ME 556 - HOMEWORK ASSIGNMENT #1

Solutions

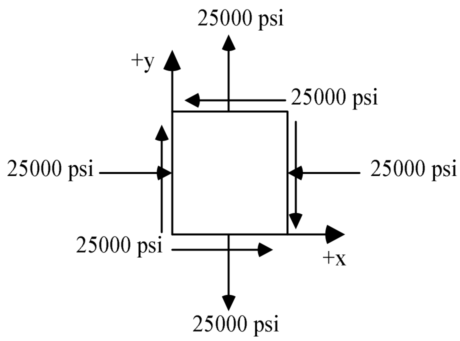
1. The stress state at a point of interest on a structure is known to be:

σ_{xx} = -25000 psi σ_{yy} = 25000 psi τ_{xy} = -25000 psi

For each of the following coordinate systems (i) determine the three in-plane stress components, and (ii) draw a "rough sketch" of the stress components on a *properly oriented* element. Make sure your sketch is neat and easily interpreted.

SOLUTION: Mohr's circle of stress for this problem is summarized on a following page. Stress states corresponding to parts (a), (b), (c), and (d) are shown in black, blue, red, and brown fonts, respectively. Alternatively, the stress transformation equations discussed during the review lectures can be applied in each case

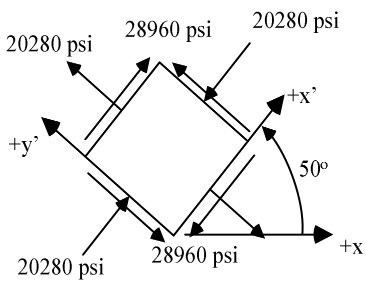
(a) The x-y coordinate system: $\sigma_{xx} = -25000\,psi$, $\sigma_{yy} = +25000\,psi$, $\tau_{xy} = -25000\,psi$



(b) The x'-y' coordinate system, 50° CCW from the x-y coordinate system:

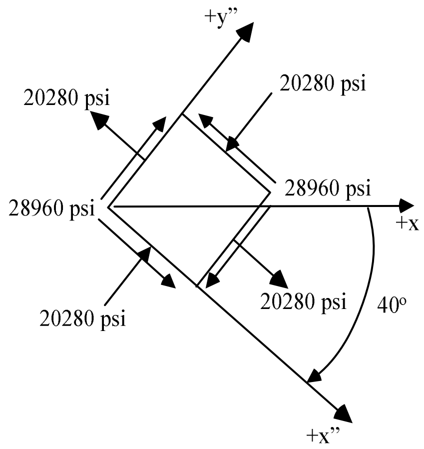
$$\begin{Bmatrix}\sigma_{x'x'}\\ \sigma_{y'y'}\\ \tau_{x'y'}\end{Bmatrix} = \begin{bmatrix}\cos^2\theta & \sin^2\theta & 2\cos\theta\sin\theta\\ \sin^2\theta & \cos^2\theta & -2\cos\theta\sin\theta\\ -\cos\theta\sin\theta & \cos\theta\sin\theta & (\cos^2\theta-\sin^2\theta)\end{bmatrix}\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \tau_{xy}\end{Bmatrix}$$

$$= \begin{bmatrix}\cos^2(50°) & \sin^2(50°) & 2\cos(50°)\sin(50°)\\ \sin^2(50°) & \cos^2(50°) & -2\cos(50°)\sin(50°)\\ -\cos(50°)\sin(50°) & \cos(50°)\sin(50°) & (\cos^2(50°)-\sin^2(50°))\end{bmatrix}\begin{Bmatrix}-25ksi\\ 25ksi\\ -25ksi\end{Bmatrix} = \begin{Bmatrix}-20280\ psi\\ +20280\ psi\\ +28960\ psi\end{Bmatrix}$$



(c) The x"-y" coordinate system, 40° CW from the x-y coordinate system:

$$\begin{Bmatrix} \sigma_{x'x'} \\ \sigma_{y'y'} \\ \tau_{x'y'} \end{Bmatrix} = \begin{bmatrix} \cos^2(-40°) & \sin^2(-40°) & 2\cos(-40°)\sin(-40°) \\ \sin^2(-40°) & \cos^2(-40°) & -2\cos(-40°)\sin(-40°) \\ -\cos(-40°)\sin(-40°) & \cos(-40°)\sin(-40°) & (\cos^2(-40°)-\sin^2(-40°)) \end{bmatrix} \begin{Bmatrix} -25ksi \\ 25ksi \\ -25ksi \end{Bmatrix} = \begin{Bmatrix} 20280psi \\ -20280psi \\ -28960psi \end{Bmatrix}$$

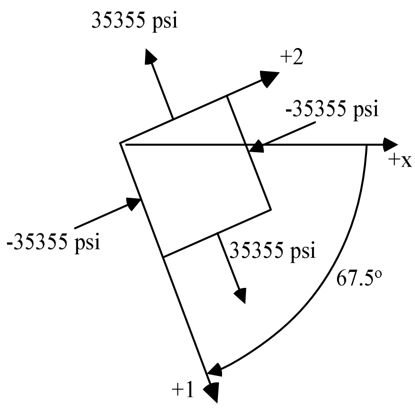


(d) The principal stress coordinate system: Using Eqs 1.12 and 1.14a:

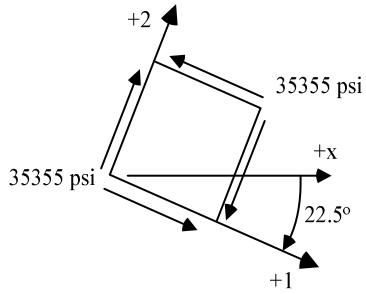
$$\sigma_1, \sigma_2 = \frac{\sigma_{xx}+\sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} = \frac{-25ksi+25ksi}{2} \pm \sqrt{\left(\frac{-25ksi-25ksi}{2}\right)^2 + (-25ksi)^2} = \pm 35{,}355\,psi$$

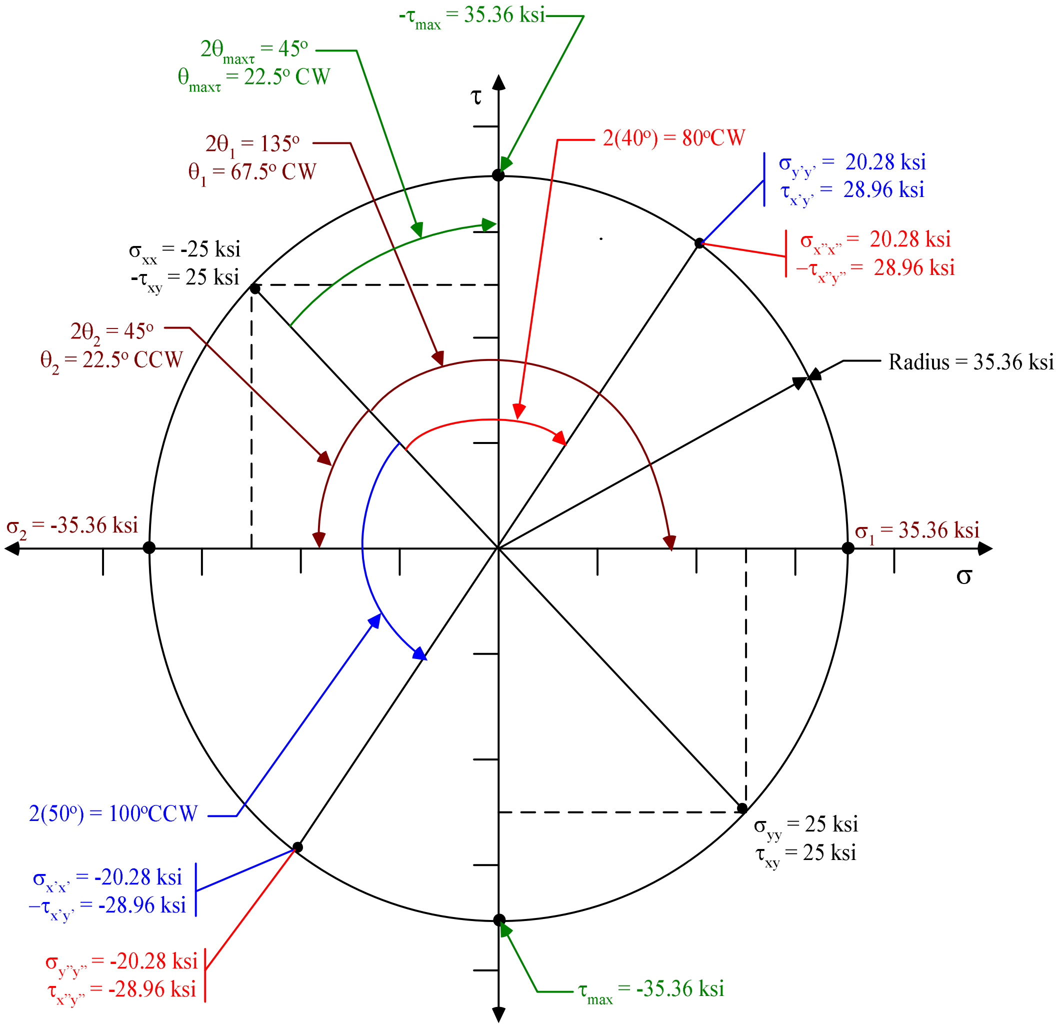
$$\theta_1 = \frac{1}{2}\tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_{xx}-\sigma_{yy}}\right) = \frac{1}{2}\tan^{-1}\left(\frac{2(-25ksi)}{-25ksi--25ksi}\right) = 22.5°$$

Inspection of Mohr's circle shows that this is the angle from the x-axis to the +2-axis. Hence, the angle from the x-axis to the +1 axis is (22.5°-90°) = -67.5°



(e) From Mohr's circle, the maximum shear stress exists in a coordinate system 22.5° CW from the x-axis , with a magnitude given by: $\tau_{\max} = \frac{1}{2}(\sigma_1 - \sigma_2) = \frac{1}{2}(35355\,psi + 35355\,psi) = 35355\,psi$





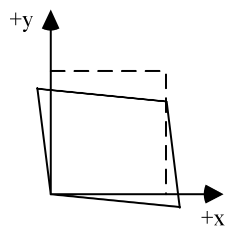
2. The strain state at a point of interest on a structure is measured to be:

ε_{xx} = 3000 $\mu m/m$ $\qquad$ ε_{yy} = -3000 $\mu m/m$ $\qquad$ γ_{xy} = -3000 μrad

For each of the following coordinate systems (i) determine the three in-plane strain components, and (ii) draw a "rough sketch" of both the initial square element and the deformed element, *properly oriented*. Make sure your sketch is neat and easily interpreted.

<u>SOLUTION</u>: Mohr’s circle of strain for this problem is summarized on a following page. Strain states corresponding to parts (a), (b), (c), and (d) are shown in black, blue, red, and brown fonts, respectively. Alternatively, the strain transformation equations discussed during the review lectures can be applied in each case.

(a) The x-y coordinate system: $\begin{aligned}\varepsilon_{xx} &= 3000\mu m/m \\ \varepsilon_{yy} &= -3000\mu m/m \\ \gamma_{xy} &= -3000\mu rad\end{aligned}$

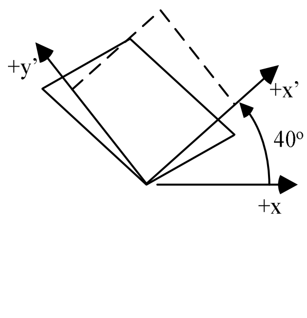


(b) The x'-y' coordinate system, 40° CCW from the x-y coordinate system:

$$\begin{Bmatrix} \varepsilon_{x'x'} \\ \varepsilon_{y'y'} \\ \gamma_{x'y'}/2 \end{Bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\cos\theta\sin\theta \\ \sin^2\theta & \cos^2\theta & -2\cos\theta\sin\theta \\ -4os\theta\sin\theta & \cos\theta\sin\theta & (\cos^2\theta - \sin^2\theta) \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy}/2 \end{Bmatrix}$$

$$= \begin{bmatrix} \cos^2(40°) & \sin^2(40°) & 2\cos(40°)\sin(40°) \\ \sin^2(40°) & \cos^2(40°) & -2\cos(40°)\sin(40°) \\ -\cos(40°)\sin(40°) & \cos(40°)\sin(40°) & (\cos^2(40°) - \sin^2(40°)) \end{bmatrix} \begin{Bmatrix} 3000\mu m/m \\ -3000\mu m/m \\ -1500\mu rad \end{Bmatrix} = \begin{Bmatrix} -956\mu m/m \\ 956\mu m/m \\ -3215\mu rad \end{Bmatrix}$$

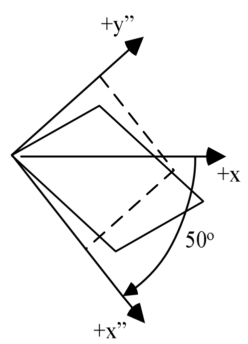
$$\begin{Bmatrix} \varepsilon_{x'x'} \\ \varepsilon_{y'y'} \\ \gamma_{x'y'} \end{Bmatrix} = \begin{Bmatrix} -956\mu m/m \\ 956\mu m/m \\ -6430\mu rad \end{Bmatrix}$$



(c) The x"-y" coordinate system, 50° CW from the x-y coordinate system:

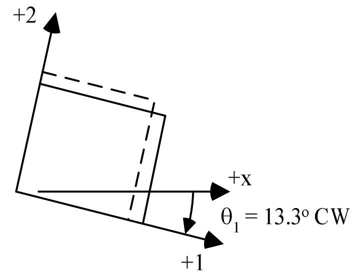
$$\begin{Bmatrix} \varepsilon_{x''x''} \\ \varepsilon_{y''y''} \\ \gamma_{x''y''}/2 \end{Bmatrix} = \begin{bmatrix} \cos^2(-50°) & \sin^2(-50°) & 2\cos(-50°)\sin(-50°) \\ \sin^2(-50°) & \cos^2(-50°) & -2\cos(-50°)\sin(-50°) \\ -\cos(-50°)\sin(-50°) & \cos(-50°)\sin(-50°) & (\cos^2(-50°)-\sin^2(-50°)) \end{bmatrix} \begin{Bmatrix} 3000\mu m/m \\ -3000\mu m/m \\ -1500\mu rad \end{Bmatrix} = \begin{Bmatrix} 956\mu m/m \\ -956\mu m/m \\ 3215\mu rad \end{Bmatrix}$$

$$\begin{Bmatrix} \varepsilon_{x''x''} \\ \varepsilon_{y''y''} \\ \gamma_{x''y''} \end{Bmatrix} = \begin{Bmatrix} 956\mu m/m \\ -956\mu m/m \\ 6430\mu rad \end{Bmatrix}$$



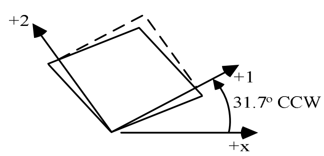
(d) The principal strain coordinate system. From Mohr’s circle:

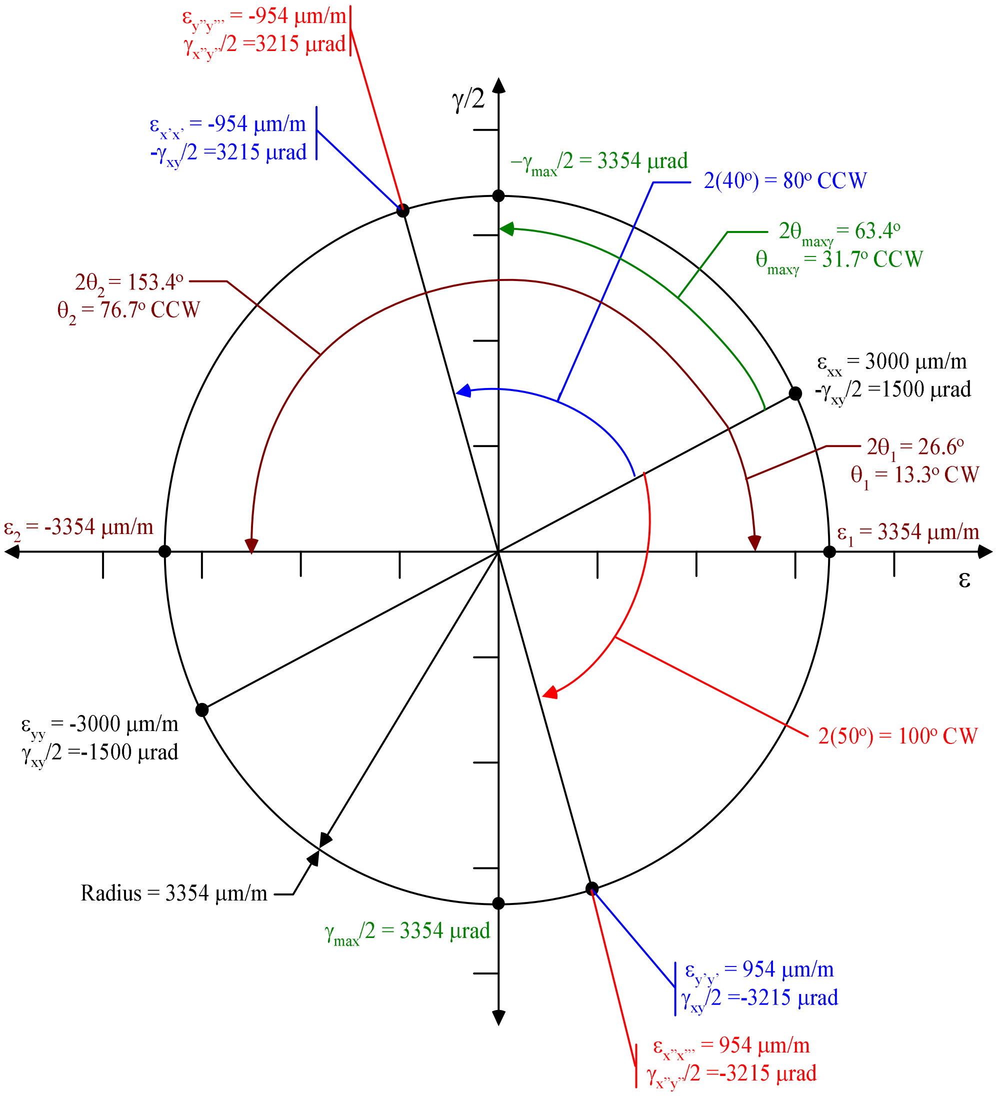
$$\varepsilon_{11} = 3354\mu m/m$$
$$\varepsilon_{22} = -3354\mu m/m$$
$$\theta_1 = 13.3°CW$$



(e) The maximum shear strain coordinate system: From Mohr’s circle:

$$\gamma_{max} = -6708 rad$$
$$\theta_{max} = 31.7°CCW$$





ε_y''y'' = -954 μm/m
γ_x''y''/2 =3215 μrad
γ/2
ε_x'x' = -954 μm/m
-γ_xy/2 =3215 μrad
−γ_max/2 = 3354 μrad
2(40°) = 80° CCW
2θ_maxγ = 63.4°
θ_maxγ = 31.7° CCW
2θ_2 = 153.4°
θ_2 = 76.7° CCW
ε_xx = 3000 μm/m
-γ_xy/2 =1500 μrad
2θ_1 = 26.6°
θ_1 = 13.3° CW
ε_2 = -3354 μm/m
ε_1 = 3354 μm/m
ε
ε_yy = -3000 μm/m
γ_xy/2 =-1500 μrad
2(50°) = 100° CW
Radius = 3354 μm/m
γ_max/2 = 3354 μrad
ε_y'y' = 954 μm/m
γ_xy/2 =-3215 μrad
ε_x''x'' = 954 μm/m
γ_x''y''/2 =-3215 μrad

3. An isotropic material is known to have the following elastic properties:

$E = 205GPa$ $\qquad \nu = 0.285$ $\qquad G = 79.8GPa$

What stress tensor must be applied to this material in order to induce a uniaxial strain $\varepsilon_{yy} = 2000\mu m/m$?

SOLUTION: Apply Hooke's law using the specified material properties and strain tensor:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{(1+\nu)(1-2\nu)} \begin{bmatrix} (1-\nu) & \nu & \nu & 0 & 0 & 0 \\ \nu & (1-\nu) & \nu & 0 & 0 & 0 \\ \nu & \nu & (1-\nu) & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{(1-2\nu)}{2} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \frac{205x10^9}{(1.285)(0.430)} \begin{bmatrix} (0.715) & (0.285) & (0.285) & 0 & 0 & 0 \\ (0.285) & (0.715) & (0.285) & 0 & 0 & 0 \\ (0.285) & (0.285) & (0.715) & 0 & 0 & 0 \\ 0 & 0 & 0 & (0.215) & 0 & 0 \\ 0 & 0 & 0 & 0 & (0.215) & 0 \\ 0 & 0 & 0 & 0 & 0 & (0.215) \end{bmatrix} \begin{Bmatrix} 0 \\ 2000\mu m/m \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} 211.5MPa \\ 530.5MPa \\ 211.5MPa \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$