SOME AMPLIFIER CHARACTERISTICS EXPERIMENTALISTS NEED TO KNOW

I. Background Discussion: Periodic Waveforms and Fourier Series Expansion

• A fundamental goal of experimental stress analysis is to measure the "response" of a structure to external loading. Some common requirements are to measure the displacement, acceleration, or strains induced within a structure by the expected service loading, for example.

• These physical phenomena are typically measured using a "transducer" of some sort (strain gage w/Wheatstone bridge, accelerometer, LVDT, etc) that transforms the physical phenomenon of interest into an analogous electrical signal. The magnitude of the physical phenomenon is (usually) represented by the voltage level of the analogous signal.

• If the load(s) applied to the structure changes with time, then the physical phenomena being measured (displacement, acceleration, strain, etc) will also change with time...a plot of the measurement versus time is called a "waveform".

• The type of waveforms encountered in practice may be classified into two main categories:

I. "Deterministic" Waveforms ...waveforms for which knowledge of past behavior permits prediction of future behavior. Deterministic waveforms are describable in a mathematical sense. Two types of deterministic waveforms can be defined:

a) periodic waveforms - signals that continue "indefinitely", and furthermore repeat themselves *in their entirety* over some characteristic length of time called the "fundamental period", *T*. An example could be the strain induced in the crankshaft of an internal combustion engine, and in this case the period would likely be associated with the engine speed. A typical periodic waveform is shown in Figure 1.

b) aperiodic waveforms - signals that do not continue indefinitely (example: damped oscillation of a cantilever beam)

II. "Non-deterministic" Waveforms...waveforms for which knowledge of the past only permits a "guess" of future behavior. Non-deterministic waveforms can only be described mathematically in a probabilistic sense. Two types of non-deterministic waveforms can be defined:

a) stationary - a random signal whose mean value does not change with time (interior noise levels in a passenger automobile traveling at highway speeds)

b) non-stationary - a random signal whose mean value changes with time

• Amplifier characteristics are described based on how they modify periodic waveforms

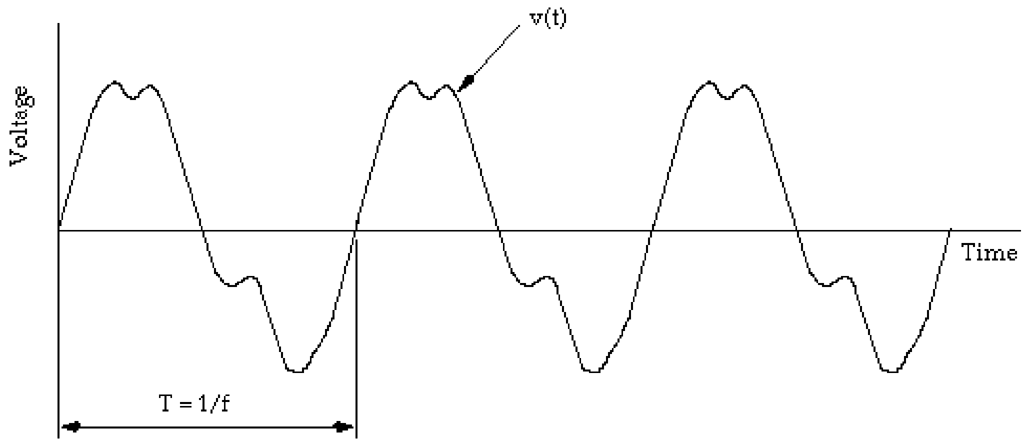


Figure 1: A Typical Periodic Waveform

Mathematical Description of a Sinusoidal Waveform

• The characteristic features used to describe any periodic waveform can be traced to the properties of a sinusoidal waveform (which is itself a simple periodic waveform):

Referring to Figure 2:

Amplitude = A (volts, say)

Period = T (time/cycle; seconds/cycle, say)

Frequency = $f = 1/T$ (cycles/time; most common is cycles/sec = 1 Hertz = 1 Hz)

Angular frequency = $\omega = 2\pi f = 2\pi / T$ (radians/time; most common is radians/sec)

Phase Lag = λ (time; seconds, say)

Phase angle = $\phi = 2\pi f\lambda = 2\pi\lambda / T$ (radians)
(The phase angle is the fraction of a cycle that a sinusoidal signal "lags behind"...)

Based on these definitions, a sinusoid can be expressed mathematically in any of several equivalent forms, for example:

$$v(t) = A\sin\left[2\pi f(t-\lambda)\right] = A\sin\left[\frac{2\pi}{T}(t-\lambda)\right] = A\sin(\omega t - \phi)$$

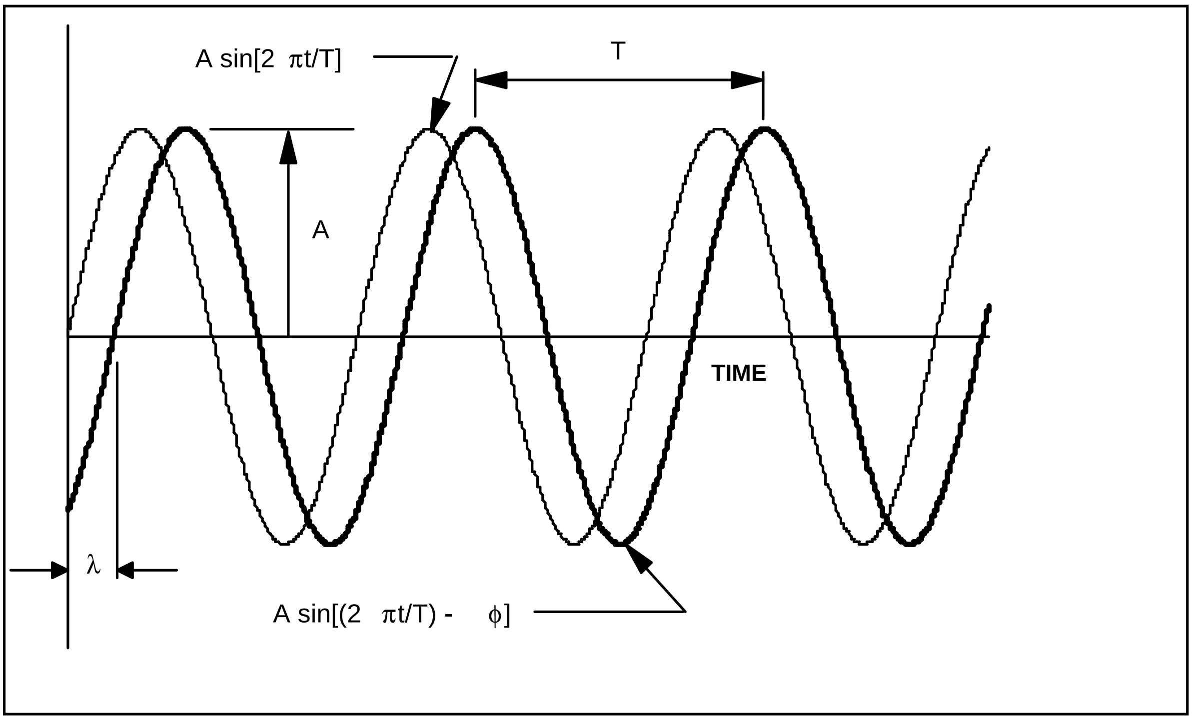


Figure 2: A Sinusoidal Periodic Waveform

Fourier Series Expansion

Any periodic waveform can be represented as an infinite series of sine and cosine terms, by means of a Fourier series expansion:

$$v(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega_o t) + b_n \sin(n\omega_o t)\right]$$

where:

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} v(t)\cos(n\omega_o t)dt, \quad n = 0, 1, 2, 3,...$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} v(t)\sin(n\omega_o t)dt, \quad n = 1, 2, 3,...$$

$$\omega_o = \frac{2\pi}{T} = 2\pi f = \text{fundamental angular frequency}$$

$T =$ fundamental period

$f =$ fundamental frequency

Fourier Series Expansion of a Square Wave

For a square wave referenced to the time axis as shown in Figurer 3 (i.e., note that the origin has been selected such that the wave is an "odd" function), the Fourier series expansion becomes:

$$v(t) = \sum_{n=1}^{\infty}\left[\frac{4A}{n\pi}\sin(2\pi nft)\right] = \sum_{n=1}^{\infty}\left[\frac{4A}{n\pi}\sin(n\omega_o t)\right], \quad n = 1, 3, 5, 7,.... \qquad (1)$$

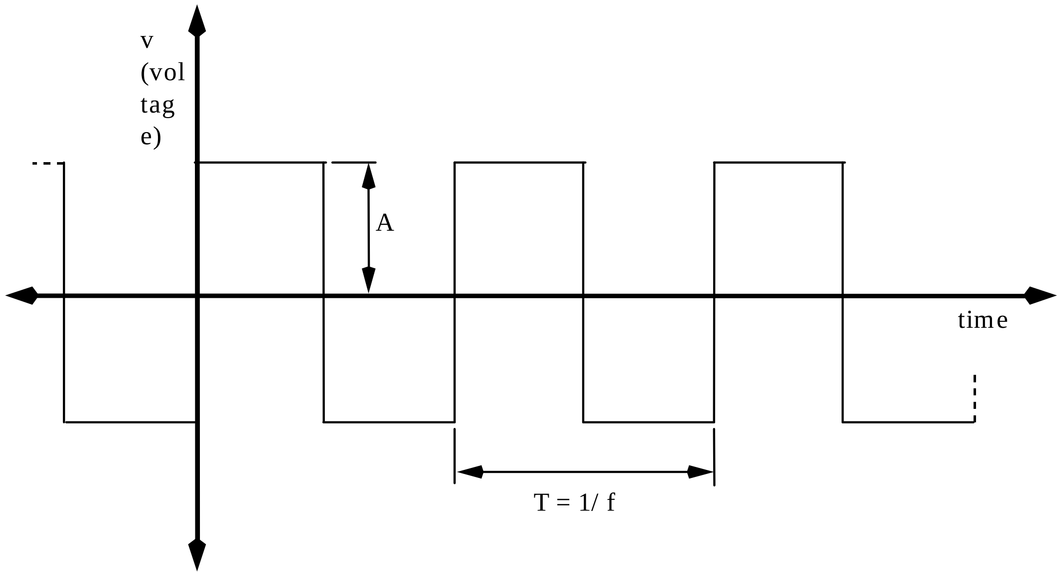


Figure 3: A Periodic Square Wave

The Fourier series expansion (i.e., Eq 1) shows that the square wave can be represented as the sum of an infinite number of sine terms, all of which are "in phase" (i.e., $\phi_n = 0$ at all frequencies). Figure 3 is a depiction of a square wave in the "time domain". Equivalently, a square wave can be plotted in the "frequency domain," as shown in Figure 4:

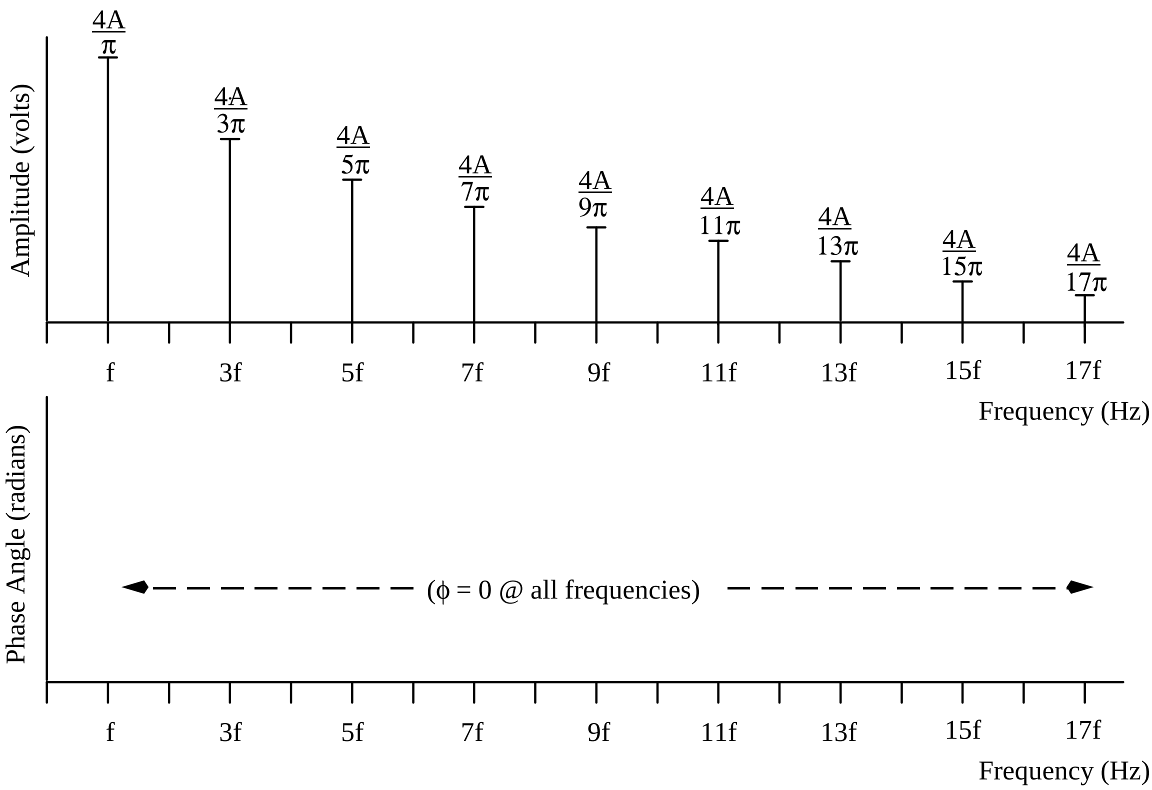


Figure 4: A Square Wave in the Frequency Domain

Suppose we wish to "reconstruct" a square wave, based on knowledge of (amplitude,phase) pairs at specified frequencies. Since we cannot use an *infinite* number of terms, we must truncate the series expansion. For example, if only one term is used (n = 1), Eq (1) becomes:

$$v(t) = \frac{4A}{\pi}\sin(\omega_o t)$$

A plot of a 1-term expansion (with A = 1 volt) over one cycle is shown in Figure 5, and is not very satisfactory. That is, since only one term was used the series expansion of the "square" wave is nothing more than a pure sinusoidal wave. If two terms are used instead (n = 1, 3), Eq (1) becomes:

$$v(t) = \frac{4A}{\pi}\sin(\omega_o t) + \frac{4A}{3\pi}\sin(3\omega_o t)$$

As shown in Figure 5 the two-term expansion is an improvement over the 1-term expansion. Additional plots are shown in Figure 5 for a 10-term expansion (n = 1, 3, 5, ...19) and a 100-term expansion (n = 1, 3, 5, ...199). Clearly, the Fourier series representation of a square wave is dramatically improved as the number of terms used in increased. That is, the representation becomes better and better as terms at higher and higher frequencies are included.

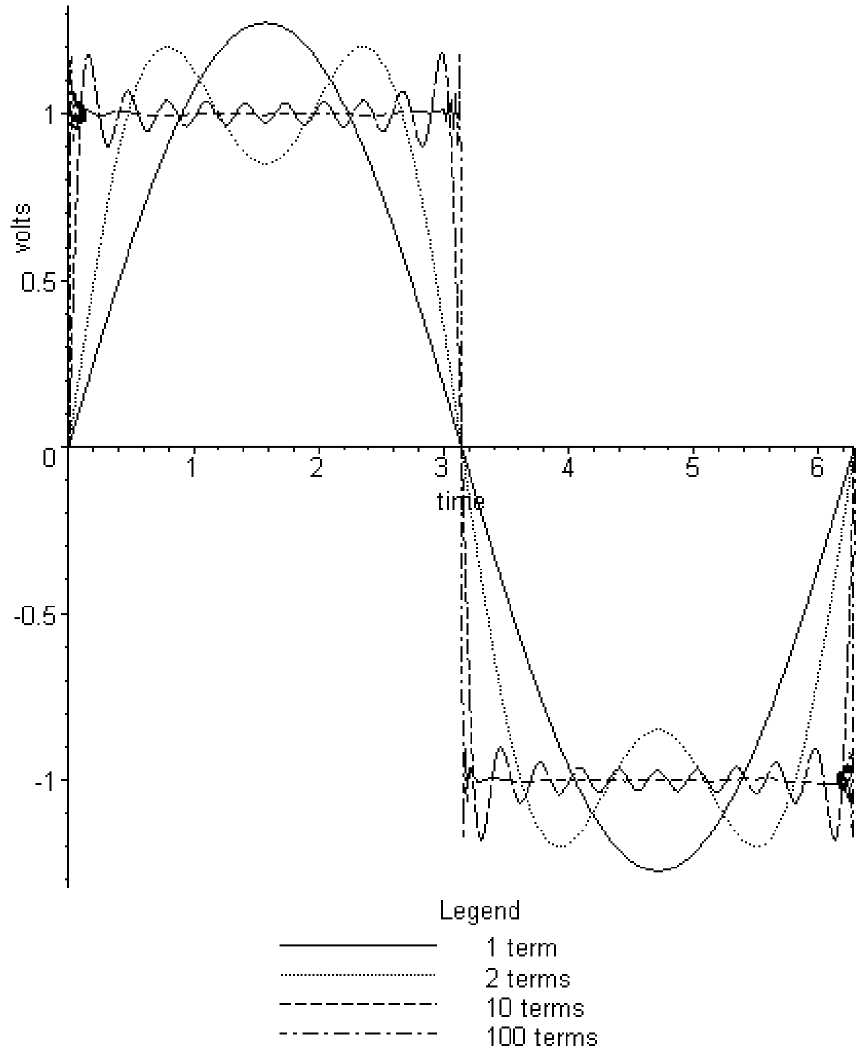


Figure 5: Fourier Series Expansion of a Square Wave, Based on a Different number of Terms

II. Some Basic Amplifier Characteristics

• Amplifier Gain and Phase: Consider a sinusoidal voltage given by:

$$v_i(t) = A_i \sin(\omega t) \quad (2)$$

If this voltage is input into an amplifier, it will (in general) be modified in two ways:

(a) The input amplitude (A_i) will be increased, and

(b) A phase lag (λ), or equivalently, a phase angle (ϕ) will be introduced...generally, phase angle increases with angular frequency, ω

Thus, the output signal produced by the amplifier and corresponding to the input signal given by Eq (2) can be written:

$$v_o(t) = A_o \sin(\omega t - \phi)$$

(Note: the amplitude of the output signal is usually greater than the amplitude of the input signal, $A_o > A_i$, and hence the input signal is said to be "amplified". In contrast, if the amplitude of the output signal is less than the amplitude of the input signal, $A_o < A_i$, the input signal is said to be "attenuated".)

The gain of the amplifier (sometimes called the “linear gain”) is defined as:

$$Gain = \left|\frac{A_o}{A_i}\right|$$

Amplifier gain is most often reported in terms of decibels:

$$Gain\ dB = 10\log_{10}\left(\left|\frac{A_o}{A_i}\right|^2\right) = 20\log_{10}\left|\frac{A_o}{A_i}\right|$$

For example, if a sinusoidal voltage signal with an amplitude of 10 mV is amplified and the resulting output signal has an amplitude of 1 V, the amplifier gain is 100, or equivalently, the gain is 40 dB.

An "ideal" amplifier would

(a) exhibit a constant (i.e., "flat") gain at all frequencies and

(b) introduce no phase angle ($\phi = 0$ at all frequencies).

Real amplifiers cannot achieve this ideal performance, and in practice both the gain and phase angle of an amplifier vary with frequency. Idealized plots of typical amplifier gain and phase angle characteristics plotted agains the log of frequency are shown in Figure 6:

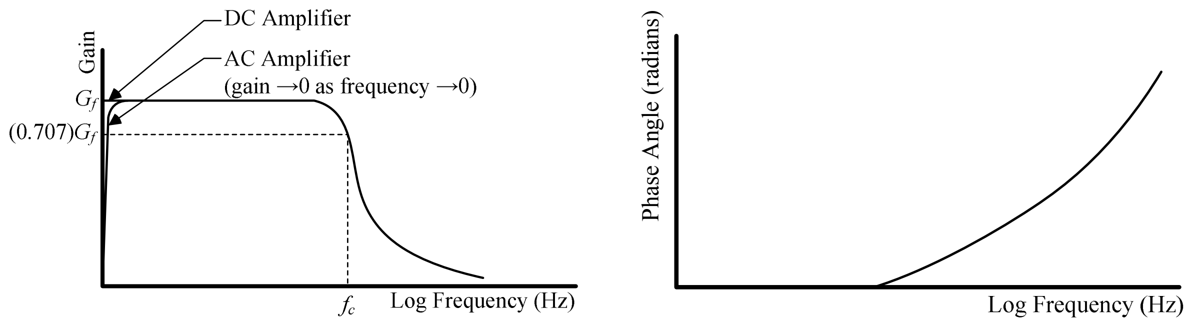


Figure 6: Idealized plot of typical amplifier gain and phase characteristics

Note that the gain of an "AC amplifier" is zero at low frequency (i.e., gain → 0 as $f \to 0\,Hz$), whereas for DC amplifier the gain remains constant at zero frequency; a DC amplifier will amplify a DC voltage.

The combined plots of gain and phase are called "Bode plots" and illustrate the so-called "transfer function" of the amplifier. In general, amplifier gain is constant ("flat") at low frequencies, but decreases at high frequencies, eventually becoming zero at very high frequencies. Also, an amplifier typically introduces little/no phase angle at low frequencies, but introduces a significant phase angle at high frequencies.

In practice, the non-ideal behavior of real amplifiers implies that:

(a) (in the time domain) depending on the frequencies involved, the "shape" of the amplified output signal may not resemble the "shape" of the input signal,

or equivalently,

(b) (in the frequency domain) the amplitudes of the frequency components present in the amplified output signal may not be in the same proportion as in the input signal, and furthermore the phase angles of high-frequency components may be shifted relative to the low-frequency components

In short, the non-ideal amplifier characteristics shown in Figure 6 may result in distortion of the amplified output signal.

The range of frequencies overwhich the gain of an amplifier is more-or-less constant (flat) must be known in order to properly select an amplifier for use. The most common measure of this frequency range is the so-called "cutoff frequency", f_c, which is defined as frequency at which the gain has decreased to a value of:

$$(1/\sqrt{2})(Gain\ at\ low\ frequency) = 0.707G_f$$

The "frequency response" of an amplifier typically refers to the cutoff frequency, f_c. Alternatively, if gain is reported in terms of decibels, then the gain at the cutoff frequency has decreased by 3 dB compared to the gain at low frequencies...that is:

$$20\log_{10}\left(\frac{0.707G_f}{G_f}\right) = -3dB$$

Hence, the cutoff frequency is also called the "3dB down frequency". The cutoff frequency is illustrated in Figure 6.

• Maximum Output Voltage and "Chopping": An idealized plot of output voltage vs input voltage is shown in Figure 7. As indicated, any real amplifier can output some maximum voltage level, v_o^{max}, regardless of the gain setting and/or input voltage. Below v_o^{max} the input and output voltages are linearly related by the amplifier gain; that is, below v_o^{max} the slope of the v_o vs v_i curve equals the amplifier gain. In most cases the gain of an amplifier is adjustable. Therefore, taken together the gain setting and v_o^{max} dictate the maximum input voltage: $v_i^{max} = v_o^{max} / gain$.

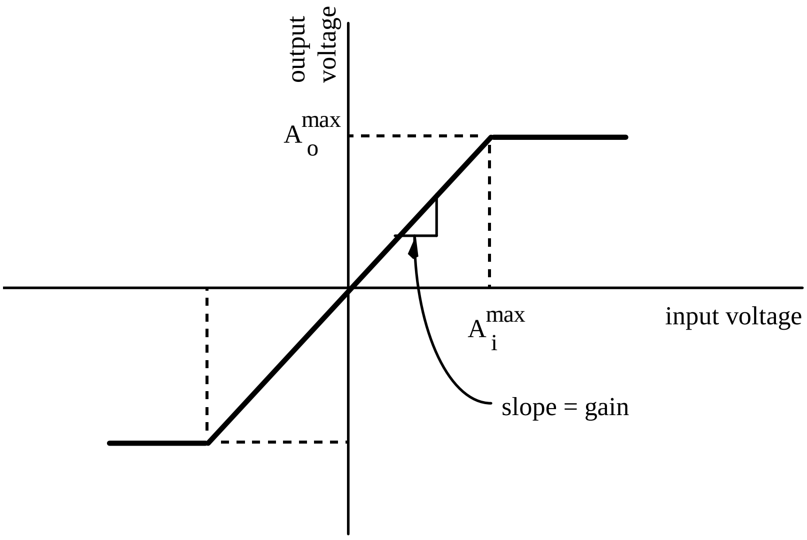
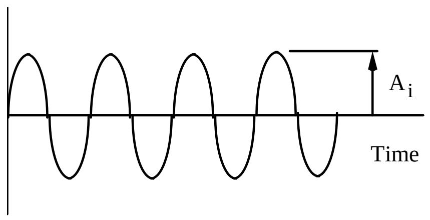


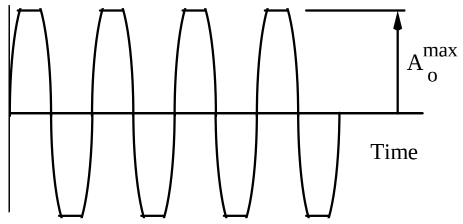
Figure 7: Idealized plot of output voltage vs input voltage

Suppose: (a) a sinusoidal signal is input into an amplifier and
(b) the amplifier gain setting is such that the maximum output signal is exceeded.

In such a case the amplified signal is distorted via "chopping". This phenomenon is illustrated in Figure 8.



Input Signal



"Chopped" Output Signal

Figure 8: Illustration of the "chopping" phenomenon

• <u>Signal-to-Noise (S/N) Ratio:</u> As discussed above, for a given gain setting there is a maximum input voltage ($v_i^{\max}$) that can be amplified without chopping. Conversely, there is also a *minimal* input voltage, $v_i^{\min}$, that can be sensed by an amplifier. That is, at voltage levels below $v_i^{\min}$ the input voltage cannot be distinguished from extraneous/spurious system "noise". An (idealized) plot of $v_i^{\max}$ and $v_i^{\min}$ as a function of frequency is shown in Figure 9. As indicated, $v_i^{\min}$ often exhibits modest "peaks" at discrete frequencies, usually associated with the frequency of the electrical power source used to operate the amplifier (in the U.S.A., 60 Hz).

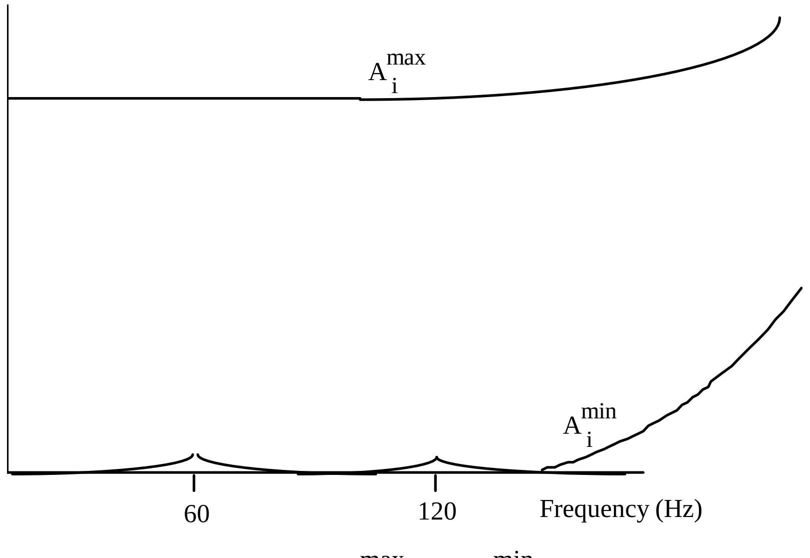


Figure 9: Idealized plot of $v_i^{\max}$ and $v_i^{\min}$ as a function of frequency

The signal-to-noise (*S/N*) ratio is defined as the *power* ratios associated with the maximum and minimum input voltages, expressed in decibles:

$$S / N = 10\,\mathbf{log}_{10}\left(\frac{A_i^{\mathbf{max}}}{A_i^{\mathbf{min}}}\right)^2 = 20\,\mathbf{log}_{10}\left(\frac{A_i^{\mathbf{max}}}{A_i^{\mathbf{min}}}\right)$$

Noe that since:

(a) $v_i^{\max}$ depends on gain setting, and

(b) both $v_i^{\max}$ and $v_i^{\min}$ depend on frequency

the *S/N* ratio depends on both gain and frequency....often the *S/N* ratio is reported at maximum gain and at 1000 Hz, but occasionally the *S/N* ratio will be reported at other gain settings and/or other frequencies.

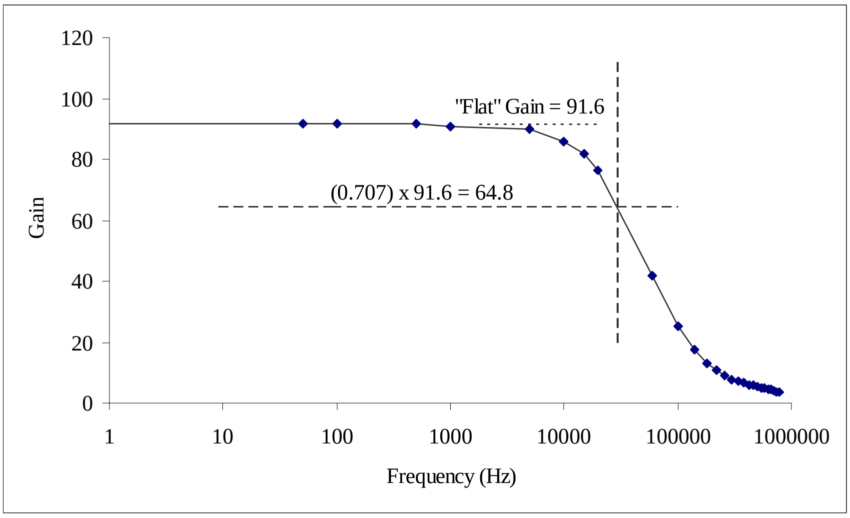
Example: Distortion of a Square Wave Due to Amplifier Gain/Phase Characteristics

A photo of a 1960's-era battery-powered strain gage amplifier produced by Ellis Associates and known as the "BAM-1" is shown in Figure 10. The gain and phase characteristics of this amplifier measured at the UW are shown in Figure 11. The amplifier has a flat gain of about 91.6, and a cut-off frequency of about 29.4 kHz. The amplifier introduces a measureable phase angle at frequencies higher than about 5000 Hz. (Note: modern strain gage amplifiers may exhibit higher *or lower* cutoff frequencies. For example, the strain gage sensor cards used with the System 5000 data acquisition system (controlled by the Strain Smart software package) has a cut-off frequency of only 5 Hz)

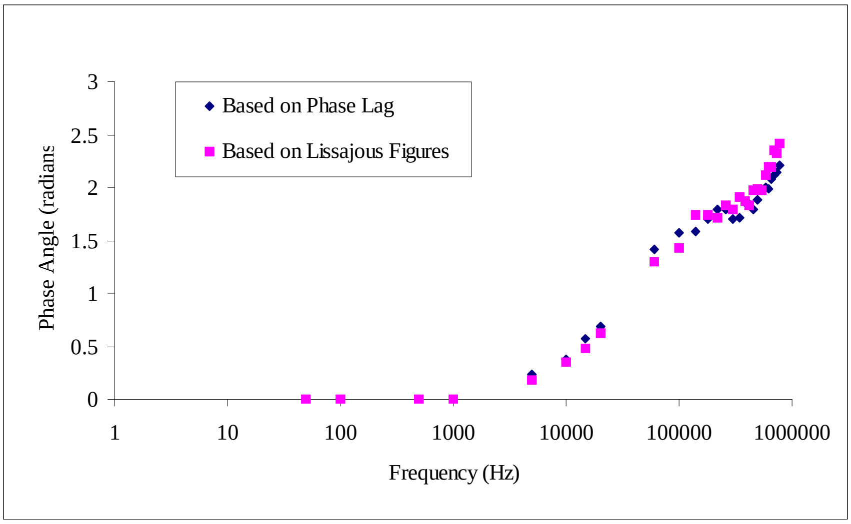
Suppose the BAM-1 amp is used to amplify a 50 mV, 20 kHz square wave. Since the 20 kHz square wave contains frequency components of significant amplitude well above the cut-off frequency of 29.4 kHz, and since the amplifier introduces a phase angle that increases at frequencies above 5000 Hz, the output signal will be distorted and will not resemble a square wave. The amplified signal produced by the BAM-1 can be predicted by combining the plot of the input square wave in the frequency domain (as previously shown in Figure 4, where in this case A = 50 mV and f = 20 kHz) with the gain and phase of the BAM-1 amplifier shown in Figure 11.

The first 20 terms of the Fourier series expansion for both the input and output signals are summarized in Table 1. As implied in Table 1, the predicted output signal is obtained by multiplying the amplitude of a given frequency component by the amplifier gain at that frequency, and by adding the phase angle introduced by amplifier. For example, from Figure 4 it is seen that the third term in Fourier series expansion for the 50 mV, 20 kHz square wave is a component at (5)(20 kHz) = 100 kHz, with an amplitude of $4A/5\pi = 4(50mV)/5\pi$. From Figure 11 it is seen that at 100 kHz the gain and phase angle of the BAM-1 amplifier is 25.0 and 1.5 radians, respectively. Hence, the third term in the Fourier series expansion for the output signal is a component at 100 kHz with an amplitude $25.0(4)(50mV)/5\pi = 1.0/\pi$ V and phase angle of 1.5 radians.

Figure 10: An Ellis BAM-1 strain gage amplifier



(a) Gain characteristics



(b) Phase characteristics

Figure 11: Gain and phase characteristics for a BAM-1 strain gage amplifier (measured in 1999)

The last two columns in Table 1 represent the amplified output signal in the frequency domain. The amplified signal can be "reconstructed" in the time domain by summing all terms and plotting the signal over one period. Such a plot is shown in Figure 11

Prediction of the distorted square wave may seem to be an academic exercise. After all, the fact that the amplifier has distorted the input square wave is immediately obvious. The point of this demonstration is as follows: in actual practice the *shape of the input waveform is not known*. Hence, if the BAM-1 amplifier were used to monitor a waveform that, unbeknownst to the user, contains frequency components of significant amplitude at or above the cut-off frequency, then the measured waveform will not be an accurate representation of the actual phenomenon that the user is attempting to measure.

Table 1: Summary of first 20 terms of the Fourier series expansion of 50 mV, 20 kHz square wave amplified using BAM-1 strain gage amplifier

	Input Signal		Amplifier Characteristics		Output Signal	
Frequency (kHz)	Amplitude (mV)	Phase (radians)	Gain	Phase Angle (radians)	Amplitude (V)	Phase Angle (radians)
20	$4(50)/\pi$	0	76.4	0.66	$15.3/\pi$	0.66
60	$4(50)/3\pi$	0	42.0	1.4	$2.8/\pi$	1.4
100	$4(50)/5\pi$	0	25.0	1.5	$1.0/\pi$	1.5
140	$4(50)/7\pi$	0	17.5	1.7	$0.50/\pi$	1.7
180	$4(50)/9\pi$	0	13.2	1.7	$0.29/\pi$	1.7
220	$4(50)/11\pi$	0	11.0	1.8	$0.20/\pi$	1.8
260	$4(50)/13\pi$	0	9.1	1.8	$0.14/\pi$	1.8
300	$4(50)/15\pi$	0	7.8	1.8	$0.10/\pi$	1.8
340	$4(50)/17\pi$	0	7.2	1.8	$0.085/\pi$	1.8
380	$4(50)/19\pi$	0	6.6	1.9	$0.070/\pi$	1.9
420	$4(50)/21\pi$	0	6.0	1.9	$0.057/\pi$	1.9
460	$4(50)/23\pi$	0	5.8	1.9	$0.050/\pi$	1.9
500	$4(50)/25\pi$	0	5.4	1.9	$0.043/\pi$	1.9
540	$4(50)/27\pi$	0	5.0	2.0	$0.037/\pi$	2.0
580	$4(50)/29\pi$	0	4.8	2.1	$0.033/\pi$	2.1
620	$4(50)/31\pi$	0	4.6	2.1	$0.030/\pi$	2.1
660	$4(50)/33\pi$	0	4.3	2.1	$0.026/\pi$	2.1
700	$4(50)/35\pi$	0	4.2	2.2	$0.024/\pi$	2.2
740	$4(50)/37\pi$	0	3.8	2.2	$0.021/\pi$	2.2
780	$4(50)/39\pi$	0	3.7	2.3	$0.019/\pi$	2.3

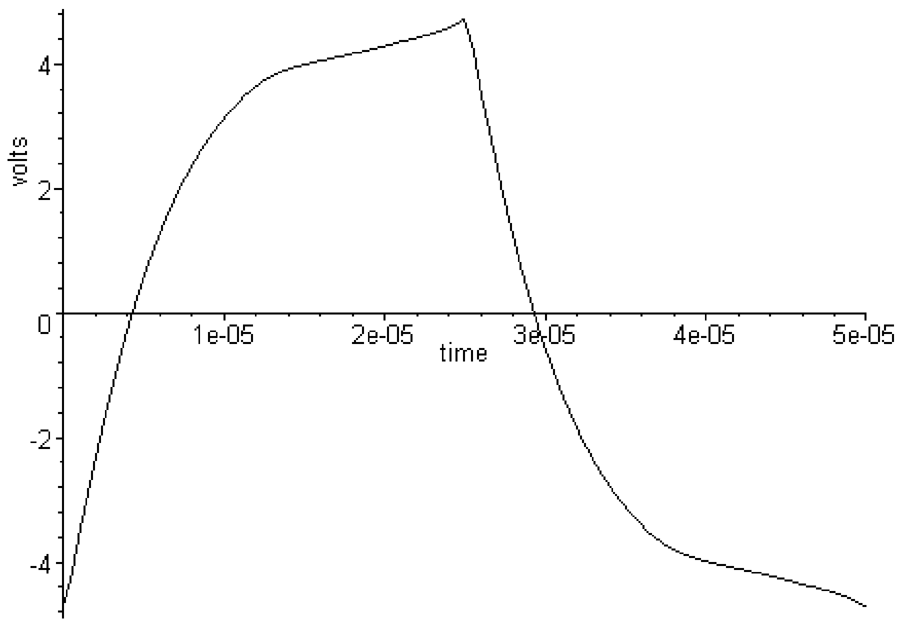


Figure 12: Predicted shape of of 50 mV, 20 kHz square wave amplified using a BAM-1 strain gage amplifier