Chapter 6: Mechanical Properties II

Outline

• Elastic recovery during plastic deformation
• Compressive, shear, and torsional deformation
• Hardness
• Variability of material properties
• Design/safety factors
Concepts of stress and strain

- **Tension tests**
  - engineering stress \( \sigma = \frac{F}{A_0} \)
  - engineering strain \( \varepsilon = \frac{l_i - l_0}{l_0} = \frac{\Delta l}{l_0} \)

- **Compression tests**
Elastic Deformation

1. Initial

2. Small load

3. Unload

Elastic means reversible!

\[ F \]
\[ \delta \]

Linear-elastic

Non-Linear-elastic
Plastic Deformation (Metals)

1. Initial

2. Small load
   - bonds stretch
   - & planes shear

3. Unload
   - planes still sheared

Plastic means permanent!

\[ \delta_{\text{elastic + plastic}} \]

\[ \delta_{\text{plastic}} \]

\[ \delta \]

\[ F \]
Elastic Strain Recovery

Adapted from Fig. 6.17, *Callister 7e.*
Other Elastic Properties

- Elastic **Shear modulus, \( G \):**
  \[ \tau = G \gamma \]

- Elastic **Bulk modulus, \( K \):**
  \[ P = -K \frac{\Delta V}{V_0} \]

- Special relations for isotropic materials:
  \[ G = \frac{E}{2(1 + \nu)} \]
  \[ K = \frac{E}{3(1 - 2\nu)} \]
Useful Linear Elastic Relationships

• Simple tension:

\[ \delta = \frac{FL_o}{EA_o} \quad \delta_L = -\nu \frac{Fw_o}{EA_o} \]

• Simple torsion:

\[ \alpha = \frac{2ML_o}{\pi r_o^4 G} \]

• Material, geometric, and loading parameters all contribute to deflection.
• Larger elastic moduli minimize elastic deflection.
Hardness

- Resistance to permanently indenting the surface.
- Large hardness means:
  - resistance to plastic deformation or cracking in compression.
  - better wear properties.

E.g., 10 mm sphere apply known force measure size of indent after removing load

Smaller indents mean larger hardness.

Increasing hardness:
- Most plastics
- Brasses
- Al alloys
- Easy to machine steels
- File hard
- Cutting tools
- Nitrided steels
- Diamond
Hardness: Measurement

• **Rockwell**
  - No major sample damage
  - Each scale runs to 130 but only useful in range 20-100.
  - Minor load 10 kg
  - Major load 60 (A), 100 (B) & 150 (C) kg
    • A = diamond, B = 1/16 in. ball, C = diamond

• **HB = Brinell Hardness**
  - $TS \text{ (psia)} = 500 \times HB$
  - $TS \text{ (MPa)} = 3.45 \times HB$
### Table 6.5 Hardness Testing Techniques

<table>
<thead>
<tr>
<th>Test</th>
<th>Indenter Description</th>
<th>Shape of Indentation</th>
<th>Load</th>
<th>Formula for Hardness Number&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brinell</td>
<td>10-mm sphere of steel or tungsten carbide</td>
<td></td>
<td></td>
<td>[ \text{HB} = \frac{2P}{\pi D \left[ D - \sqrt{D^2 - d^2} \right]} ]</td>
</tr>
<tr>
<td>Vickers microhardness</td>
<td>Diamond pyramid</td>
<td></td>
<td></td>
<td>[ \text{HV} = 1.854P/d_1^2 ]</td>
</tr>
<tr>
<td>Knoop microhardness</td>
<td>Diamond pyramid</td>
<td></td>
<td></td>
<td>[ \text{HK} = 14.2P/l^2 ]</td>
</tr>
<tr>
<td>Rockwell and</td>
<td>Diamond cone, diameter: (1/16), (1/8), (1/4), (1/2) in.</td>
<td></td>
<td>60 kg</td>
<td>Rockwell</td>
</tr>
<tr>
<td>Superficial Rockwell</td>
<td>Rockwell and Superficial Rockwell</td>
<td></td>
<td>100 kg</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stee spheres</td>
<td></td>
<td>150 kg</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>15 kg</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>30 kg</td>
<td>Superficial Rockwell</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>45 kg</td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> For the hardness formulas given, \( P \) (the applied load) is in kg, while \( D \), \( d \), \( d_1 \), and \( l \) are all in mm.

Hardness tests (continued)

- Rockwell and superficial rockwell
- $20 < \text{hardness} < 100$

### Table 6.5a Rockwell Hardness Scales

<table>
<thead>
<tr>
<th>Scale Symbol</th>
<th>Indenter</th>
<th>Major Load (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Diamond</td>
<td>60</td>
</tr>
<tr>
<td>B</td>
<td>$\frac{1}{16}$ in. ball</td>
<td>100</td>
</tr>
<tr>
<td>C</td>
<td>Diamond</td>
<td>150</td>
</tr>
<tr>
<td>D</td>
<td>Diamond</td>
<td>100</td>
</tr>
<tr>
<td>E</td>
<td>$\frac{1}{8}$ in. ball</td>
<td>100</td>
</tr>
<tr>
<td>F</td>
<td>$\frac{1}{16}$ in. ball</td>
<td>60</td>
</tr>
<tr>
<td>G</td>
<td>$\frac{1}{8}$ in. ball</td>
<td>150</td>
</tr>
<tr>
<td>H</td>
<td>$\frac{1}{16}$ in. ball</td>
<td>60</td>
</tr>
<tr>
<td>K</td>
<td>$\frac{1}{8}$ in. ball</td>
<td>150</td>
</tr>
</tbody>
</table>

### Table 6.5b Superficial Rockwell Hardness Scales

<table>
<thead>
<tr>
<th>Scale Symbol</th>
<th>Indenter</th>
<th>Major Load (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15N</td>
<td>Diamond</td>
<td>15</td>
</tr>
<tr>
<td>30N</td>
<td>Diamond</td>
<td>30</td>
</tr>
<tr>
<td>45N</td>
<td>Diamond</td>
<td>45</td>
</tr>
<tr>
<td>15T</td>
<td>$\frac{1}{8}$ in. ball</td>
<td>15</td>
</tr>
<tr>
<td>30T</td>
<td>$\frac{1}{8}$ in. ball</td>
<td>30</td>
</tr>
<tr>
<td>45T</td>
<td>$\frac{1}{8}$ in. ball</td>
<td>45</td>
</tr>
<tr>
<td>15W</td>
<td>$\frac{1}{8}$ in. ball</td>
<td>15</td>
</tr>
<tr>
<td>30W</td>
<td>$\frac{1}{8}$ in. ball</td>
<td>30</td>
</tr>
<tr>
<td>45W</td>
<td>$\frac{1}{8}$ in. ball</td>
<td>45</td>
</tr>
</tbody>
</table>
Hardness tests

- Brinell: 10-mm sphere of steel or tungsten carbide
- Knoop and Vickers microhardness
- Hardness conversion
Correlation between hardness and tensile strength

- Relations between hardness and tensile strength for steel, brass, and cast iron.
- For most steels:
  TS (MPa) = 3.45xHB
  TS (psi) = 500xHB
Example

- Estimate the Brinell and Rockwell hardness for brass

**Figure 6.12** The stress–strain behavior for the brass specimen.
Error in a scientific measurement usually does not mean a mistake or blunder. Instead, the terms "error" and "uncertainty" both refer to unavoidable imprecision in measurements.

Error analysis may seem tedious; however, without proper error analysis, no valid scientific conclusions can be drawn.

http://phys.columbia.edu/~tutorial/
Variability in Material Properties

- Elastic modulus is a material property.
- Critical properties depend largely on sample flaws (defects, etc.). Large sample to sample variability.
- Statistics
  - Mean
    \[ \bar{X} = \frac{\sum x_n}{n} \]
  - Standard Deviation
    \[ s = \left[ \frac{\sum (x_i - \bar{X})^2}{n-1} \right]^{\frac{1}{2}} \]

where \( n \) is the number of data points.
Example

- Determine the average and standard deviation of tensile strength

<table>
<thead>
<tr>
<th>Sample Number</th>
<th>Tensile Strength (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>520</td>
</tr>
<tr>
<td>2</td>
<td>512</td>
</tr>
<tr>
<td>3</td>
<td>515</td>
</tr>
<tr>
<td>4</td>
<td>522</td>
</tr>
</tbody>
</table>

\[
\overline{TS} = \frac{\sum_{i=1}^{4} (TS)_i}{4} = \frac{520 + 512 + 515 + 522}{4} = 517 \text{ MPa}
\]

\[
s = \left[ \frac{\sum_{i=1}^{4} ((TS)_i - \overline{TS})^2}{4 - 1} \right]^{1/2} = \left[ \frac{(520 - 517)^2 + (512 - 517)^2 + (515 - 517)^2 + (522 - 517)^2}{4 - 1} \right]^{1/2} = 4.6 \text{ MPa}
\]
Design or Safety Factors

- Design uncertainties mean we do not push the limit.
- Factor of safety, \( N \)

\[
\sigma_{\text{working}} = \frac{\sigma_y}{N}
\]

Often \( N \) is between 1.2 and 4

- Example: Calculate a diameter, \( d \), to ensure that yield does not occur in the 1045 carbon steel rod below. Use a factor of safety of 5.

1045 plain carbon steel:
\( \sigma_y = 310 \text{ MPa} \)
\( TS = 565 \text{ MPa} \)

\[
\frac{220,000N}{\pi\left(d^2 / 4\right)} = \frac{\sigma_y}{N}
\]

\[
d = 0.067 \text{ m} = 6.7 \text{ cm}
\]
Summary

• **Stress** and **strain**: These are size-independent measures of load and displacement, respectively.

• **Elastic** behavior: This reversible behavior often shows a linear relation between stress and strain. To minimize deformation, select a material with a large elastic modulus ($E$ or $G$).

• **Plastic** behavior: This permanent deformation behavior occurs when the tensile (or compressive) uniaxial stress reaches $\sigma_y$.

• **Toughness**: The energy needed to break a unit volume of material.

• **Ductility**: The plastic strain at failure.

• **Hardness**: Resistance to permanently indenting the surface.

• **Safety**: Design uncertainties mean we do not push the limit