ME 556 HOMEWORK PROBLEM #3
Due Thursday, October 20

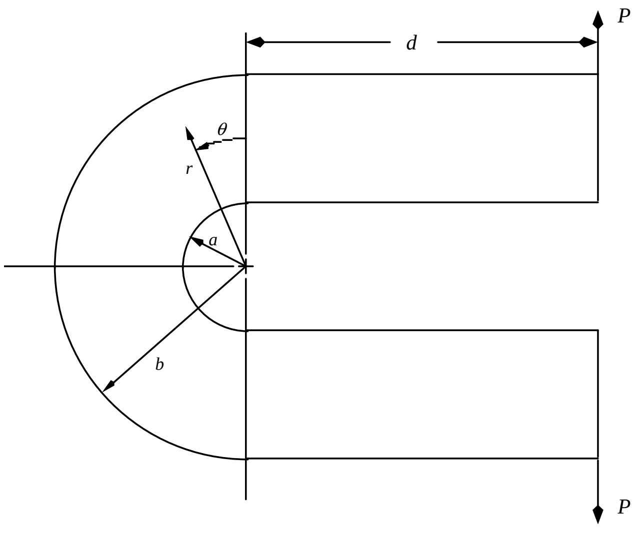
Consider the U-shaped curved beam shown below (with rectangular cross-section and thickness *h*), which is loaded by force *P* as indicated. A free-body diagram reveals that a moment $M = Pd$ and shear force $V = P$ are applied to the "curved section" of the beam, as shown.

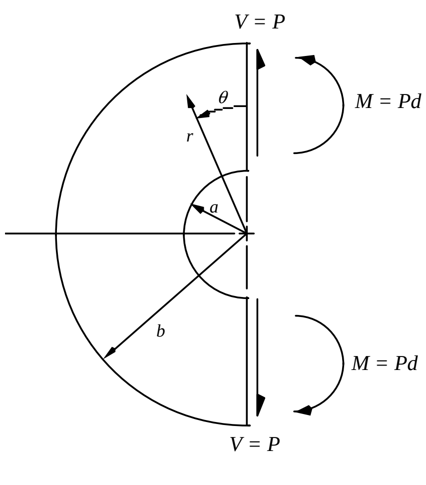
Based on elasticity solutions for a curved beam (described on the following pages), develop a computer-based methodology* to generate plots of the following quantities along a cross-section located at an arbitrary angular position, θ:

(a) The maximum principal stress, σ_1, vs radial position, *r*.
(b) The minimum principal stress, σ_2, vs radial position, *r*.
(c) The angle between {the line of action of force *P*} and {the +*1*-axis}, vs radial position, *r*.

Use the following dimensions and load, and submit plots for the cross-section at $\theta = 45°$:

a = 1.0 in b = 3.0 in h = 0.125 in d = 7 in P = 240 lbf





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* The "computer methodology" may well involve several tools; i.e., the use of FORTRAN, BASIC, C, MatLab, Mathematica,EXCEL etc. Note: You will use this program during lab #3; during the lab a U-shaped curve beam with dimensions different than those considered during this assignment will be studied.

BACKGROUND INFORMATION

Two different elasticity solutions must be superimposed to solve this problem (of course, a finite-element analysis could also be performed, but that is another topic!). The first solution is for a curved beam subjected to a bending moment only, while the second solution is for a curved beam subjected to a shear force only. These two solutions are described separately below:

I. A semi-circular beam (with rectangular cross-section and thickness h) loaded by a pure bending moment M is shown below. Using the theory of elasticity it can be shown* that the radial, tangential, and shear stresses induced in the beam are given by:

$$\sigma_r = \frac{-4M}{Nh}\left[\frac{a^2b^2}{r^2}\ln\left(\frac{b}{a}\right) + b^2\ln\left(\frac{r}{b}\right) + a^2\ln\left(\frac{a}{r}\right)\right]$$

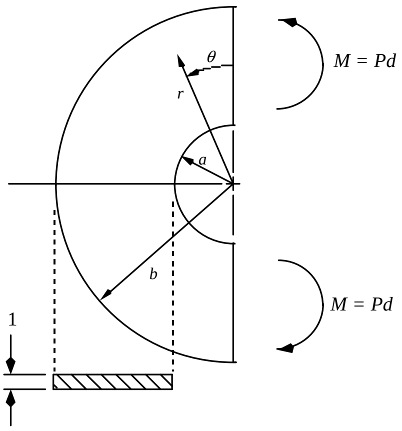
$$\sigma_\theta = \frac{-4M}{Nh}\left[\frac{-a^2b^2}{r^2}\ln\left(\frac{b}{a}\right) + b^2\ln\left(\frac{r}{b}\right) + a^2\ln\left(\frac{a}{r}\right) + b^2 - a^2\right]$$

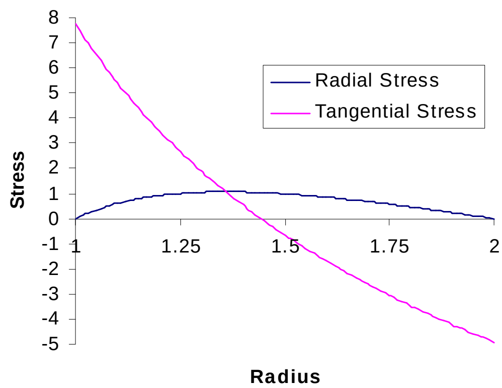
$$\tau_{r\theta} = 0$$

where:

$$N = \left(b^2 - a^2\right)^2 - 4a^2b^2\left(\ln\frac{b}{a}\right)^2$$

Note that since $\tau_{r\theta} = 0$ the radial and tangential axes are everywhere the principal axes. Also note that since stresses do not depend on θ, stresses vary only with *r*. For example, the radial and tangential stresses for the specific case of *a* =1.0, *b*= 2.0, *h*= 1.0, and *M* = 1.0 are plotted below; these same stresses are induced at any angular position $0° < \theta < 180°$.





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*Timoshenko and Goodier, Theory of Elasticity, 3rd Edition, McGraw-Hill, Article 29. NOTE: These authors use "log" to denote the natural logarithm. Also, this problem is similar to Prob 3.24 in the Shukla and Dally textbook.

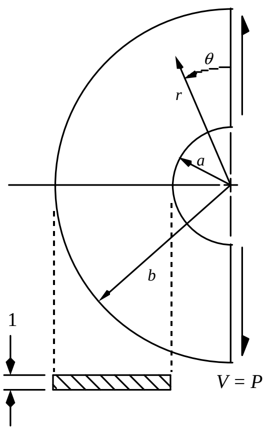
II A semi-circular beam (with rectangular cross-section and thickness *h*) loaded by a shear force *V* is shown below. In this case it can be shown* that the radial, tangential, and shear stresses induced are given by:

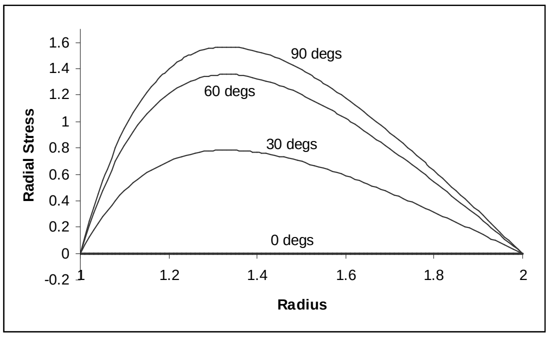
$$\sigma_r = \left(2Ar - \frac{2B}{r^3} + \frac{D}{r}\right)\sin\theta \qquad \sigma_\theta = \left(6Ar + \frac{2B}{r^3} + \frac{D}{r}\right)\sin\theta$$

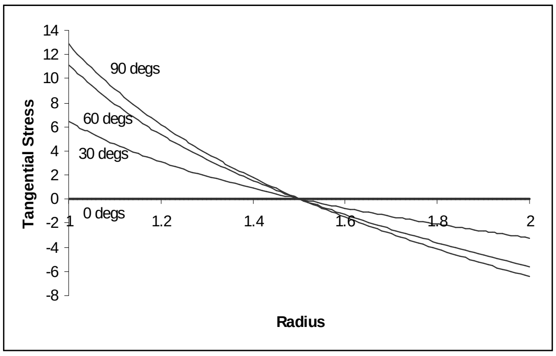
$$\tau_{r\theta} = -\left(2Ar - \frac{2B}{r^3} + \frac{D}{r}\right)\cos\theta$$

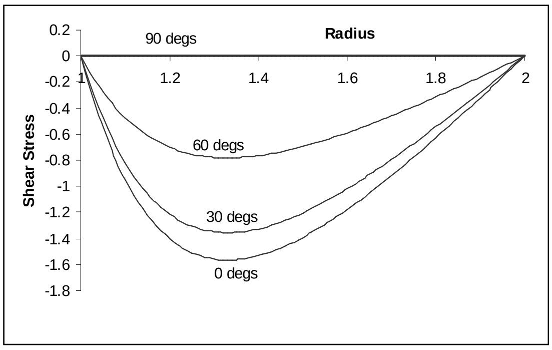
where: $A = \dfrac{-V}{2Eh}$ $\qquad B = \dfrac{Va^2b^2}{2Eh}$ $\qquad D = \dfrac{V}{Eh}\left(a^2 + b^2\right)$ $\qquad E = a^2 - b^2 + \left(a^2 + b^2\right)\ln\left(\dfrac{b}{a}\right)$

Note that since $\tau_{r\theta} \neq 0$ (in general) the radial and tangential axes are not the principal axes (in general). Also note that stresses depend on both *r* and θ. For example, σ_r , σ_θ , and $\tau_{r\theta}$ for the specific case of *a* =1.0, *b* = 2.0, *h* =1.0, and *V* = 1.0 are plotted below for several values of θ.









*Timoshenko and Goodier, Theory of Elasticity, 3rd Edition, McGraw-Hill, Article 33. NOTE: These authors use "log" to denote the natural logarithm. Also, this problem is similar to Prob 3.25 in the Shukla and Dally textbook.