ME599: ADVANCED MECHANICS OF COMPOSITE MATERIALS
TAKEHOME MIDTERM EXAM SPRING QTR 2003 PROF. M. TUTTLE

SOLUTION

Note: This problem can be solved following several different viable (and mathematically equivalent) solutions paths. Only one solution path is described here. In essence, the approach follows the analysis presented in Section 8.6 of the Coursepak (see also the solution to Problem 1 of Homework set 3).

I. Preliminary discussion

A. The external bending moment M_o^b will induce:

- a stress resultant $N_{xx}\big|_{tf}$ and moment resultant $M_{xx}\big|_{tf}$, acting at the midplane of the flange laminate, and
- a stress resultant $N_{xx}\big|_{w}$ in the web laminate

Also, $N_{xx}\big|_{w}$ will vary over the web, and the maximum value will occur at the bottom of the web (i.e., at $z = z_b$).

B. Based on how the beam was formed:

- The web is a symmetric 16-ply graphite-epoxy laminate with the following stacking sequence: $[(0_2/90/0)_s]_s$. Although the overall beam dept is known (75 *mm* = 0.075 *m*), the height of the web laminate (length *h*) must be calculated based on the thickness of the flange laminate.

- The flange is a symmetric 22-ply graphite-epoxy laminate with the following stacking sequence: $[(0_2/90/0)_s/30/\pm 45]_s$. The overall width of the beam (as well as the width of the flange laminate) is *b* = 60 *mm* = 0.060 *m*. Since each ply has a thickness of 0.125 *mm* = 0.000125 *m*, the thickness of the flange laminate is $t_{tf} = 22(0.000125m) = 0.002750m$. Therefore, the height of the web laminate is: $h = 0.075m - 0.002750m = 0.072250m$.

C. Program BEAM was used to determine the location of the neutral axis and the effective flexural rigidity of the T-Beam (an excessive number of significant digits will be reported throughout this document to minimize confusion due to round-off issues):

$$z_b = 0.0546394(m)$$

$$\overline{IE} = 16.9951E3(N - m^2)$$

D. As per the discussion in the CoursePak (see page 57), the curvature induced by the pure bending moment is:

$$\kappa_{xx} = \frac{M_o^b}{\overline{IE}} = \frac{M_o^b}{16.9951E3}$$

Note from the figure on the exam that the externally applied bending moment is negative: $M_o^b < 0$. The negative bending moment tends to cause the upper portion of the T-beam to experience tensile stress/strain, while the lower portion tends to experience compressive stress/strain.

II. Determining the Bending Moment Necessary to Cause Failure of the Flange Laminate

A. Based on the analysis methodology presented in Section 8.6 of the Coursepak (see also the solution to Problem 1 of Homework set 3), the stress resultant (N_{xx}) and bending moment resultant (M_{xx}) induced at the midplane of the top flange of the T-beam are:

$$N_{xx}\big|_{tf} = (z_b - h - \frac{t_{tf}}{2})\frac{M_o^b}{(a_{11}^{tf})(\overline{IE})}$$

$$M_{xx}\big|_{tf} = \frac{M_o^b}{(d_{11}^{tf})(\overline{IE})}$$

The values of a_{11}^{tf} and d_{11}^{tf} for a $[(0_2/90/0)_s/30/\pm 45]_s$ laminate can be determined using program CLT:

$$a_{11}^{tf} = 0.40910E-8\ (m/N)$$
$$d_{11}^{tf} = 0.54947E-2\left(\frac{1}{N\text{-}m}\right)$$

Substituting all known numerical values for this problem, we have:

$$N_{xx}\big|_{tf} = \left[(.0546394) - (0.07225) - \left(\frac{0.00275}{2}\right)\right]\left[\frac{M_o^b}{(0.40910E-8)(16.9951E3)}\right]$$

$$N_{xx}\big|_{tf} = -273.0698 M_o^b$$

$$M_{xx}\Big|_{tf} = \frac{M_o^b}{(d_{11}^{tf})(\overline{IE})} = \frac{M_o^b}{(0.54947E-2)(16.9951E3)}$$

$$M_{xx}\Big|_{tf} = 0.01070859 M_o^b$$

The values of N_{xx} and M_{xx} induced in the flange laminate when $M_o^b = -1N-m$ is applied are therefore:

$$N_{xx}\big|_{tf} = 273.0698\ N/m$$

$$M_{xx}\Big|_{tf} = -0.01070859\ N-m/m$$

Ply stresses ($\sigma_{11}, \sigma_{22}, \tau_{12}$) induced by these loads were determined using CLT; stresses in plies of particular interest are summarized in Table 1. Also, in each case a scaling factor (*fac*) is calculated based on the appropriate failure strength. The scaling factor indicates how much M_o^b must be increased in order to cause failure of a given ply. For example, Table 1 indicates that a stress $\sigma_{11} = 0.16185\ MPa$ is induced in ply 1 (a $0°$ ply) by the loads $N_{xx} = 273.0698 N/m$ and $M_{xx} = -0.01070859\ N-m/m$. This stress level is very low compared to the fiber failure strength $\sigma_{11}^{fT} = 1500 MPa$. If the applied loads were increased by a factor of 9268, then the fiber stress in ply 1 would equal the fiber failure strength. That is, fiber failures in ply 1 would occur if the external bending moment were increased to $M_o^b = -9268\ N-m$ (assuming other plies had not failed at lower values of M_o^b).

As indicated in Table 1, ply 3 (a $90°$ ply) is predicted to fail at the lowest value of M_o^b. Specifically, matrix failures are predicted to occur in ply 3 if the magnitude of the external bending moment is increased to 4444 *N-m* (that is, if $M_o^b = -4444\ N-m$). The stress and moment resultants in the flange at this load level are:

Table 1: Failure Analysis – Top Flange Laminate (using $M_o^b = -1\,N-m$)

Ply Number	Fiber Angle	σ_{11} (Pa)	$fac = \frac{\sigma_{11}^{fT}}{\sigma_{11}}$ (or) $\frac{\sigma_{11}^{fC}}{\lvert\sigma_{11}\rvert}$	σ_{22} (Pa)	$fac = \frac{\sigma_{22}^{yT}}{\sigma_{22}}$ (or) $\frac{\sigma_{22}^{yC}}{\lvert\sigma_{22}\rvert}$	τ_{12} (Pa)	$fac = \frac{\tau_{12}^{y}}{\lvert\tau_{12}\rvert}$
1	0	.16185E6	9268	.44686E3	111892	-.2002E4	37463
3	90	-.31469E5	38133	.11250E5	***4444***	.19927E4	37637
9	30	.93468E5	16048	.37624E4	13290	-.10611E5	7068
10	45	.44594E5	33637	.64595E4	7741	-.11056E5	7458
11	-45	.76699E5	19557	.45729E4	10934	.10992E5	6823
12	-45	.76190E5	19688	.45323E4	11032	.10929E5	6862
13	45	.43300E5	34642	.63246E4	7906	-.10866E5	6902
14	30	.89970E5	16672	.36136E4	13837	-.10326E5	7263
20	90	-.30521E5	39317	.10018E5	4992	.19133E4	39199
22	0	.14093E6	10644	.17437E3	286747	-.19040E4	39391

$$N_{xx}\Big|_{tf} = (z_b - h - \frac{t_{tf}}{2})\frac{M_o^b}{(a_{11}^{tf})(\overline{IE})}$$

$$N_{xx}\Big|_{tf} = \left[(.0546394) - (0.07225) - \left(\frac{0.00275}{2}\right)\right]\left[\frac{-4444}{(0.40910E-8)(16.9951E3)}\right]$$

$$N_{xx}\Big|_{tf} = 1213.5\, kN$$

$$M_{xx}\Big|_{tf} = \frac{M_o^b}{(d_{11}^{tf})(\overline{IE})} = \frac{-4444}{(0.54947E-2)(16.9951E3)}$$

$$M_{xx}\Big|_{tf} = -47.701\, N-m/m$$

These results can be confirmed using program CLT….for example, performing an analysis in which $N_{xx} = 1213.5\, kN/m$ and $M_{xx} = -47.701\, N-m/m$ are applied to a $[(0_2/90/0)_s/30/\pm 45]_s$ graphite-epoxy laminate (with the specified elastic properties), it will be found that the matrix stress in ply 3 is precisely equal to the tensile yield stress: $\sigma_{22}\big|_{ply3} = 50MPa = \sigma_{22}^{yT}$.

III. Determining the Bending Moment Necessary to Cause Failure of the Web Laminate

A. The stress resultant (N_{xx}) induced in the web laminate by a pure bending moment M_o^b is (see page 52 of the CoursePak):

$$N_{xx}\Big|_w = z\left(\frac{\kappa_{xx}}{a_{11}^w}\right) = z\left(\frac{M_o^b}{(a_{11}^w)(\overline{IE})}\right)$$

The stress resultant will be a maximum at $z = z_b$:

$$N_{xx}\Big|_w^{\max} = z_b\left(\frac{\kappa_{xx}}{a_{11}^w}\right) = z_b\left(\frac{M_o^b}{(a_{11}^w)(\overline{IE})}\right)$$

The value of a_{11}^{w} for the $[(0_2/90/0)_s]_s$ web laminate can be determined using program CLT:

$$a_{11}^{w} = 0.48040E-8\,(m/N)$$

Substituting all known values the maximum stress resultant induced in the web laminate is:

$$N_{xx}\Big|_{w}^{\max} = (0.0546394)\left(\frac{M_o^b}{(0.48040E-8)(16.9951E3)}\right)$$

$$N_{xx}\Big|_{w}^{\max} = 669.2359 M_o^b$$

The maximum value of N_{xx} induced in the web when $M_o^b = -1N-m$ is applied is therefore:

$$N_{xx} = -669.2359\; N/m$$

Ply stresses induced by this loads were determined using CLT and summarized in Table 2. Also, in each case a scaling factor (*fac*) is calculated based on the appropriate failure strength. The scaling factor indicates how much M_o^b must be increased in order to cause failure of a given ply. For example, Table 2 indicates that a stress $\sigma_{11} = -0.43555\; MPa$ is induced in all $0°$ plies by the load $N_{xx} = -669.2359\; N/m$. If the applied loads were increased by a factor of 2755, then the fiber stress in the $0°$ plies would equal the fiber compressive strength. That is, compressive fiber failures all the $0°$ plies would occur if the external bending moment were increased to $M_o^b = -2755\; N-m$. The maximum stress resultant in the web laminate at this load level is:

$$N_{xx}\Big|_{w}^{\max} = z_b\left[\frac{M_o^b}{(a_{11}^{w})(\overline{EI})}\right] = (0.0546394m)\left[\frac{(-2755)}{(0.48040E-8)(16.9951E3)}\right]$$

$$N_{xx}\Big|_{w}^{\max} = -1844\; kN/m$$

This result can be confirmed using program CLT….for example, performing an analysis in which $N_{xx} = -1844\; kN/m$ is applied to a $[(0_2/90/0)_s]_s$ graphite-epoxy laminate (with the specified elastic properties), it will be found that the fiber stresses in all $0°$ plies are precisely equal to the compressive fracture stress…for all $0°$ plies:

$\sigma_{11} = -1200 MPa = \sigma_{11}^{fC}$

Table 2: Failure Analysis – Web Laminate (using $M_o^b = -1\,N-m$)

Ply Numbers	Fiber Angle	σ_{11} (Pa)	$fac = \frac{\sigma_{11}^{fT}}{\sigma_{11}}$ (or) $\frac{\sigma_{11}^{fC}}{\lvert\sigma_{11}\rvert}$	σ_{22} (Pa)	$fac = \frac{\sigma_{22}^{yT}}{\sigma_{22}}$ (or) $\frac{\sigma_{22}^{yC}}{\lvert\sigma_{22}\rvert}$
1,2,4,5,7,8,9,10, 12,13,15,16	0	-.43555E6	***2755***	-.61173E4	16347
3,6,11,14	90	.18352E5	81735	-.31810E5	3144

IV. Summary and Conclusion

To summarize the analysis presented above, first-ply failure of the flange laminate is predicted at $M_o^b = -4444\ N-m$ due to a tensile matrix failure in ply 3. In contrast, first-ply failure in the web laminate is predicted at $M_o^b = -2755\ N-m$ due to a compressive fiber failure in the $0°$ plies. Hence,

FIRST-PLY FAILURE OF THE T-BEAM IS PREDICTED TO OCCUR WHEN THE EXTERNAL BENDING MOMENT IS INCREASED TO $M_o^b = -2755\ N-m$.