A “home-made” three-axis load cell will be calibrated during this lab experiment. The L-shaped load cell was originally used to support a prototype version of a muon detector chamber designed for use in the large particle accelerator at CERN in Geneva. The two arms were instrumented with strain gages so as to be sensitive to three orthogonal force components, with directions defined as shown below.

During the lab the load cell will be mounted directly to a wall plate in a pillar located in MEB 123. Four different tests will be conducted, where orientation of the load cell will be varied from one test to the next as follows:

In all cases the load cell will be mounted to the wall pillar plate by means of four bolts, and two 40 lbf weights (80 lbf total) will be applied to the load cell. The objective of the experiment is to (a) determine the calibration matrix for the load cell (defined in the following discussion), and (b) determine the angle $\alpha$ associated with the fourth test.
Procedure:

Watch the “Three-Axis_Load_Cell” video available on the desktop of the computer located in MEB 123 as well as the course website. As described in the video, output from three Wheatstone bridge circuits will be monitored using the StrainSmart system. The system is set up to record measurements at a rate of 10 scans/sec. The video will describe how to:

(a) Mount the load cell in each of the four orientations depicted on the previous page,
(b) To balance the three Wheatstone bridge circuits,
(c) Apply a load of 80 lbf by placing two 40 lbf weights on a load pan, and
(d) To initiate/record a scan.

Record the strains induced by the 80 lbf load by recording scans for “about” 1 second – that is, record “about” 10 scans. Then create an Excel file using the StrainSmart software package, calculate the average strain measured for each of the four orientations, and enter the average strains in the appropriate tables on the following page.

Determine the calibration matrix, $C_{ij}$, and the inverse of the calibration matrix, $c_{ij}$. Enter the values of these matrices on the following page.

Determine the orientation angle $\alpha$ used during the fourth test, and enter values on the following page.

Lab Report

Submit a brief lab report, containing:
- basic description of the test set-up and equipment
- any discussion of the lab experiment or results that you deem appropriate
- the following page, including the data/calculations just described.
ME556 Lab: Calibration of a Three-Axis Load Cell

Name:______________________________________________Date: __________________

• Calibration data:

<table>
<thead>
<tr>
<th></th>
<th>Ave strain in x-dir (µε)</th>
<th>Ave strain in y-dir (µε)</th>
<th>Ave strain in z-dir (µε)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80 lb. load in X-dir</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80 lb. load in Y-dir</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80 lb. load in Z-dir</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

• Calibration matrix, where $\varepsilon_i = C_{ij}F_j$:

\[
C_{ij} = \begin{bmatrix}
( & ) & ( & ) & ( & ) \\
( & ) & ( & ) & ( & ) \\
( & ) & ( & ) & ( & ) \\
\end{bmatrix}
\]

• Inverse of the calibration matrix, where $F_i = c_{ij}\varepsilon_j$

\[
c_{ij} = \begin{bmatrix}
( & ) & ( & ) & ( & ) \\
( & ) & ( & ) & ( & ) \\
( & ) & ( & ) & ( & ) \\
\end{bmatrix}
\]

• Strains measured when load is applied at angle $\alpha$:

<table>
<thead>
<tr>
<th>80 lb load applied at angle $\alpha$</th>
<th>Ave strain in x-dir (µε)</th>
<th>Ave strain in y-dir (µε)</th>
<th>Ave strain in z-dir (µε)</th>
</tr>
</thead>
</table>

• Based on these measured strains, $\alpha =$ ____________ degs
CALIBRATION OF A 3-AXIS LOAD CELL

DISCUSSION

Strain Gages and Wheatstone Bridge Circuits:

Three four-arm bridge circuits are used in this load cell. That is, one bridge is used to measure loads in the x-, y-, and z-directions. Micro-Measurement CEA-13-125UW-120 uniaxial gages are used in all cases. These are mounted and connected so that each bridge is sensitive to only one of the three force components, and is insensitive to bending moments. This will be further discussed below.

The Calibration Matrix:

Although, in principle, each strain bridge should be sensitive to only one load component, in practice there is some crosstalk. The effect of this can be accounted for by using a calibration matrix:

Denote the strain measurements recorded using the x-, y-, and z-bridge circuits, and caused by calibration load $P_x$, as $e_{xx}$, $e_{xy}$, and $e_{xz}$. Similarly, the strain measurements recorded using the x-, y-, and z-bridge circuits and caused by $P_y$ and $P_z$ are denoted $(e_{yx}, e_{yy},$ and $e_{yz})$ and $(e_{zx}, e_{zy},$ and $e_{zz})$, respectively.

Now consider the strains induced by an unknown load vector $F_i$. The strain measurement recorded by the x-bridge circuit, $\varepsilon_x$, caused by $F_i$ is given by:

$$\varepsilon_x = \frac{e_{xx}}{P_x} F_x + \frac{e_{yx}}{P_y} F_y + \frac{e_{zx}}{P_z} F_z$$

A similar equations can be written for $\varepsilon_y$ and $\varepsilon_z$. In matrix form we have

$$\varepsilon_i = C_{ij} F_j$$

Where $C_{ij}$ is the 3x3 calibration matrix. Once the calibration matrix is known then an unknown force can be measured. That is, if an unknown force is applied then the x-, y- and z- bridge circuits will return a measurement $(\varepsilon_x, \varepsilon_y, \text{and } \varepsilon_z)$, and the force applied is determined as:

$$F_i = C_{ij}^{-1} \varepsilon_j = c_{ij} \varepsilon_j$$

Elimination of Load Cell Sensitivity to Bending Moments:

Consider a cantilever prismatic beam equipped with four strain gages wired into a full Wheatstone bridge, as shown below.
Since the beam has rectangular cross-section, then strains induced at gages \( G_1 \) and \( G_2 \) are equal in magnitude but opposite in algebraic sign. These strains are due to the bending moment induced at distance \( x \) from the transverse force \( F \). Let the strains at gage sites 1 and 2 be given by:

\[
\varepsilon|_{G_1} = (C)(F)(x) \\
\varepsilon|_{G_2} = -(C)(F)(x)
\]

Where \( C \) is a constant given by Young’s modulus and the dimensions of the beam cross-section.

Similarly, the strains at gages \( G_3 \) and \( G_4 \) are equal in magnitude but opposite in algebraic sign. Also, as drawn the magnitude of strain at gages \( G_3 \) and \( G_4 \) is larger than at gages \( G_1 \) and \( G_2 \). Let the strains at gage sites 3 and 4 be given by:

\[
\varepsilon|_{G_3} = (C)(F)(x + d) \\
\varepsilon|_{G_4} = -(C)(F)(x + d)
\]

Due to the nature of the Wheatstone bridge (“opposite arms add, adjacent arms subtract”), the output from the bridge will be proportional to:

\[
V_o \propto |\varepsilon|_{G_1} - \varepsilon|_{G_2} + \varepsilon|_{G_3} - \varepsilon|_{G_4}| = -2CFd
\]

Note that, since both \( d \) and \( C \) are constants, the output from the bridge is strictly proportional to force \( F \). In particular, note that the output is not a function of the position of force \( F \); that is, the output is independent of distance \( x \). This implies that a couple applied at the end of the beam would produce no output from the bridge. Also, if the beam has a rectangular cross-section and the gages are placed on the neutral axis for bending in the plane at right angles to that shown, then there will be no output for a force component perpendicular to the figure. Finally, an axial force component will produce the same tensile strain in all four gauges and, thus, will produce no bridge output.