Measurement of Residual Stress/Strain

(Using Strain Gages and the Hole Drilling Method)

Summary of Discussion in Section 8.9

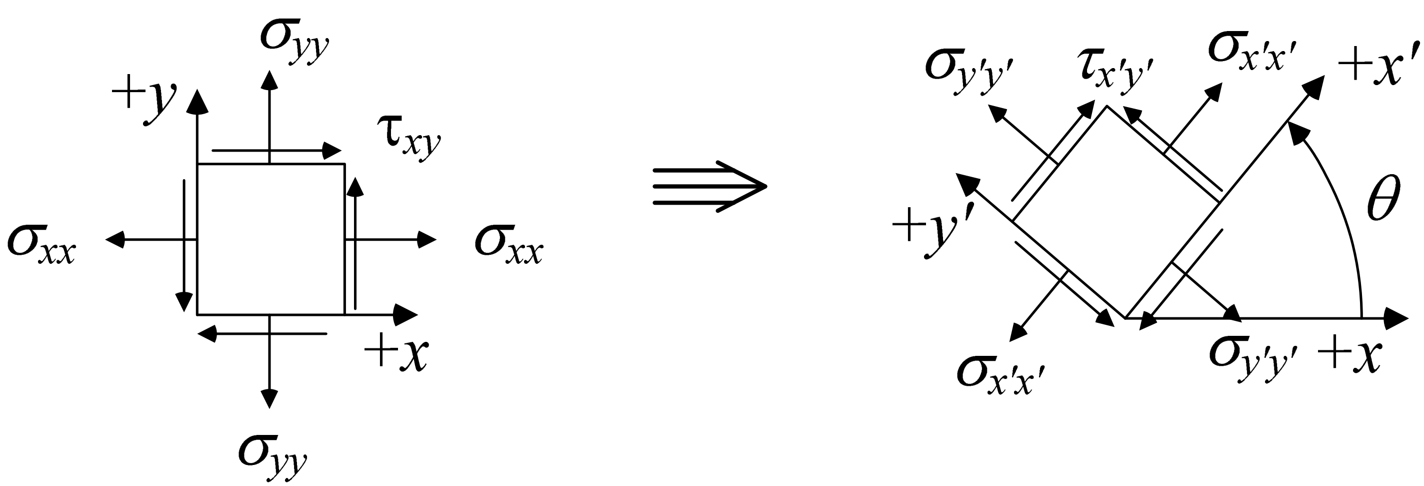
The Hole Drilling Method Is Based On:

(a) Stress transformation equations

$$\sigma_{x'x'} = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\cos\theta\sin\theta$$

$$\sigma_{y'y'} = \sigma_{xx}\sin^2\theta + \sigma_{yy}\cos^2\theta - 2\tau_{xy}\cos\theta\sin\theta$$

$$\tau_{x'y'} = \left(\sigma_{yy} - \sigma_{xx}\right)\cos\theta\sin\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$



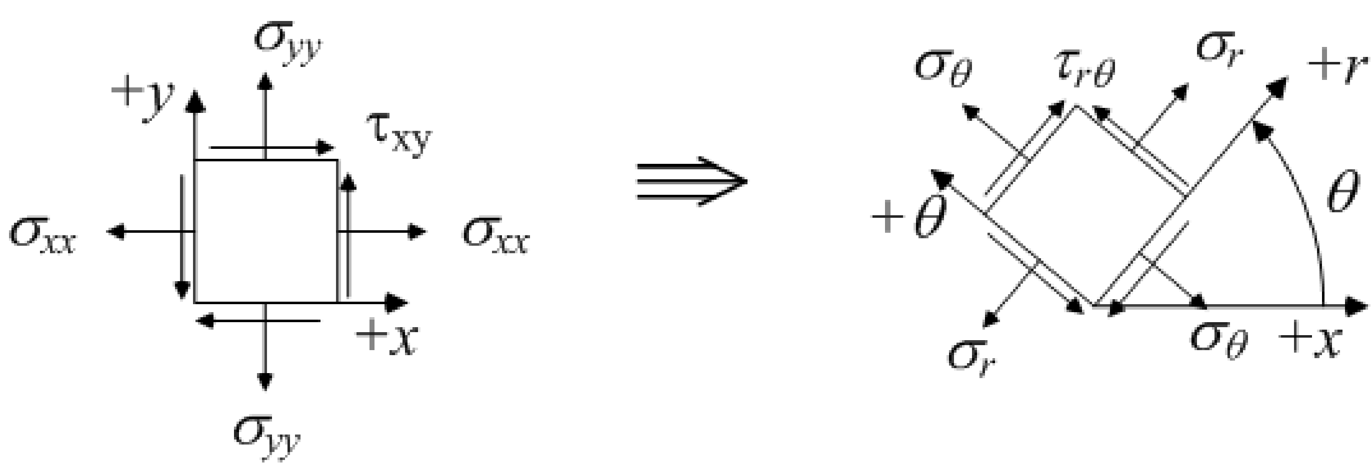
The Hole Drilling Method Is Based On:

(a) Stress transformation equations…label the new axes "r" and "θ" (instead of x' and y')

$$\sigma_r = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\cos\theta\sin\theta$$

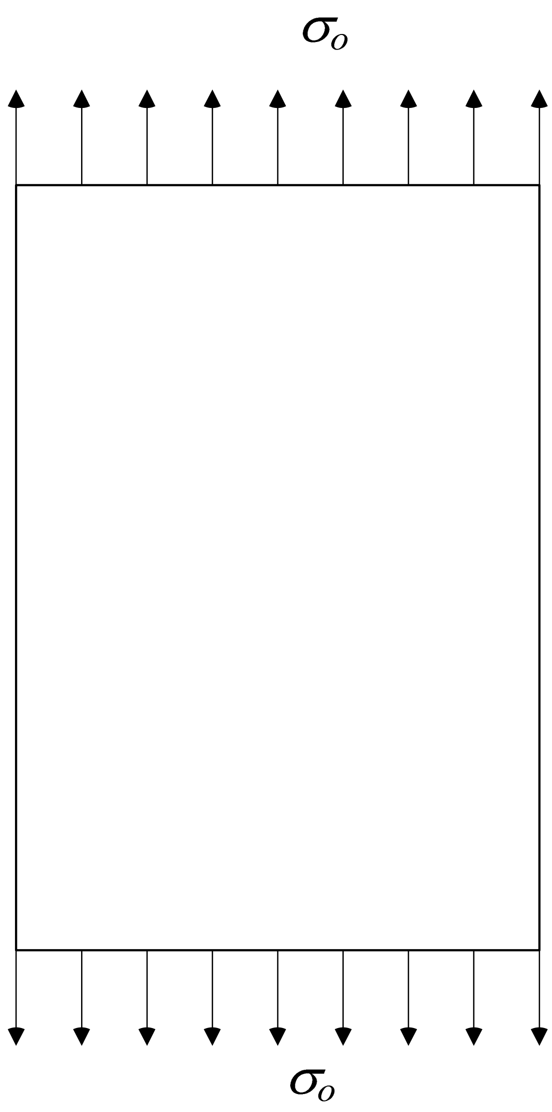
$$\sigma_\theta = \sigma_{xx}\sin^2\theta + \sigma_{yy}\cos^2\theta - 2\tau_{xy}\cos\theta\sin\theta$$

$$\tau_{r\theta} = \left(\sigma_{yy} - \sigma_{xx}\right)\cos\theta\sin\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$



The Hole Drilling Method Is Based On:

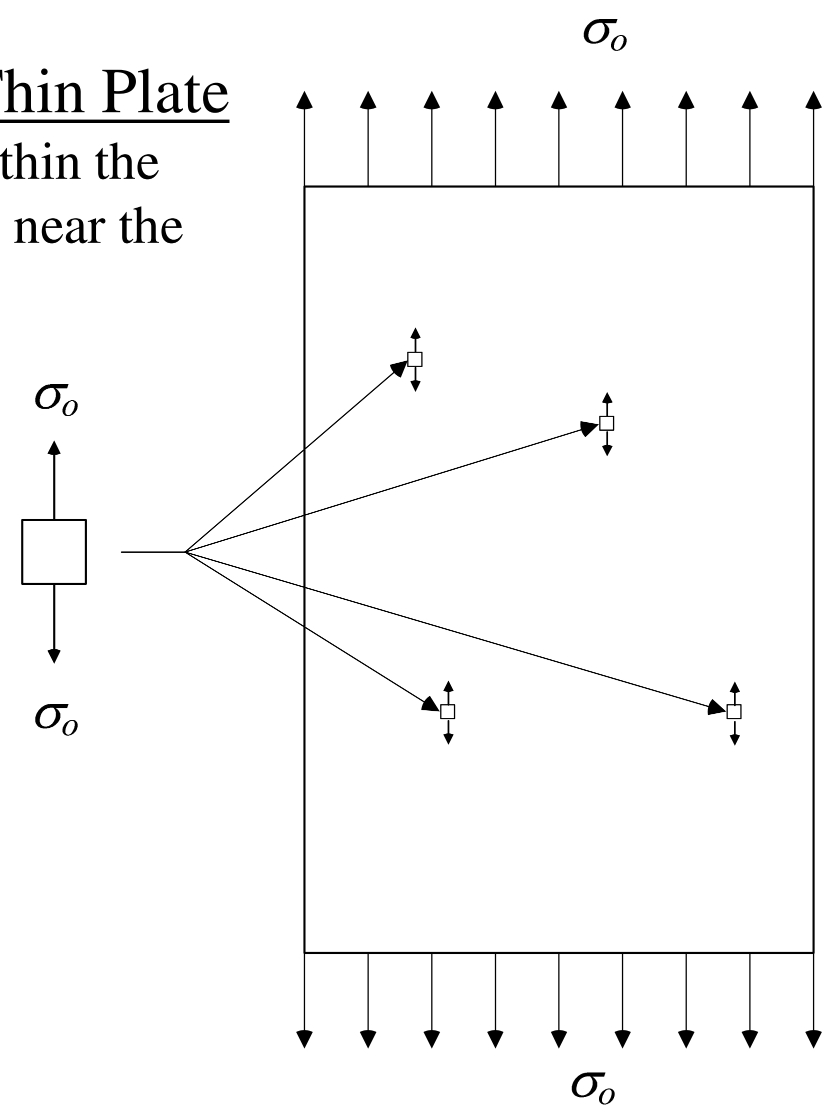
(b) Stresses in a Uniformly-Loaded Thin Plate



The Hole Drilling Method Is Based On:

(b) Stresses in a Uniformly-Loaded Thin Plate

The state of stress is identical at all points within the plate (ignoring possible stress concentrations near the loaded ends)



The Hole Drilling Method Is Based On:

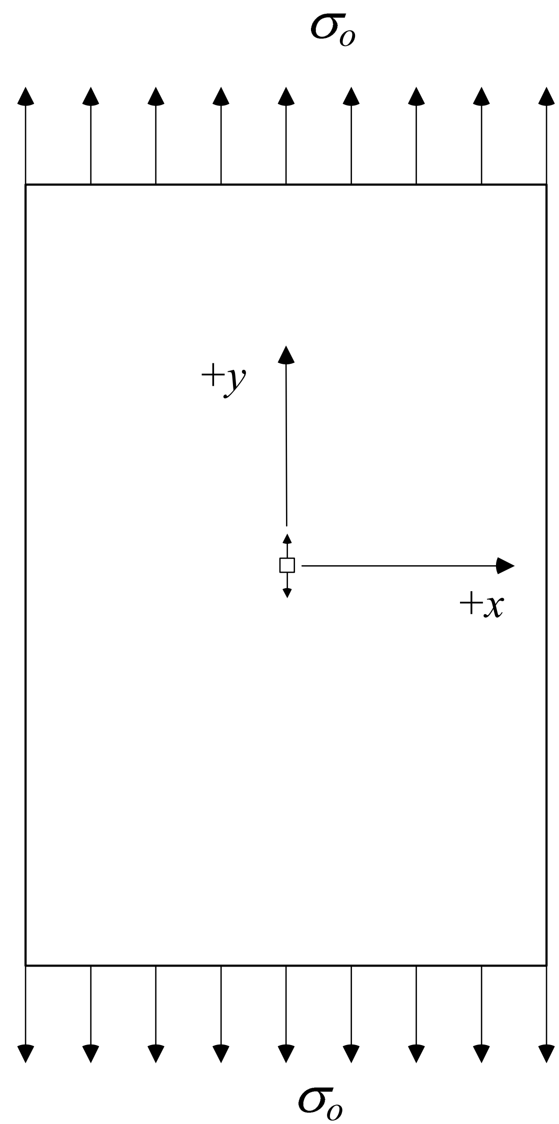
(b) Stresses in a Uniformly-Loaded Thin Plate

Define an *x*-*y* coordinate system at the center of the plate:

$$\sigma_{xx} = 0$$

$$\sigma_{yy} = \sigma_o$$

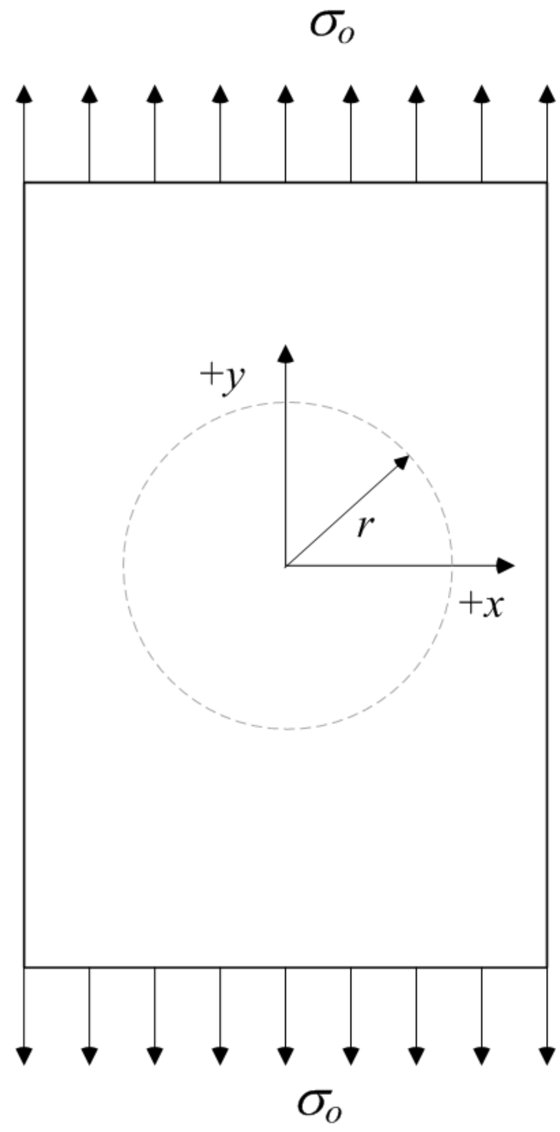
$$\tau_{xy} = 0$$



The Hole Drilling Method Is Based On:

<u>(b) Stresses in a Uniformly-Loaded Thin Plate</u>

Rotate the stress element along the circumference of an (imaginary) circle of radius *r*



The Hole Drilling Method Is Based On:

<u>(b) Stresses in a Uniformly-Loaded Thin Plate</u>

Rotate the stress element along the circumference of an (imaginary) circle of radius r…at any angle θ:

$$\sigma_r = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + 2\tau_{xy}\cos\theta\sin\theta$$

$$\sigma_\theta = \sigma_{xx}\sin^2\theta + \sigma_{yy}\cos^2\theta - 2\tau_{xy}\cos\theta\sin\theta$$

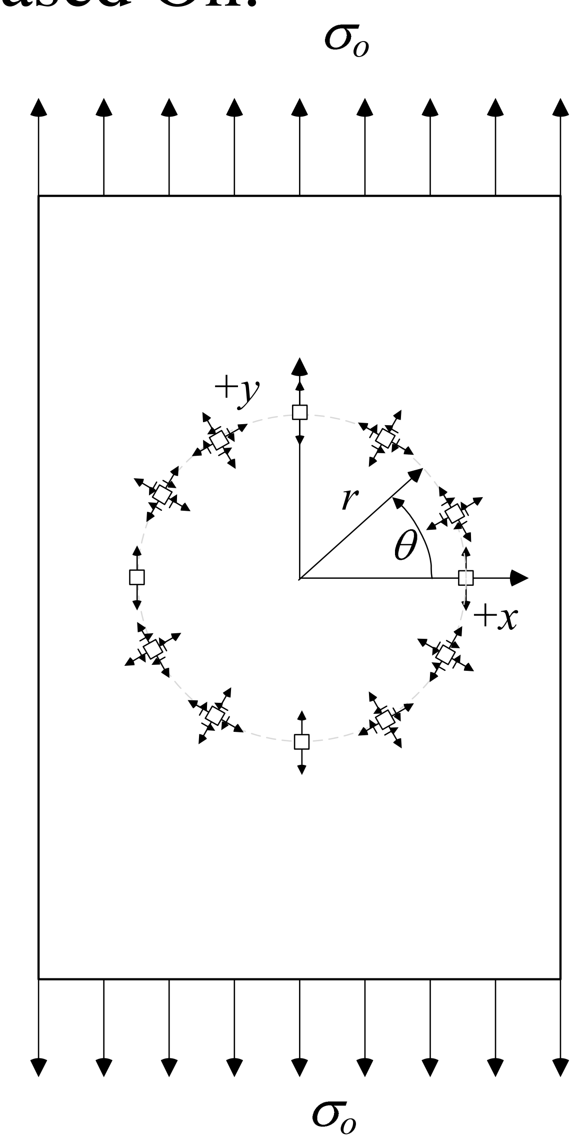
$$\tau_{r\theta} = \left(\sigma_{yy} - \sigma_{xx}\right)\cos\theta\sin\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$

Since $\sigma_{xx} = \tau_{xy} = 0$, $\sigma_{yy} = \sigma_o$

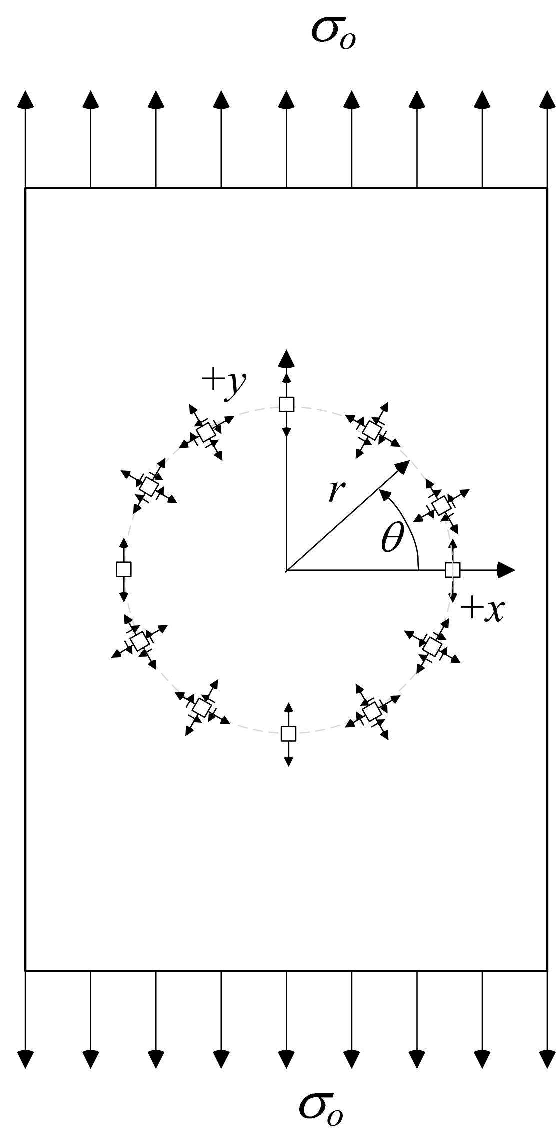
$$\sigma_r = \sigma_o\sin^2\theta$$

$$\sigma_\theta = \sigma_o\cos^2\theta$$

$$\tau_{r\theta} = \sigma_o\cos\theta\sin\theta$$



The Hole Drilling Method Is Based On:



(b) Stresses in a Uniformly-Loaded Thin Plate

Using trig identities:

$$\sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

$$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$$

$$\cos\theta\sin\theta = \frac{1}{2}\sin 2\theta$$

The stress components can be written:

$$\sigma_r = \frac{1}{2}\sigma_o(1-\cos 2\theta)$$

$$\sigma_\theta = \frac{1}{2}\sigma_o(1+\cos 2\theta)$$

$$\tau_{r\theta} = \frac{1}{2}\sigma_o\sin 2\theta$$

eqs (e), pg 69
eqs (8.44), pg 244

The Hole Drilling Method Is Based On:

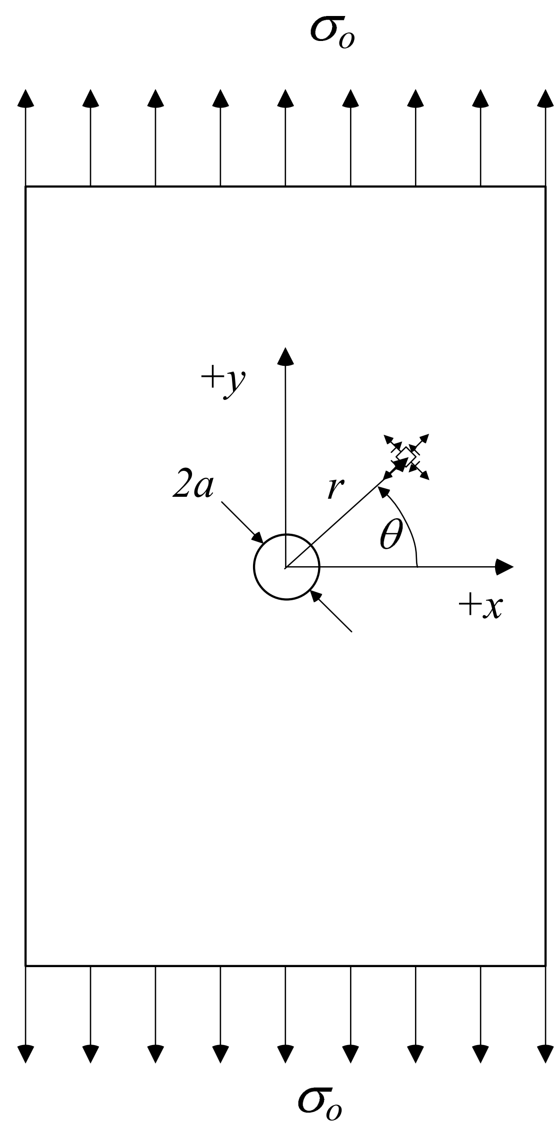
(c) The Kirsch Solution

• (Derived in section 3.13): The stresses induced at any point (r,θ) in an infinitely large thin plate with hole of radius a, subjected to a remote uniaxial stress σ_o, are given by (where $r \geq a$):

$$\sigma_r = \frac{\sigma_o}{2}\left\{\left(1-\frac{a^2}{r^2}\right)\left[1+\left(\frac{3a^2}{r^2}-1\right)\cos 2\theta\right]\right\}$$

$$\sigma_\theta = \frac{\sigma_o}{2}\left[\left(1+\frac{a^2}{r^2}\right)+\left(1+\frac{3a^4}{r^4}\right)\cos 2\theta\right] \qquad \text{eqs (3.42)}$$

$$\tau_{r\theta} = \frac{\sigma_o}{2}\left[\left(1+\frac{3a^2}{r^2}\right)\left(1-\frac{a^2}{r^2}\right)\sin 2\theta\right]$$



(…an Aside..) Stress Concentration Near a Circular Hole

- Stresses along the x-axis in an infinite plate predicted by the Kirsch solution:

$$\sigma_{rr} = \sigma_{xx} = \frac{\sigma_o}{2}\left(1 - \frac{a^2}{x^2}\right)\frac{3a^3}{x^2}$$

$$\sigma_{\theta\theta} = \sigma_{yy} = \frac{\sigma_o}{2}\left(2 + \frac{a^2}{x^2} + \frac{3a^4}{x^4}\right)$$

$$\tau_{r\theta} = \tau_{xy} = 0$$

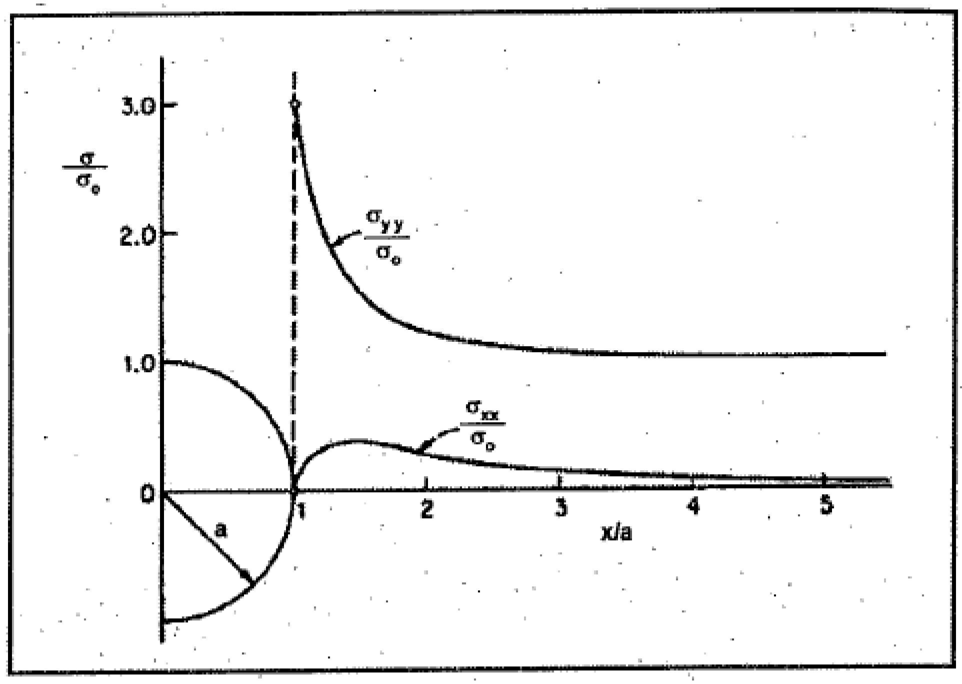


Figure 3.6: Distribution of σ_{xx}/σ_o and σ_{yy}/σ_o along the x-axis

(…an Aside…) Stress Concentration Near a Circular Hole

- Stresses at the edge of the hole (at $x = a$):

$$\sigma_{rr} = \sigma_{xx} = 0$$

$$\sigma_{\theta\theta} = \sigma_{yy} = 3\sigma_o$$

$$\tau_{r\theta} = \tau_{xy} = 0$$

- The stress concentration factor for a circular hole in an infinite plate:

$$K_t = \frac{\sigma_{yy}}{\sigma_o} = 3$$

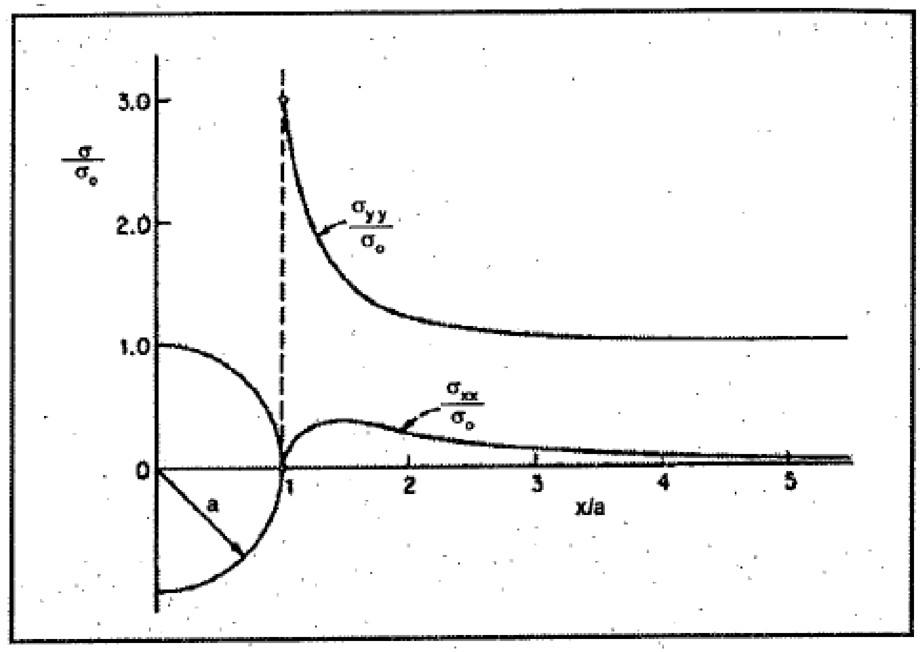
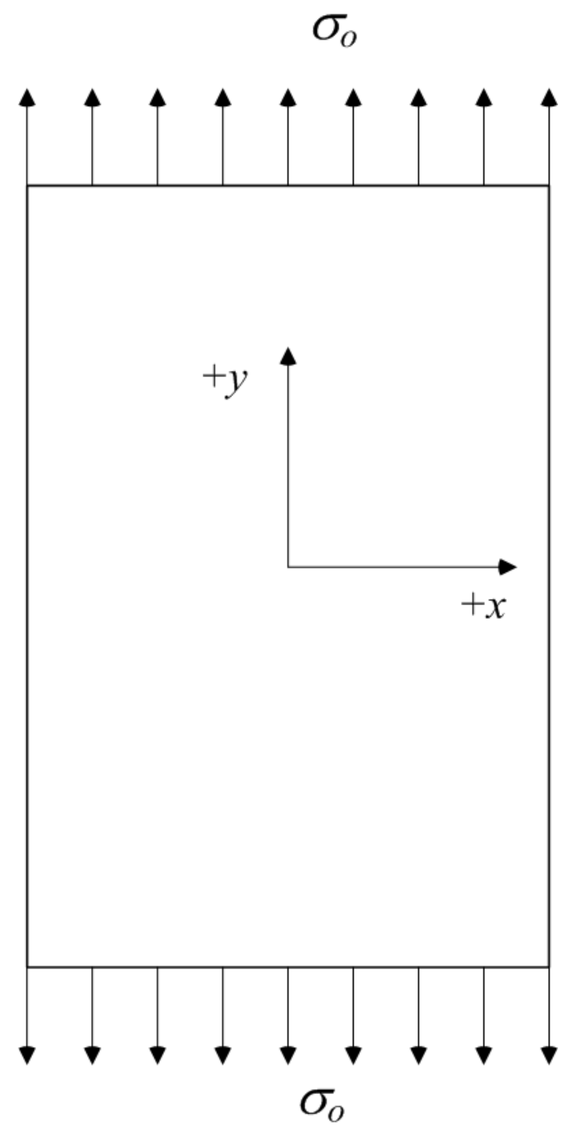


Figure 3.6: Distribution of σ_{xx}/σ_o and σ_{yy}/σ_o along the x-axis

The Hole Drilling Method

<u>Assume:</u> A *uniaxial* residual stress exists in a thin plate (Note a uniaxial residual stress field is unusual…will generalize this discussion later…)



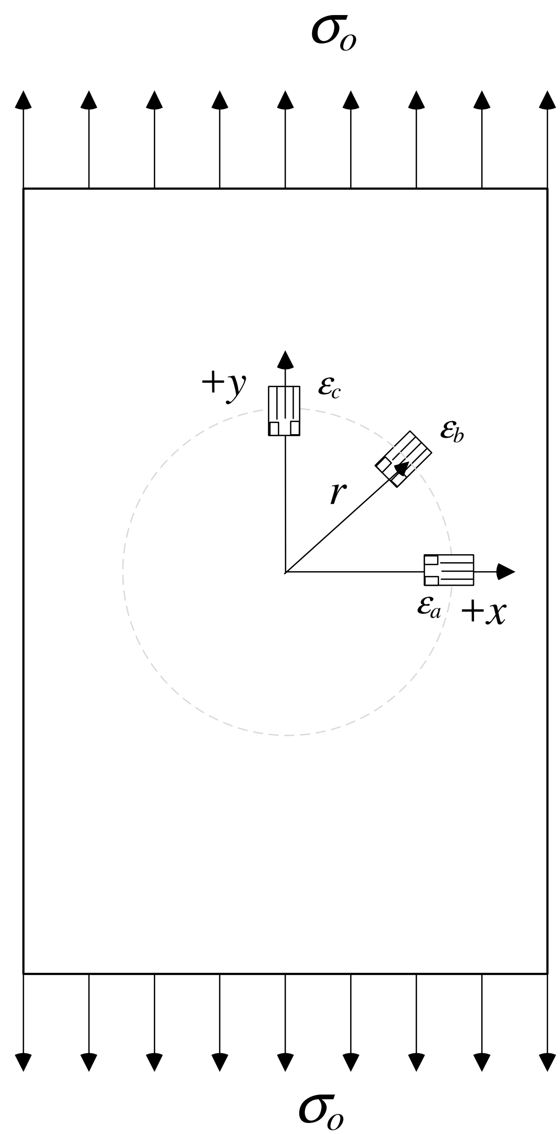
The Hole Drilling Method

<u>Assume:</u> A *uniaxial* residual stress exists in a thin plate (Note a uniaxial residual stress field is unusual…will generalize this discussion later…)

<u>Hole-drilling Procedure</u>:

a) Special 3-element strain gage rosette bonded to the plate…Note:

- gage elements arranged at constant radius r in a circular pattern
- gage circuits *are balanced in this condition*…since residual stresses are present, the measured zero strain does not correspond to zero stress



The Hole Drilling Method

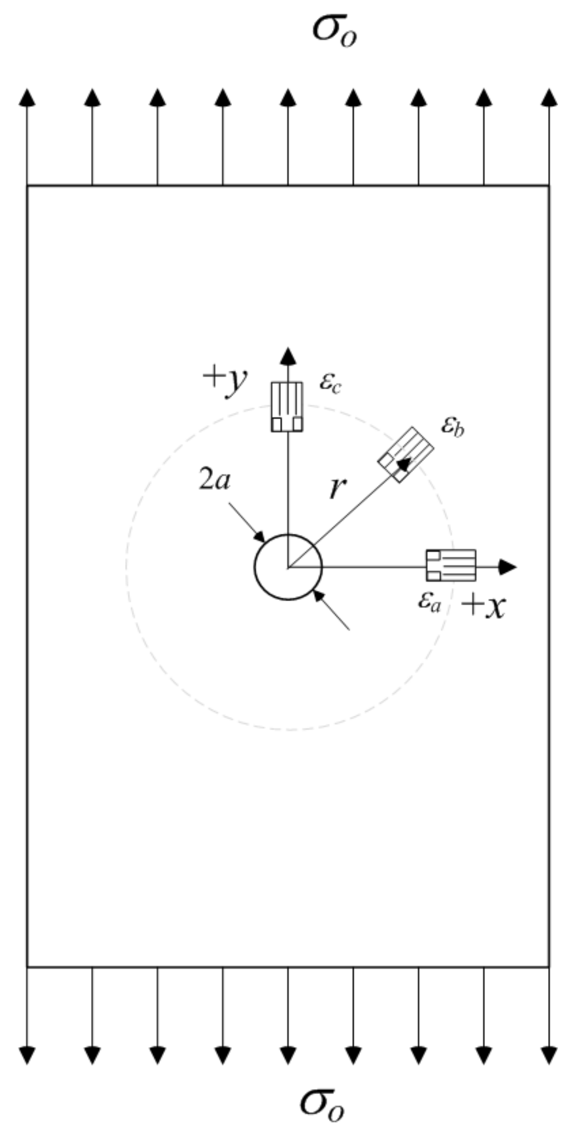
<u>Assume:</u> A *uniaxial* residual stress exists in a thin plate (Note a uniaxial residual stress field is unusual…will generalize this discussion later…)

<u>Hole-drilling Procedure</u>:

a) Special 3-element strain gage rosette bonded to the plate…Note:

- gage elements arranged at constant radius *r* in a circular pattern
- gage circuits *are balanced in this condition*…since residual stresses are present, the measured zero strain does not correspond to zero stress

- A hole with radius *a* is drilled through the center of the rosette, releasing the residual stresses that existed in the hole volume



The Hole Drilling Method

• The strain gages respond to the *change* in the stress field, which is given by:

(eqs 3.42) – (eq 8.44)

uniform stress ↗ (eqs 3.42); plate with hole ↖ (eq 8.44)

• Making the substitution and performing algebraic simplifications, the stress field that the strain gages respond to is given by:

$$\sigma_r = \frac{-\sigma_o a^2}{2r^2}\left[1+\left(\frac{3a^2}{r^2}-4\right)\cos 2\theta\right]$$

$$\sigma_\theta = \frac{\sigma_o a^2}{2r^2}\left(1+\frac{3a^2}{r^2}\cos 2\theta\right) \qquad \text{eqs (8.45)}$$

$$\tau_{r\theta} = \frac{-\sigma_o a^2}{2r^2}\left[\left(\frac{3a^2}{r^2}-2\right)\sin 2\theta\right]$$

The Hole Drilling Method

• The change in radial and tangential strains caused by (careful!) drilling of the hole can be obtained by substituting the change in stress (i.e., eqs 8.45) into Hooke's Law for plane stress:

$$\varepsilon_r = \frac{1}{E}\left(\sigma_r - \nu\sigma_\theta\right) \qquad \varepsilon_\theta = \frac{1}{E}\left(\sigma_\theta - \nu\sigma_r\right)$$

• Making this substitution and performing algebraic simplifications:

$$\varepsilon_r = \frac{-\sigma_o a^2(1+\nu)}{2Er^2}\left[1 + \frac{3a^2}{r^2}\cos 2\theta - \frac{4\cos 2\theta}{1+\nu}\right]$$

$$\varepsilon_\theta = \frac{\sigma_o a^2(1+\nu)}{2Er^2}\left[1 + \frac{3a^2}{r^2}\cos 2\theta - \frac{4\nu\cos 2\theta}{1+\nu}\right] \qquad \text{eqs (8.46)}$$

The Hole Drilling Method

• The change in radial and tangential strains caused by (careful!) drilling of the hole can be obtained by substituting the change in stress (i.e., eqs 8.45) into Hooke's Law for plane stress:

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• Making this substitution and performing algebraic simplifications:

$$\varepsilon_r = \frac{-\sigma_o a^2 (1+\nu)}{2Er^2}\left[1 + \frac{3a^2}{r^2}\cos 2\theta - \frac{4\cos 2\theta}{1+\nu}\right]$$

Radial strains measured by the 3-element rosette, *if* residual stress is uniaxial and aligned with *y*-axis

$$\varepsilon_\theta = \frac{\sigma_o a^2 (1+\nu)}{2Er^2}\left[1 + \frac{3a^2}{r^2}\cos 2\theta - \frac{4\nu\cos 2\theta}{1+\nu}\right]$$

eqs (8.46)

The Hole Drilling Method

Practical Application

- The residual stress field is usually biaxial, involving:
  - two unknown principal residual stresses σ_1^R and σ_2^R
  - unknown orientation of principal axis, θ_1 (or θ_2)

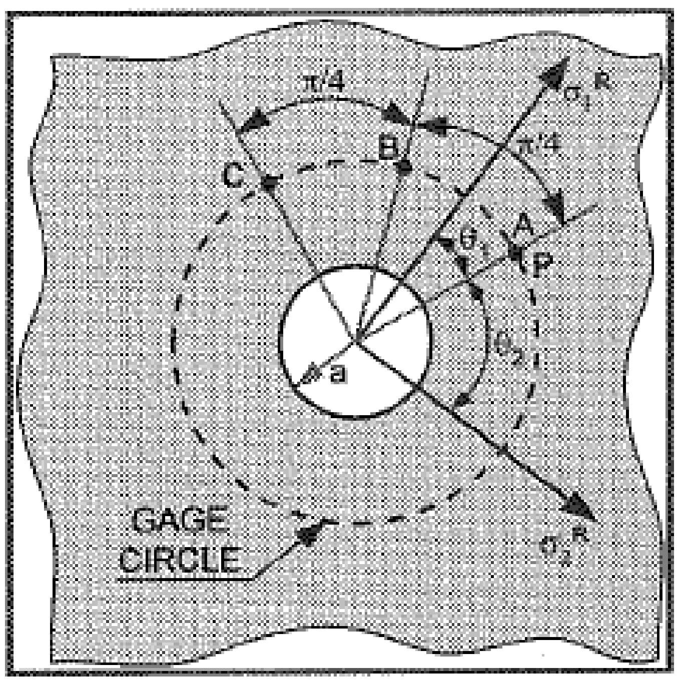


Figure 8.13

The Hole Drilling Method

Practical Application

- The predicted strains induced by a uniaxial stress σ_o (eqs 8.26) can be used to separately calculate:
  - strains induced by principal stress σ_1^R
  - strains induced by principal stress σ_2^R (acting at 90º w/r/t σ_1^R)

The Hole Drilling Method

Practical Application

- The predicted strains induced by a uniaxial stress σ_o (eqs 8.26) can be used to separately calculate:
  - strains induced by principal stress σ_1^R
  - strains induced by principal stress σ_2^R (acting at 90º w/r/t σ_1^R)
- Applying the principal of superposition, using Hooke's law and the strain transformation equations we find:

$$\sigma_1^R = \frac{\varepsilon_A + \varepsilon_C}{4C_1} + \frac{\sqrt{2}}{4C_2}\sqrt{(\varepsilon_A - \varepsilon_B)^2 + (\varepsilon_B - \varepsilon_c)^2}$$

$$\sigma_2^R = \frac{\varepsilon_A + \varepsilon_C}{4C_1} - \frac{\sqrt{2}}{4C_2}\sqrt{(\varepsilon_A - \varepsilon_B)^2 + (\varepsilon_B - \varepsilon_c)^2}$$

eqs (8.53)

$$\tan 2\theta = \frac{\varepsilon_A - 2\varepsilon_B + \varepsilon_C}{\varepsilon_C - \varepsilon_A}$$

$$C_1 = -\frac{1+\nu}{2E}\left(\frac{a}{r}\right)^2 \qquad C_2 = -\frac{1+\nu}{2E}\left(\frac{a}{r}\right)^2\left[-3\left(\frac{a}{r}\right)^2 + \frac{4}{1+\nu}\right]$$

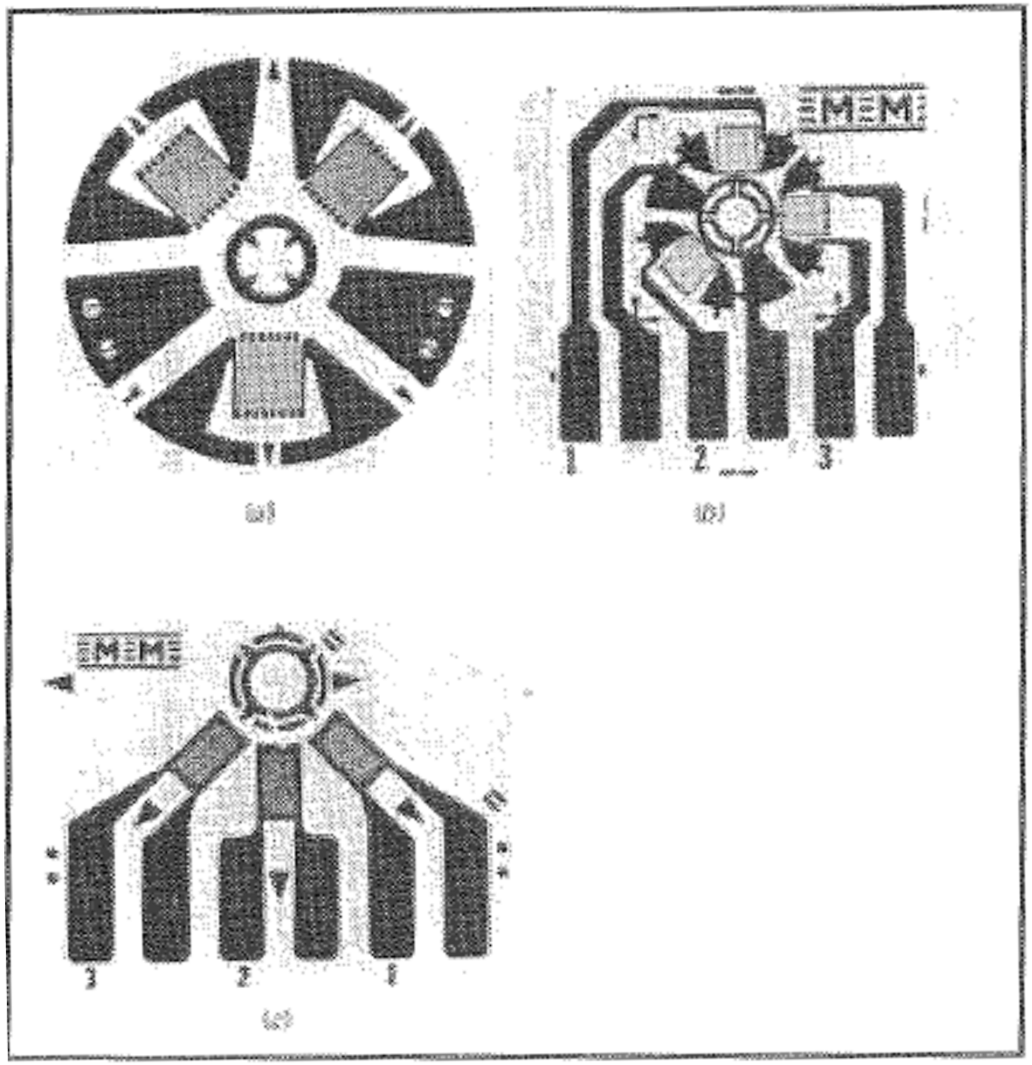


Fig 8.14: Residual Stress (Strain) Gages produced by Micro-Measurements

GAGE PATTERN AND DESIGNATION (Actual Size Shown. Insert Desired S-T-C No. in Spaces Marked XX.)	RES. IN OHMS	LIST PKG. PRICE (FIVE GAGES)	DIMENSIONS (inches / *millimetres*)					
			GAGE LENGTH	GRID CTR'LINE DIA.	TYPICAL HOLE DIA.		MATRIX	
					Min.	Max.	Length	Width
EA-XX-031RE-120 EA-XX-031RE-120/SE 	120 ±0.2% 120 ±0.4%	73.00 104.00	0.031 *0.79*	0.101 *2.56*	0.03 *0.8*	0.04 *1.0*	0.29 *7.4*	0.29 *7.4*
			Due to small pattern size, measurement error can be magnified by slight mislocation of drill hole. Pattern not recommended for general-purpose applications.					
N2K-XX-030RR-350/DP 	350 ±0.4%	225.00	0.030 *0.76*	0.170 *4.32*	0.090 *2.3*	0.100 *2.5*	0.37 *9.4*	0.37 *9.4*
			Special six-element configuration that provides somewhat higher output than three-element designs.					
EA-XX-062RE-120 EA-XX-062RE-120/SE 	120 ±0.2% 120 ±0.4%	73.00 104.00	0.062 *1.57*	0.202 *5.13*	0.06 *1.5*	0.08 *2.0*	0.42 *10.7*	0.42 *10.7*
			Most widely used RE pattern for general-purpose residual stress measurement applications.					
EA-XX-125RE-120 EA-XX-125RE-120/SE 	120 ±0.2% 120 ±0.4%	126.00 162.00	0.125 *3.18*	0.404 *10.26*	0.12 *3.0*	0.16 *4.1*	0.78 *19.8*	0.78 *19.8*
			Larger version of the 062RE pattern.					
CEA-XX-062UL-120 	120 ±0.4%	86.00	0.062 *1.57*	0.202 *5.13*	0.06 *1.5*	0.08 *2.0*	0.50 *12.7*	0.62 *15.7*
			Fully encapsulated with large copper-coated soldering tabs. Same pattern geometry as 062RE pattern.					
CEA-XX-062UM-120 	120 ±0.4%	91.00	0.062 *1.57*	0.202 *5.13*	0.06 *1.5*	0.08 *2.0*	0.38 *9.6*	0.48 *12.2*
			Fully encapsulated with large copper-coated soldering tabs and special trim alignment marks. Trim line spaced 0.068 in *(1.73 mm)* from hole center. Limitations may exist in data reduction equations.					

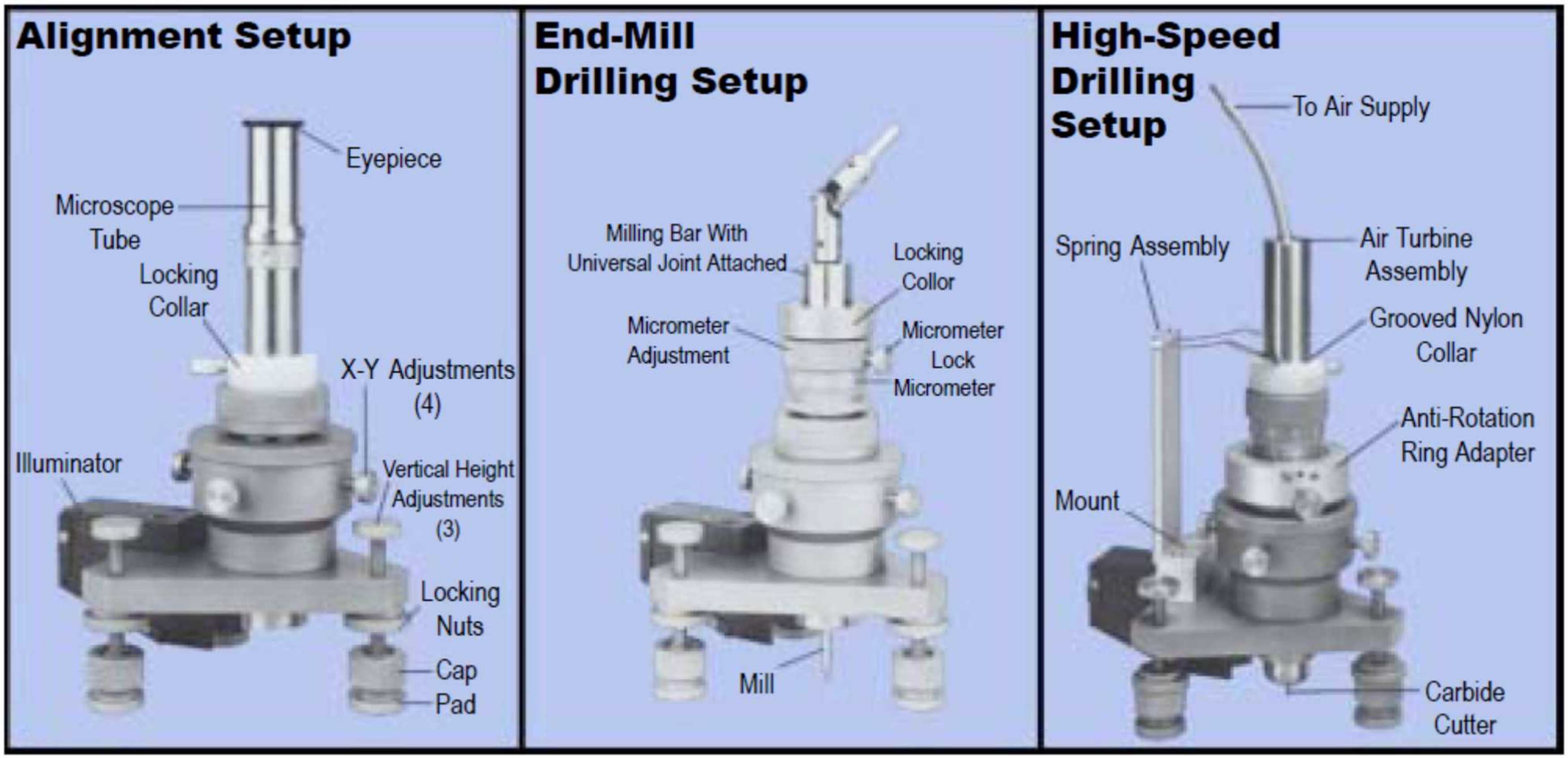
Residual Stress (Strain) Gages produced by Micro-Measurements

References:

(1) ASTM E837: "Standard Test Method
for Determining Residual Stresses by the Hole-Drilling Strain-Gage Method"

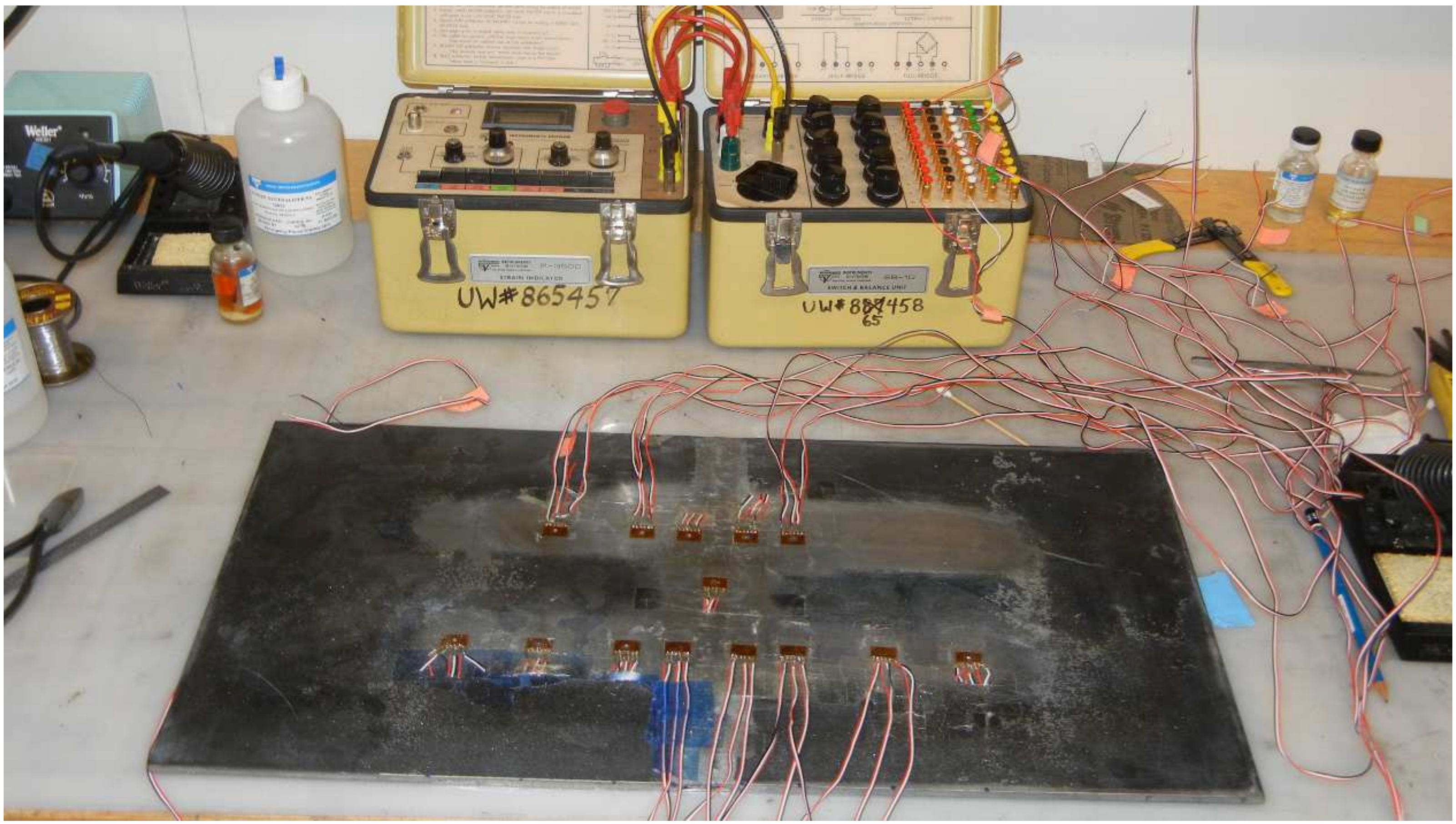
(2) M-M TN-503: "Measurement of Residual Stresses by the Hole Drilling Stain Gage Method"

Alignment and Hole-Drilling Setup for Residual Stress/Strain Measurement

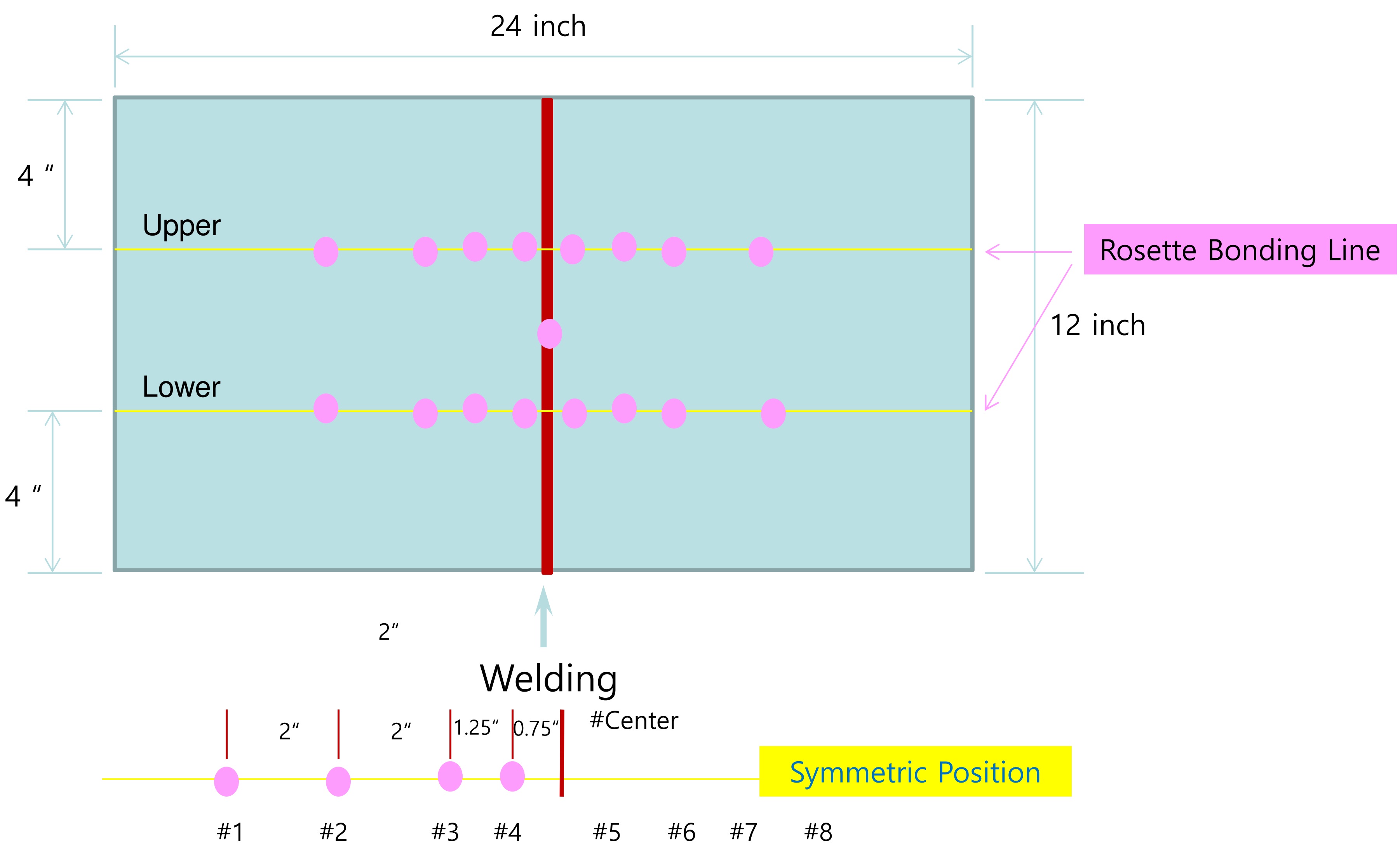


(Former) ME556 Lab Experiment: Residual Stress Measurement

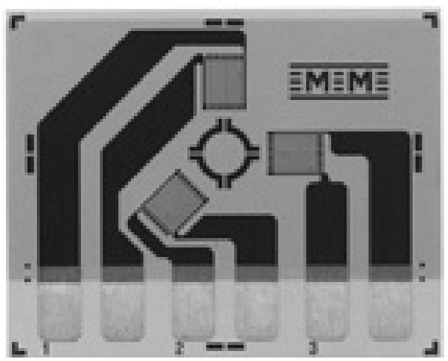
<u>Overall View of Set-up:</u> Welded Hot Rolled Mild Steel Plate (ASTM A36), Residual Strain Gage Rosettes, P-3500 Strain Gage Amplifier, SB-10 Ten-Channel Switch and Balance Unit



Specimen : Welded Hot Rolled Steel Plate (Intended Rosette Pattern)



Rosettes:
Before (on Right)
and
After (on Left)
Center Hole is Drilled



↑ ↑ ↑

Element number: 3 2 1

Definition of gage element 1, 2, and 3