of the energy of the system with $"a"$ may be equated to the change of the slip band energy with $R$ and following the Griffith approach as

$$\frac{dU}{dR} = \frac{\mu (n_d b)^2}{2R} = G = \frac{\tau^2}{\mu} \pi a$$

(21)

$$n_d b = (2\pi a R)^{0.5} \frac{\tau}{\mu}.$$ 

(22)

By making the crack extension per cycle proportional to the displacement along the slip band, the crack growth rate was defined as

$$\frac{da}{dN} = f_1(2\pi a R)^{0.5} \frac{\tau}{\mu}$$

(23)
The value of $R$, the effective slip band length, depends on the distance to the next barrier and also on the strength of the barrier which in turn depends on the ratio of the resolved shear stresses along the slip band plane on the two grains separated by the barrier. The argument used by Chan and Lankford [61] to define the change in crack tip plastic strain, was used to define $R$ in eq. (23), i.e.

$$ R = D \left[ 1 - \phi \left( \frac{D - X}{D} \right) \right]^m $$

(24)

where $D$ is the distance from barrier to barrier and $\phi$ is a function related to the relative crystallographic orientation of the two grains; the grain (A) where the crack develops and grain (B) in which the crack will next propagate. Function $\phi$ was defined, in terms of the threshold shear stresses along the slip bands, as

$$ \phi = 1 - \frac{\tau_B}{\tau_A} $$

(25)

$\phi = 1$ when the slip orientation of the next grain is most unfavourable and the crack will arrest. If $\phi = 0$, it corresponds to the case where the orientation of the two grains is similar, i.e. $\tau_A = \tau_B$. The combination of eqs (23)–(25) gives the following:

$$ \frac{da}{dN} = f_t (2\pi aD)^{0.5} \left( 1 - \frac{\phi(D - X)}{D} \right)^m \frac{\tau}{\mu}. $$

(26)

A comparison of the experimental and predicted growth behaviour by eq. (26) is depicted in Fig. 13.

Fig. 13. Application of the model to various short crack cases in banded ferrite-pearlite microstructure of 0.4% C steel; solid line represents predicted growth behaviour [50].
de los Rios et al. in their both models [2, 50], proposed that crack growth rate is proportional to the strength of the slip band [2] and bulk energy considerations, assuming that the local energy, i.e. the energy at the slip band, is equal to the crack extension energy for crack propagation. A second consideration of the model is the nature and strength of the barrier that should be overcome to propagate the crack into the next grain [50]. Both models are successful in predicting growth rate for individual cracks studied from replicas.

In smooth samples there are usually more than one cracks, each growing at different growth rate and facing grain or second phase barriers of different orientation. Failure occurs when these cracks combine into one dominant crack.

2.5. Model based on grain boundary effect

Hobson [64] used a statistical approach to accommodate this factor and proposed two equations, one for “short” crack growth and the other for “physically” small cracks. The short crack growth equation is

$$\frac{da_s}{dN} = C_2(d - a_s)$$

(27)

where “$a_s$” = surface crack length, $C_2$ = material constant in short crack region and $d$ represents the distance to the first microstructural barrier affecting crack growth, which may be a grain boundary or any other metallurgical obstacle. This equation assumes that crack arrest, temporary or permanent, will occur at the grain boundary, when $a = d$ i.e. $(da_s)/(dN) = 0$, depending upon the applied stress level. This equation is dimensionally correct.

The crack growth behaviour of the crack length greater than the length “$d$” was expressed by the following equation:

$$\frac{da_s}{dN} = C_3a_s - D_1$$

(28)

where $C_3$ is the material constant and is a function of stress/strain and $D_1$ represents crack growth threshold. To calculate the value of $D_1$, it was assumed that at the fatigue limit $(da_s)/(dN) = 0$ and that this happens when crack length equals the microstructural barrier distance “$d$”. Taking $a_s = d$, at fatigue limit $(\sigma_{F1})$ eq. (28) becomes

$$0 = C_3(\sigma_{F1})^\mu d - D_1$$

$$D_1 = C_3^\mu(\sigma_{F1})^\mu d.$$  

(29)

Application of eqs (27) and (28), on 0.4% C steel tested at low and high strain rates and on C–Mn steel tested under uniaxial loading is shown in Figs 14, 15 and 16.

Hobson calculated the value of “$d$” for individual crack by plotting the values of $(da_s)/(dN) = 0$ against average $a_s$ on a linear scale. Then a least-square fit was performed on those data points where the growth rate was decreasing in relation to crack length. The value of “$d$” was taken to be that value of $a_s$, where the extrapolated least-square equation intersected the crack length axis. For a physical small crack, crack growth data corresponding to crack lengths greater than the parameter “$d$” was used to derive the physical small crack growth expression. It was assumed that the growth of cracks longer than “$d$” was not affected by microstructure, which results in shorter lifetime prediction than the actual fatigue life. The method used to calculate the value of “$d$” is a numerical procedure, which gives the value of “$d$” different to the actual microstructural length, i.e. the grain size.

The assumptions and method proposed by Hobson yields good agreement with experimental data if the microstructural effect on the short crack behaviour is considered in the first grain only. It has been observed by the author and reported by other researchers that the crack growth behaviour is affected by the microstructures even when the crack is longer than one grain diameter [10, 67–69].
2.6. Model based on plastic zone interaction

Navarro and de los Rios[71] proposed a model which can describe the short and physically small crack growth behaviour by a single equation. The usual behaviour of short fatigue cracks, acceleration and deceleration, is described by successive blocking of the plastic zone by the grain boundaries and the subsequent initiation of the slip in the next grain. Crack growth was defined by

$$\frac{da}{dN} = f_2 \phi_1.$$  \hspace{1cm} (30)

Factor $f_2$ represents the degree of irreversibility of slip on each stress cycle and can be equated to the fraction of dislocation which are drawn into the crack during each reversal of stress cycle. \(\phi_1\) is the crack tip displacement and is expressed as

$$\phi_1 = \frac{2\kappa}{G} \sqrt{\frac{1 - n^2}{n}} \sigma a$$  \hspace{1cm} (31)

where $G$ is the shear modulus, $\kappa = 1$ for screw dislocation and $1 - \nu$ for edge dislocations, "$a$" is the crack length and $\sigma$ is the applied stress. As the crack grows, while the plastic zone is blocked by the grain boundary, the value of "$n$" increases in magnitude up to a critical value ($n_c$), where the stress concentration is sufficiently high to activate a dislocation source in the next grain; this is defined as

$$n_c = \cos \left( \frac{\pi \sigma - \sigma_{Li}}{2 \sigma_{comp}} \right)$$  \hspace{1cm} (32)

$$\sigma_{Li} = \frac{\sigma_{\text{eff}}}{\sqrt{i}}$$
where $\sigma_{\text{comp}}$ is the comparison stress, which is equal to flow stress within the plastic zone, and $\sigma_{\text{FL}}$ is the fatigue limit. As slip initiates in the next grain the plastic zone extends across to the grain boundary. The stress concentration ahead of the new extended plastic zone is defined as

$$n_s = n_c \frac{i}{i+2}$$

(33)

where $i = 1, 3, 5, \ldots$, the number of the half grains. The transfer of slip is repeated grain after grain; crack growth accelerate (after slip is produced in the next grain) and decelerates (as the crack approaches the grain boundary) during the process until failure. However, as the crack approaches failure, the acceleration and deceleration in growth rate decrease.

A schematic of the Navarro–Rios model is shown in Fig. 17, which represents the deceleration and acceleration behaviour at grain boundary and soon after crossing the grain boundary, respectively, along with upper and lower curves. Figures 18 [71] and 19 [72] show the comparison of the experimental and calculated growth behaviour by the Navarro–Rios model. The theory of the Navarro–Rios model is based on the assumption that the microstructural features will affect the growth rate until failure of the specimen. However, it has been observed by the author and reported by other researchers that after a certain length, cracks are not influenced by microstructure. Hussain et al. [67] observed that effect of microstructure is prominent up to the crack length equal to 3–4 grains diameter. Taylor and Knott [10] observed that growth characteristics of those cracks which are shorter than 10 times the grain diameter, depending on the surrounding microstructure. Keiro et al. [68] reported that when the surface crack length is longer than three grain diameters, crack growth rates are not influenced by the microstructure. According to Daebuler and Thompson [69], when a crack changes from stage I to stage II it is no longer affected by microstructure.

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Fig. 15. Experimental and predicted short crack growth rates. Different symbols represent different cracks and solid line shows predicted growth behaviour[66].
2.7. Two-stage micromechanics model

On the basis of the observation discussed above, Hussain et al. [67] proposed a two-stage short crack growth model. The first part of the model deals with short crack growth behaviour dependent on the surrounding microstructure. The stress concentration in this region was defined as in eq. (32). The second part of the model represents physically small crack region (where growth rate is not influenced by its surrounding microstructure). The stress concentration ahead of the crack tip will always attain a value sufficiently high to overcome the barrier strength and therefore the growth rate will be independent of the microstructure. The value of $n_c = n$, which can be calculated using Bilby et al.'s relationship [73], as

$$ n = \cos \left( \frac{\pi}{2} \right) \frac{\sigma}{\sigma_{\text{comp}}} $$

In the short crack region (first stage), where crack growth decreases as the crack approaches the grain boundary and increases when slip is activated in the next grain, the value of $n$ is defined by the position of the crack tip relative to the grain boundary. This is expressed by eqs (32) and (33).

The crack growth rate in terms of the surface crack length is expressed as

$$ \frac{da_s}{dN} = f_3 \phi^* $$

![Fig. 16. Experimental (solid curves) and predicted (dashed curve) short crack growth behaviour [65].](image-url)
where

\[ \phi^* = \frac{\kappa \sqrt{1 - n^2}}{G} \frac{\Delta \sigma \cdot a_l}{n}. \]  \hspace{1cm} (36)

In the Hussain–Rios[67] two-stage model, the expected deceleration and acceleration in the crack growth, as the crack approaches grain boundaries and where slip is transmitted to the next grain, is assumed to occur solely for the first three to four grains. The model predicts not only the author's experimental observations but also supports the findings of others that short crack growth is affected by the grain boundary up to a certain number of grains. The model can be used to predict short fatigue crack growth behaviour in uniaxial as well as in torsional loading. The constant \( \kappa = 1 \) for screw dislocation (uniaxial loading) and \( \kappa = 1 - \nu \) for edge dislocation (torsional loading). The experimental crack growth data and predicted curve is shown in Figs 20 and 21, both in torsional and uniaxial loading.

The Hussain–Rios[67] model was latter modified to define growth behaviour of individual crack. The crack length with minimum growth rate was considered to be the length \( (d) \) between the barriers to growth. This length was found different for different cracks and it varied within the one crack depending upon the size of the grains through which it propagated. To accommodate this experimental observation, the parameter “\( n \)” for a particular length, in short crack region, of an individual crack was defined as[74]:

Fig. 17. Schematic of oscillating crack tip plastic displacement and upper and lower limit curves[71].
Equation (37) was used in eq. (36). The factor \( f_2 \) was calculated for each stress level. The curves in Fig. 22 were calculated using eqs (35)–(37). The calculated growth rates match reasonably well with the experimental results and showed a systematic decrease in growth rate at grain boundaries.

3. SUMMARY

The propagation behaviour of long fatigue cracks is uniquely determined by the range of elastic stress intensity factor \( \Delta K \). However, when the crack length is small, the growth rate is no longer predictable from conventional \( \Delta K \)-based fracture mechanics. Short cracks with dimensions on the order of the material microstructure (grain size) are called microstructurally short/small cracks. They show anomalously high growth rates when compared with long cracks at the same \( \Delta K \) value. The assumption of macroscopic continuum in fracture mechanics breaks down for these microstructurally short cracks.

For microstructurally short cracks, the resistance to crack growth is much influenced by the material microstructure such as grain orientation and grain boundary. The crack growth rate will be different even when the crack is subjected to the same loading parameter of continuum mechanics, \( \Delta K \) and \( \Delta J \). Local parameters will be necessary to express the crack driving force [44]. Tanaka et al. [75] calculated \( \Delta \text{CTOD} \) and derived irregular growth of microstructurally small cracks by assuming the crack growth range as a power function of \( \Delta \text{CTOD} \). Cracks decelerate as they approach the grain boundary. When the crack length is larger than several
times the grain size (mechanically short crack), the resistance of the materials is regarded as homogeneous. For such cracks an appropriate choice of fracture mechanics parameters such as $\Delta K_{\text{eff}}$ or $\Delta J$ will make crack growth prediction possible. All of the crack closure mechanisms indicate that the crack wake made by fatigue is responsible for crack closure. Since a short crack has a short wake, the amount of crack closure is expected to be smaller and therefore, the growth rate is higher for short cracks. Premature crack closure plays a dominant role in anomalous, fast growth behaviour of mechanically short cracks[44].

Morris et al.[76] proposed a model of roughness-induced closure of short cracks, by considering the measured value of surface roughness. The analysis of the plasticity-induced closure of short cracks indicates that the opening stress is lower under higher amplitude of stress cycling[77]. The acceleration of crack growth due to a small amount of macro-plasticity is predictable from an increase of $\Delta K_{\text{eff}}$ value. When the amount of cyclic plasticity is large, crack closure adjustment is not enough; some elastic-plastic mechanics parameters are necessary to evaluate the crack growth rate. Among several parameters proposed, the $J$-integral appears to be most successful. The $\Delta J$ can be estimated from measurement of the cyclic hysteresis loop and crack closure[6,78]. The amount of crack closure is smaller for shorter cracks under larger plastic strain. Hoshide et al.[78] attributed the growth acceleration of microstructurally short cracks
Surface crack length, μm

Crack growth rate, μm/cycle

Eq. 32

Eq. 34

Fig. 20. Crack growth rate vs surface crack length in torsion; Δt = 319.5 MPa. The curve represents predictions using two-stage model. Different symbols represent experimental behaviour of different cracks [67].

to the difference in crack growth micromechanisms. A dominant mechanism is the intergranular fracture for short cracks, while the transgranular fracture with striations for long cracks.

Any predictive method of the crack growth behaviour of microstructurally small cracks must include the statistical nature of the interaction of cracks with the material microstructure.

Chan and Lankford [61] model is based upon the concept of greater plastic strain and the grain orientation. The reduced crack tip strain at grain boundaries causes deceleration in micro-crack growth rate. However, if the orientation of the adjacent grains are same, the model predicts little or no decrease in growth rate.

The model proposed by McEvily et al. predicts crack growth behaviour in a continuum manner, but does not deal with the usual acceleration and deceleration behaviour of short fatigue cracks. Since LEFM is unsuitable for application in short cracks, the ΔK approach becomes questionable in the analysis of short crack growth.

de los Rios et al. in their models proposed that crack growth rate is proportional to the strength of the slip band [2] and bulk energy considerations, assuming that the local energy, i.e. the energy at the slip band, should be equal to the crack extension energy for crack propagation. A second consideration of the model is the nature and strength of the barrier that should be overcome to propagate the crack into the next grain [50]. Both models are successful in predicting growth rate for individual cracks.

The assumptions and method proposed by the Hobson model yields a good agreement with experimental data if the microstructural effect on the short crack behaviour is considered in the first grain only. Hobson calculated the value of “d” for individual crack by plotting the values
Fig. 21. Crack growth rate vs surface crack length in push-pull; $\Delta \sigma = 626.8$ MPa. The curve represents predictions using two-stage model. Different symbols represent experimental behaviour of different cracks [67].

of $\frac{da}{dN}$ against average $a_s$ on a linear scale. Then a least-square fit was performed on those data points where the growth rate was decreasing in relation to crack length. The value of "$d$" was taken to be that value of $a_s$, where extrapolated least-square equation intersected the crack length axis. For physical small crack, crack growth data corresponding to crack lengths greater than the parameter "$d$" was used to derive the physical small crack growth expression. It was assumed that the growth of cracks longer than "$d$" was not affected by microstructure, which results in shorter lifetime prediction than the actual fatigue life. The method used to calculate the value of "$d$" is a numerical procedure, which gives the value of "$d$" different to the actual microstructural length i.e the grain size.

In the Navarro–Rios model the microstructural influence in the short crack regime extends into the physically small crack regimes. For longer crack lengths, the difference between the upper and lower crack growth curves is negligible and the theory is similar to Hobson's approach.

Hobson considers only one barrier in the short crack regime. It appears simpler than the Navarro-Rios model, because it considers the average growth rate, ignoring the crack growth variations near the grain boundaries. In some respects it is more difficult since two parameters for each equation have to determined, including the separation of the data into two regimes, to apply the model.

The Navarro–Rios model relies only on using the data from the physically small crack regime to derive the expression "$f_2$" in the terms of applied stress, which in turn is used to
describe the crack growth in short and physically small crack regions. The theory behind the Navarro–Rios model is based on assumption of the continuous effect of microstructures up to failure. This assumption becomes questionable in the presence of the experimental observation that microstructural effect on growth behaviour is up to a certain length of crack.

The Hussain–Rios two-stage model is in fact a combination of the Hobson and Navarro–Rios methods. Both its theory and application are simpler. In this approach the expected deceleration and acceleration in the crack growth is assumed to occur solely for the first few grains. The factor "f2" is calculated from second-stage crack data, where grain boundary effects on growth are assumed to be zero, and is used to describe the short crack growth behaviour which considers the crack tip grain boundaries interaction. The model is easier and gives fatigue life prediction comparable with experimental fatigue life. See Table 1 for a summary of the models. For a detailed comparison between the Hobson, Navarro–Rios and Hussain–Rios models see Ref.[67].

Table 1. Comparison of fatigue lifetime under torsional loading predicted by different models[67] for a final crack size of 4 mm. The constant used for the calculation are f2, C2 and C3.

<table>
<thead>
<tr>
<th>Δσ (MPa)</th>
<th>Actual life time</th>
<th>Hussain–Rios model</th>
<th>Hobson model</th>
<th>Navarro–Rios model</th>
</tr>
</thead>
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<tr>
<td></td>
<td>f2</td>
<td>Lifetime</td>
<td>C2</td>
<td>C3</td>
</tr>
<tr>
<td>417.58</td>
<td>1.35 × 10^-1</td>
<td>11 976</td>
<td>9.11 × 10^-5</td>
<td>29102</td>
</tr>
<tr>
<td>369.58</td>
<td>2.12 × 10^-2</td>
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<td>2.35 × 10^-3</td>
<td>121 198</td>
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Fig. 22. Comparison of experimental and calculated growth behaviour of short crack at different stress levels[74].
REFERENCES


(Received 29 October 1996, in final form 31 July 1997, accepted 4 August 1997)