Fluid Turbulence

Problem 9.1, page 350.

(i) In the example in the text, the Courant number is restricted as (Eqn. 9.10):

$$\frac{k^{1/2}\Delta t}{\Delta x} = \frac{1}{20}\,.$$

Suppose that Δt is instead restricted as

$$\frac{\Delta t}{\tau_\eta} = 0.1 \ \text{ with } \ \tau_\eta = \left(\frac{\nu}{\epsilon}\right)^{1/2}.$$

Then with $\mathcal{T} = k/\epsilon$, and running to time $T = 4\mathcal{T}$, the number of time steps is:

$$\begin{aligned} M &= \frac{T}{\Delta t} = \frac{4\mathcal{T}}{\Delta t} = \frac{4k}{\epsilon}\frac{1}{0.1\tau_\eta} = \frac{4k}{\epsilon}\frac{1}{0.1(\nu/\epsilon)^{1/2}} \\ &= \frac{4\cdot\frac{3}{2}u'^2}{15\nu(u'^2/\lambda_g^2)}\cdot\frac{1}{0.1(\lambda_g^2/15u'^2)^{1/2}} = \frac{6\sqrt{15}}{0.1\cdot 15}\frac{u'\lambda_g}{\nu} = 4\sqrt{15}R_\lambda\,, \end{aligned}$$

i.e., $M = 4\sqrt{15}R_\lambda$.

(ii) Using Eqn. 9.9 in the text, $N^3M \sim 0.06R_\lambda^{9/2}\cdot 4\sqrt{15}R_\lambda \doteq 0.93R_\lambda^{11/2}$, i.e., $N^3M \sim 0.93R_\lambda^{11/2}$.

(iii) Following Eqn. 9.13 in the text to compute the time in days at 1 gigaflop,

$$T_G = \frac{10^3N^3M}{10^9\cdot 60\cdot 60\cdot 24} \sim \frac{10^3\cdot 0.93R_\lambda^{11/2}}{10^9\cdot 60\cdot 60\cdot 24} = 1.08\cdot 10^{-11}R_\lambda^{11/2} \doteq \left(\frac{R_\lambda}{100}\right)^{11/2}, \text{ i.e.,}$$

$$T_G \sim \left(\frac{R_\lambda}{100}\right)^{11/2}.$$