ME556 LAB EXPERIMENT #4

A LOAD CELL BASED ON A PROVING RING

General Description: The experimental arrangement is shown in Figure 1. The "proving ring" consists of a more-or-less circular member, with inner radius of 1.25 in and outer radius of 1.50 in. The ring has a depth of 0.50 in, and is made of 2024-T4 aluminum: use E = 10.6 Msi, $\nu = 0.32$.

Four uniaxial strain gages (gages G_1 - G_4) have been bonded to the ring at the locations defined in Figure 1, and wired together in a Wheatstone bridge as shown in Figure 2. During the experiment you will make measurements using three different excitation voltage levels, V.

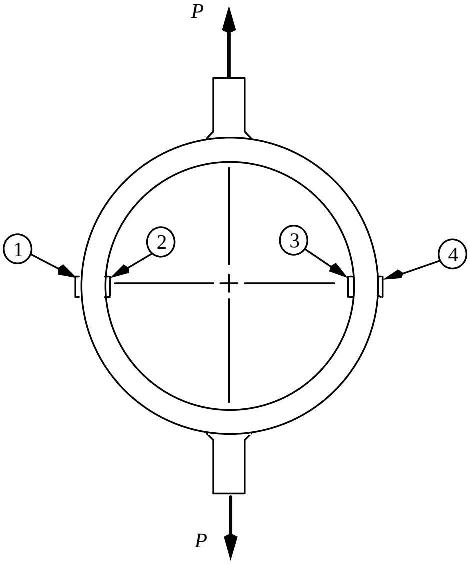


Figure 1: Strain-Gaged Proving Ring
(Gage Numbers Circled)

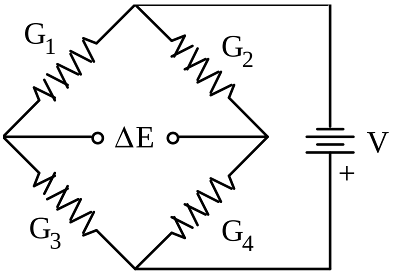


Figure 2: Wheatstone Bridge
Wiring Diagram

Performing the Test:

The steps to be followed during the test are described on the DVD labeled "ME 556 Lab #4." You will create three Excel files during the lab exercise. Transfer the pertinent portions of the data recorded in the Excel files to the "Data and Answers Sheet", attached to this handout

Lab Report:

No lab report is required. Complete the calculations and plots indicated on the "Data and Answers Sheet".

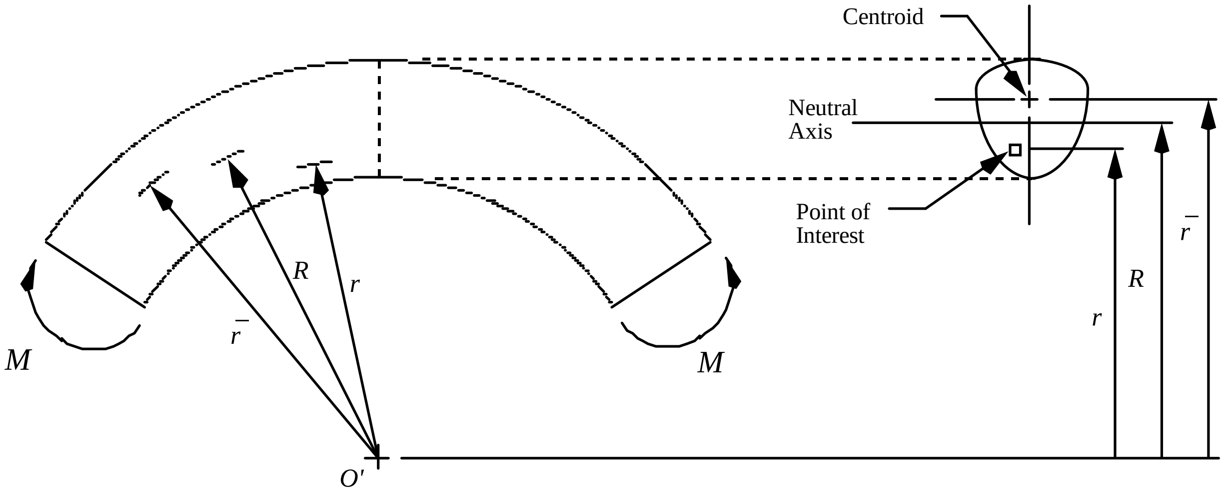
"CURVED BEAM THEORY" - REVIEW COMMENTS

In this lab exercise you will use the solution for a "curved beam" developed using fundamental beam theory, rather than the more exact theory derived from the theory of elasticity and used during Lab #3. The term "curved beam" refers to a structural member that is curved prior to the application of any loading. A curved beam subjected to a pure bending moment M is shown below. For present purposes it is assumed that:

- (a) the cross-section has an axis of symmetry that lies in the plane of loading represented by the moment M, and
- (b) the cross-section is constant along the arc of the beam (i.e., the beam is "prismatic").

Point O' is the center of curvature of the non-stressed member. The following radii are defined with respect to pt O':

$\bar{r}$ = distance from O' to the centroid of the cross-sectional area
R = distance from O' to the neutral axis of the cross-section
r = distance from O' to any point of interest in the cross-section



As in the theory for straight beams, it is assumed that *cross-sections remain plane* after the moment M is applied. Assuming also that stresses are not high enough to cause yielding (i.e., assuming linear-elastic behavior), then "it can be shown"* that the normal stress induced at any radial position r by a pure bending moment M is given by:

$$\sigma = \frac{M(R-r)}{Ar(\bar{r}-R)} \qquad (1)$$

where: A = cross-sectional area of the member, and

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* See, for example, Hibbeler, MECHANICS OF MATERIALS, 4th edition, section 6.8, Prentice Hall, ISBN 0-13-016467-4 (2000).

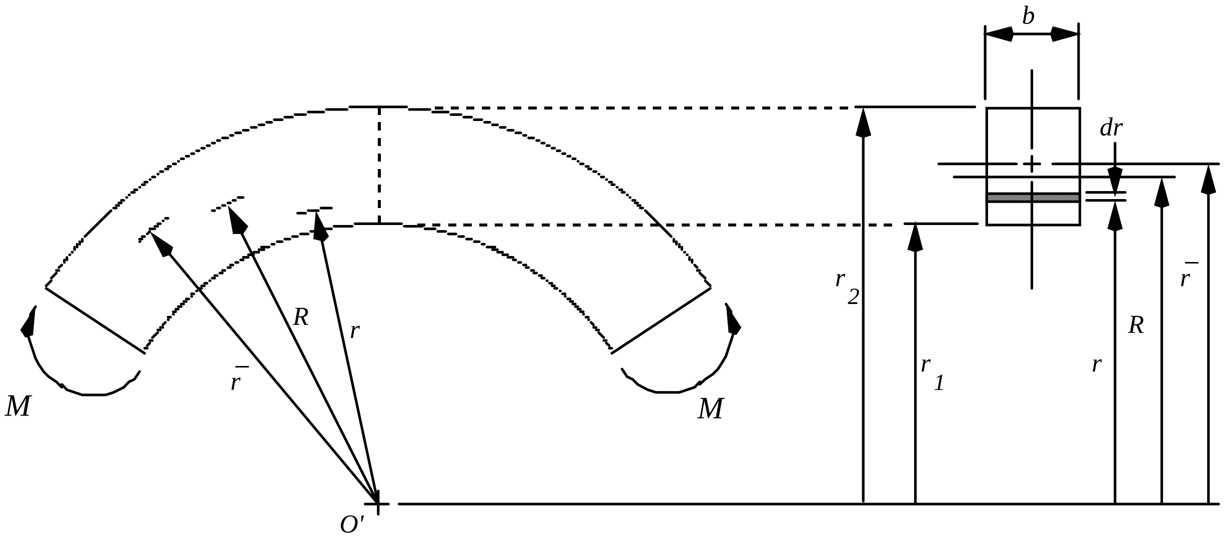
$$R = \frac{A}{\int_A \frac{dA}{r}} \qquad (2)$$

Equation (1) shows that the normal stress caused by a pure bending moment M is a hyperbolic function of r in a curved beam. As an example application of Eq (2), consider a curved beam with a rectangular cross-section, as shown below. In this case the cross-section is defined by the inner and outer radii, r_1 and r_2 respectively, and the thickness of the curved beam, length b. From the sketch it can be seen that $dA = b\,dr$. Therefore:

$$\int_A \frac{dA}{r} = \int_{r_1}^{r_2} b\frac{dr}{r} = b\,(\ln r)\Big|_{r_1}^{r_2} = b\,(\ln r_2 - \ln r_1) = b\ln\frac{r_2}{r_1}$$

The distance from the center of curvature of the member to the neutral axis is therefore given by:

$$R = \frac{b(r_2 - r_1)}{b\ln\frac{r_2}{r_1}} = \frac{(r_2 - r_1)}{\ln\frac{r_2}{r_1}}$$



A Circular Ring Subjected to Diametral Loading

A circular ring subjected to a tensile load P along a diameter is shown in the following sketch. This geometry is often referred to as a "proving ring." Commercial load cells based on a proving ring may be based on (a) sensing the change in inner diameter as a function of load P, or (b) sensing the strain induced at various points in the member using strain gages (as in this lab), again as a function of P.

A free-body diagram obtained by making an imaginary cut along the horizontal axis is also shown in the sketch. Due to symmetry considerations:

(a) The shear load at the horizontal cut must be zero (note that horizontal shear loads have therefore not been included in the free-body diagram).

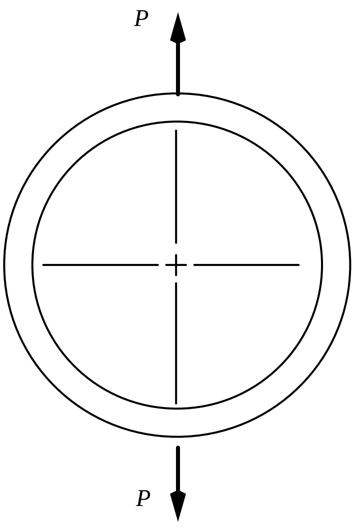
(b) An internal normal force with magnitude $(P/2)$ is induced at the horizontal cross-section.

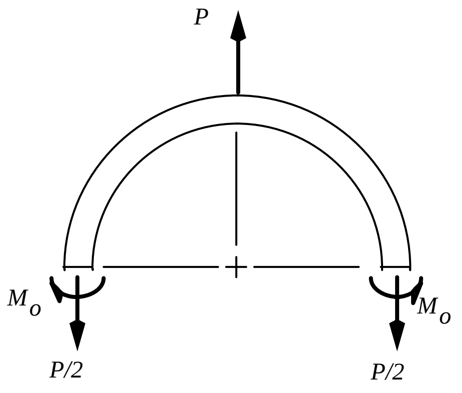
The bending moment M_o is indeterminate, and therefore cannot be determined directly from the equations of equilibrium. However, M_o can be determined using strain-energy methods. Assuming $\bar{r}/(r_2 - r_1) > 2$, then "it can be shown"[*]:

$$M_o = P\bar{r}\left(\frac{1}{2} - \frac{1}{\pi}\right)$$

Curved beam theory implies that a uniaxial state of stress is induced along the horizontal diameter. Since linear elastic behavior has been assumed, the total stress induced at the horizontal axis is the sum of the stress caused by the normal force and the stress caused by the bending moment. That is:

$$\sigma = \frac{P}{2A} + \frac{M_o(R - r)}{Ar(\bar{r} - R)}$$





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[*] See, for example, Boresi, Schmidt, and Sidebottom, ADVANCED MECHANICS OF MATERIALS, 5th edition, Section 9.6, John Wiley and Sons, ISBN 0-471-55157-0 (1993).

ME556 Lab #4
Data and Answers Sheet

Name: ________________________ Date Lab Performed: __________________

Part I: Prediction

The load cell used in this lab is intended to measure loads ranging from 0 to 200 lbf. The strain gages used have a gage factor of 2.15. What is the predicted load cell output (in mV/V)? Ignore the effects of transverse sensitivity (which will be present in the actual load cell). Attach calculation sheet(s) as appropriate to support your answer.

Predicted Load Cell Output = ________________________ mV/V

Part II: Measurements (complete the following tables by transferring the data requested from the Excel files created during the lab)

Loading ⟶

Excitation Voltage (V)	Initial Output Offset (mV)	Output Voltage at 40 lbf (mV)	Output Voltage at 80 lbf (mV)	Output Voltage at 120 lbf (mV)	Output Voltage at 160 lbf (mV)	Output Voltage at 200 lbf (mV)
0.5						
2.0						
10.0						

⟵ Unloading

Excitation Voltage (V)	Final Output Offset (mV)	Output Voltage at 40 lbf (mV)	Output Voltage at 80 lbf (mV)	Output Voltage at 120 lbf (mV)	Output Voltage at 160 lbf (mV)
0.5					
2.0					
10.0					

Part III: Data Reduction

1) Attach a single graph in which three curves of (Output Voltage) vs (Load) are plotted, where the individual curves correspond to the three excitation voltage levels used. Use symbols to indicate measured data, and use linear regression to fit a straight line to each data set.
2) Normalize each data point. That is, for each data point: (a) subtract the initial offset voltage (calculated using linear regression in step (1), and then (b) divide by the excitation voltage. Attach a plot of (normalized data) vs (applied load) on a single curve, and use linear regression to fit a straight line to the normalized data.

Based on the results obtained in step (2):

Measured Load Cell Output = ________________________ mV/V