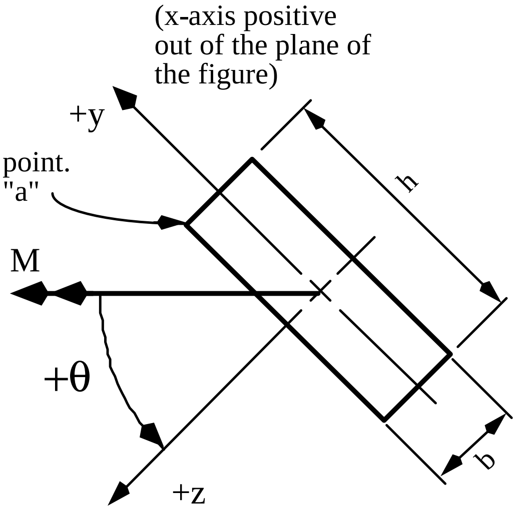
ME 556 Homework Problem #2

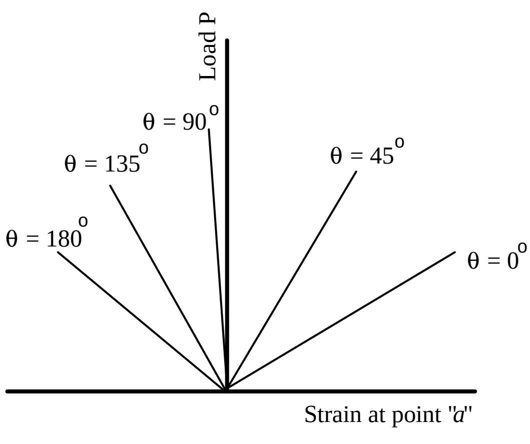
Due Thursday October 13

(Note: A brief review of "elementary beam theory" is attached; this review is intended to help with this problem...if you need help! Also, beam theory is discussed in virtually all textbooks devoted to "strength of materials", and most have a discussion of unsymmetric bending. See, for example, Gere and Timoshenko, Mechanics of Materials, 4th Edition, PWS Publishers, ISBN 0-534-93429-3, Secs 6.4, 6.5)

An aluminum beam with rectangular cross-section is subjected to 4-pt bending, and hence the central region of the beam is subjected to a pure bending moment $M = Pd/2$. As shown in the sketch below, the $y-z$ axes are defined as the principal centroidal axes of the rectangular cross-section, and the $z-$axes is oriented at an angle θ with respect to the bending moment, M. That is, the bending moment vector is perpendicular to the vertical plane, but is not aligned with a principal centroidal axis of the cross-section (unless $\theta = 0°$ or $90°$).

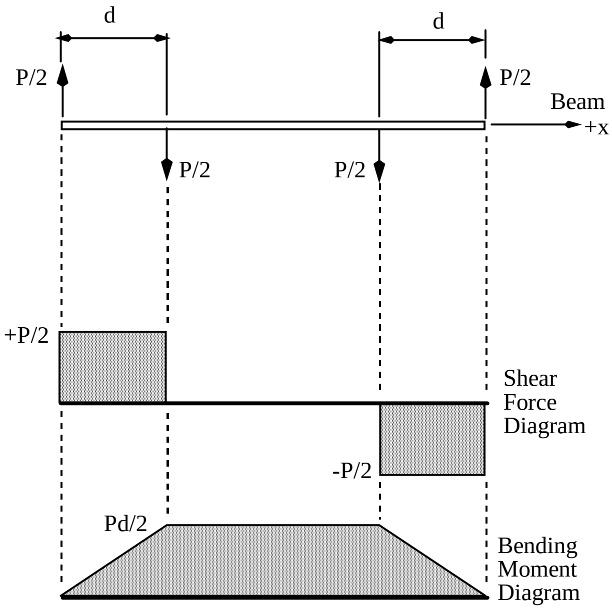
Point "*a*" is defined in the sketch, and is located at the point $y = +h/2$, $z = +b/2$. Using any appropriate software tool or package (Excel, MathCad, Mathematica, Maple, FORTRAN, etc), create a procedure to plot the predicted axial strain induced at point *a* (denoted ε_x^a, say) by a monotonically-increasing load *P*, for five orientation angles: $\theta = 0°, 45°, 90°, 135°, \text{and } 180°$. A sample plot (which may not be accurate) is shown below. Let P range from 0 to 500 *lbf*, and use the following properties and dimensions: $E = 10x10^6\ psi$, $\nu = 0.33$, $d = 5.0\ in$, $b = 0.5\ in$, $h = 1.0\ in$





"ELEMENTARY BEAM THEORY" - REVIEW COMMENTS

The term "beam" refers to a relatively long and slender structural member subjected to loads acting transversely to its' longitudinal axis. A beam subjected to "4-pt bending" is shown in the sketch below. This loading configuration places the central portion of the beam in "pure bending," i.e., in the central portion of the beam the shear force $V = 0$, and the bending moment $M = Pd/2$. Note that all loads shown lie within the vertical plane.



Elementary beam theory is principally based on the following assumptions:

1. The beam is composed of a homogenous, isotropic material.
2. The beam is composed of a linear-elastic material (no yielding occurs).
3. The "Kirchhoff hypothesis" is valid: *"A plane cross-section of the beam which is perpendicular to the axis of the beam prior to loading remains plane and perpendicular to the axis of the beam after loading."*

These assumptions ultimately lead to the conclusion that the normal stress induced parallel to the axis of the beam (σ_x) is linearly proportional to the bending moment *M*, and that the shear stress induced in the beam is linearly proportional to the shear force *V*. In the case of pure bending (i.e., if $V = 0$) the shear stress is zero and a *uniaxial* state of stress is induced in the beam. Note that the central region of a beam in 4-pt bending is in pure bending. The normal stress σ_x can be determined using the so-called "flexure-formula." The mathematical form of the flexure formula depends on whether the bending moment vector is aligned with a principal centroidal axis of the beam cross-section.

A Case In Which The Bending Moment Is Aligned With A Principal Centroidal Axis:

A rectangular beam cross-section oriented *symmetrically* about the vertical plane is shown on the following page (the vertical direction is labeled as the y-axis in this case; hence the cross-section is symmetric about the *x*-*y* plane). The fact that the beam cross-section is symmetric about the vertical plane insures that the two principal centroidal axes are vertical and horizontal. The beam is subjected to a moment vector M_z parallel to the *z*-axis, hence the bending moment vector is aligned with a principal centroidal axis. This is a case of so-called "symmetrical bending", and the axial stress induced at any distance "y" from the *z*-axis is given by:

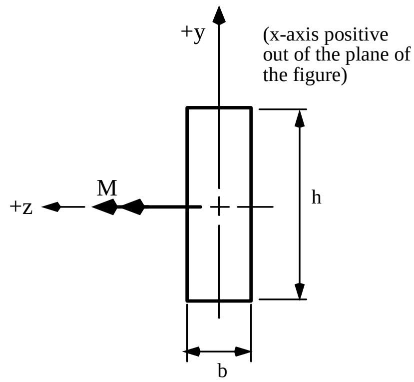
$$\sigma_x = \frac{-My}{I_z}$$

where:

I_z = moment of inertia of the cross-sectional area about the principal centroidal axis *z* (recall that for a rectangular cross-section, $I_z = bh^3/12$)

Since the beam is subjected to a uniaxial state of stress, the uniaxial form of Hooke's law applies. Hence, the axial strain (ε_x) induced at any distance y from the *z*-axis is given by:

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{-My}{EI_z}$$



A Case In Which The Bending Moment Is *NOT* Aligned With A Principal Centroidal Axis:

A rectangular beam cross-section oriented *non-symmetrically* about the vertical plane is shown on the following page. The cross-section is symmetric about both the *y*- and *z*-axes, and hence these axes are the principal centroidal axes (recall that the area moments of inertia are maximum and minimum about the principal centroidal axes). If this beam is subjected to a pure bending moment *M* acting in the horizontal plane (as shown) then the bending moment vector is *not* aligned with the principal centroidal axes. This is a case of "unsymmetrical bending", and the axial stress induced at any point in the beam cross-section is given by:

$$\sigma_x = \frac{M_y z}{I_y} - \frac{M_z y}{I_z}$$

where:

y, z = coordinates of the point of interest (measured along the *y*- and *z*- axes)

I_y, I_z = principal moments of inertia of the beam cross-sectional area about principal centroidal axes *y* and *z*

M_y, M_z = components of the total bending moment vector *M* acting about the *y*- and *z*- axes, respectively

<u>*Note*</u> that a positive angle θ is defined in accordance to the right-hand rule, and is measured *from* the bending moment vector *to* the +z-axis.

Since the beam is subjected to a uniaxial state of stress, the uniaxial form of Hooke's law applies. Hence, the axial strain (ε_x) induced at any point is given by:

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{M_y z}{EI_y} - \frac{M_z y}{EI_z}$$